Wealth Inequality and Safe Asset Demand[*]

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Abstract

We investigate the relationship between wealth inequality and safe asset demand in heterogeneous-agent economies. We find that asset bubbles emerge when agents face a sufficiently high probability of becoming extremely poor. In such cases, an asset unaffected by individual risk appears infinitely large relative to agent wealth *ex-post*, making it highly valuable *ex-ante*. This mechanism is fundamentally different from classic rational bubbles, and bubbles can arise even under stationary wealth distributions without aggregate uncertainty and growth. We use this insight to revisit how long-lived assets are priced in workhorse heterogeneous-agent models, and find some prior analyses may be incomplete due to overlooked possibility of transversality condition violation.

Keywords: Transversality Condition, Heterogeneous Agents, Wealth Inequality

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1 Introduction

When households face idiosyncratic risk, precautionary saving generates strong demand for safe assets and depresses the risk-free interest rate. While this mechanism is well understood, less is known about how the *wealth distribution* shapes safe-asset demand, potentially lowering the discount rate enough to generate asset bubbles. Clarifying this link is essential for understanding the pricing of long-lived assets such as government debt, and for evaluating fiscal sustainability in heterogeneous-agent economies.

This paper studies the relationship between wealth inequality and safe asset demand. We characterize conditions on the wealth distribution under which the present discounted value of a long-lived asset's price approaches infinity in the distant future, violating the transversality condition (TVC). In such cases, the asset price exceeds the present discounted value of cash flows, i.e., $P > PV(CF)$, permanently deviating from its fundamentals.

Our key finding is that the TVC is tightly linked to the *lower tail of the wealth distribution*, which is in turn shaped by the nature of the idiosyncratic risk. We uncover a novel mechanism for TVC violation in heterogeneous-agent economies, which is distinct from the classic rational bubble: in our setting, TVC violation requires the wealth of a significant fraction of agents to become sufficiently small relative to the average wealth in the economy. When these agents evaluate an asset that is unaffected by their idiosyncratic risk, this asset will become infinitely large relative to their own wealth *ex-post*. Anticipating this outcome *ex-ante*, they place exceptionally high value on such assets, generating excess demand that leads to TVC violation even in the absence of aggregate uncertainty or economic growth.

This result offers a new perspective for understanding how wealth distribution interacts with asset prices. Through the lens of our characterization, we revisit some workhorse heterogeneous-agent models like Constantinides and Duffie (1996); Krueger and Lustig (2010), and find that some prior analyses may be incomplete due to the possibility of TVC violation.

Analysis of Permanent Idiosyncratic Risk. We begin our analysis with the model of Brunnermeier, Merkel, and Sannikov (2022), which provides a tractable framework for modeling households facing uninsurable idiosyncratic risk. In this setting, the idiosyncratic risk is *permanent* and follows a random walk. As the wealth distribution evolves over time, the vast majority of households become very poor, while a vanishing share of households accumulates nearly all the wealth. This divergence leads to an exploding degree of wealth inequality.

Let us consider how a household in this economy prices a long-lived aggregate asset such as government debt. The value of this asset depends on its cash flow and discount rate. In standard settings including most representative-agent models, the asset's cash flow and the household's consumption level are cointegrated, which makes their covariance capturing risk premium the primary determinant of valuation. In our setting, as the household anticipates its wealth to decline far below the population average in almost all states of the world, the asset's cash flow will become very large relative to the household's size.

Due to this divergence in scale between the household and the asset, while the household's marginal utility growth correctly prices the asset's return period by period, the asset's long-run discounted value tends to infinity, leading to TVC violation for aggregate assets. In contrast, each household's TVC for its own consumption stream still holds, as it is discounted at a higher rate due to its exposure to idiosyncratic risk. As a result, the household's perceived wealth is finite, so that it will not demand more consumption goods than available in the economy.

That said, not all permanent idiosyncratic risk leads to bubbles. When the risk is permanent but sufficiently small, while households still expect their wealth to eventually vanish relative to the population average, this process occurs gradually and is outweighed by the subjective discount factor. As a result, the asset's terminal value remains finite and converges to zero in the limit.

We can also assess the condition for TVC violation by comparing the discount rate r with the growth rate g. Since households are risk-averse, the relevant discount rate r is not the risk-free rate, but the one adjusted for risk premium. We show that $r - g$ is decreasing in the magnitude of the idiosyncratic risk, as higher idiosyncratic risk strengthens the precautionary saving motive.

TVC violation occurs precisely when the idiosyncratic risk is large enough to lower r below g. Therefore, $r > g$ remains sufficient and necessary for the TVC to hold in this setting.

Analysis of Transitory Idiosyncratic Risk. The above analysis suggests that TVC violation occurs if there is sufficiently large permanent idiosyncratic risk, which makes wealth inequality explode in the limit. A natural question is whether TVC violation can still occur when the idiosyncratic risk is transitory and the wealth distribution is stationary—a setting that may be more realistic for studying asset prices.

To address this question, we introduce two additional ingredients to the model: wealth reset and death. In the first model extension, a redistribution shock resets a fixed fraction of households' wealth to the average wealth level every period. This shock generates a stationary wealth distribution in a tractable way, while preserving the total amount of resources in the economy. To our surprise, even in this setting, TVC violation can still occur. As before, the key lies in understanding the properties of the wealth distribution.

Specifically, when the intensity of the wealth reset shock is low, each household's wealth can deviate from the population average for an extended period before returning to the average. While the wealth distribution remains stationary, if the left tail of the wealth distribution is sufficiently heavy, the household anticipates large enough dispersion in its marginal utility growth, which depresses the discount rate and causes bubbles to emerge.

Conversely, when the intensity of the wealth reset shock is sufficiently high, the wealth distribution becomes more concentrated around its mean. In this case, households expect their wealth levels to be comparable to the size of the economy in most states, raising the discount rate and restoring the TVC. This property of wealth distribution is again reflected in the $r - g$ condition: $r > g$ remains a necessary and sufficient condition for the TVC to hold.

These results show that TVC violation for aggregate assets occurs not only when wealth inequality explodes, but also when the wealth distribution remains stationary but sufficiently left-skewed. Quantitatively, when we calibrate our model to U.S. post-war data, we find that the level of wealth inequality as measured by the Gini coefficient is potentially consistent with the case with TVC violation, making asset bubbles an empirically relevant possibility. That said, the Gini coefficient only offers a partial assessment. Evaluating whether the U.S. wealth distribution has the shape required to generate bubbles calls for a more detailed empirical analysis, with particular attention to the left tail of the distribution, which has received less scrutiny compared to the right tail that describes the very rich.

In the second extension, we introduce death to our model. We assume that households die at a constant rate, and new households are born to maintain a constant population size. As a result of the mortality risk, households become more impatient, consuming a larger fraction of their wealth and discounting future cash flows more heavily.

We consider two variants. First, if each newborn household inherits the wealth of a specific dying household, the wealth distribution diverges as in the permanent risk case. If the magnitude of idiosyncratic risk is large enough relative to the subjective discount rate, households are concerned about their vanishing wealth shares in scenarios where they do survive, leading to TVC violation. The only difference is that the subjective discount rate needs to be adjusted upwards to account for impatience due to the mortality risk.

Second, if each newborn household starts with the average wealth, the wealth distribution is stationary like the previous case with wealth reset. TVC violation may still occur if wealth inequality is sufficiently large, with death playing a role similar to wealth reset. Therefore, mortality risk affects TVC outcomes, but does not overturn our core intuition.

Implications for Asset Bubbles. Our findings contribute to the literature on asset bubbles by identifying and characterizing a novel class of asset bubbles. Santos and Woodford (1997) develop a general characterization of asset bubbles, with the main result showing "nonexistence of pricing bubbles in an equilibrium with the property that the economy's aggregate endowment has a finite value." Consistent with this result, our bubble arises in an economy in which the aggregate endowment has infinite value.[1] Moreover, deviating from their focus on assets with positive cash flows, we allow for cases with potentially negative cash flows, such as a government running persistent deficits. Relaxing this assumption enables us to uncover a new class of rational bubbles, which is distinct from the five examples studied in Santos and Woodford (1997). Below, we offer a brief discussion of the classic bubble types, and how they differ from our bubble.

First, a large literature examines classic rational bubbles in OLG models. In this context, bubbles arise when agents use assets for intertemporal consumption smoothing, giving value to otherwise intrinsically worthless assets. A canonical example of such a bubble is fiat money.

Second, Hirano and Toda (2025) show that asset bubbles necessarily emerge in models featuring multiple assets or sectors with unbalanced growth. These bubbles typically apply to settings like land in a growing economy or sectors with divergent growth rates.

Our bubble comprises a third, distinct type. The key driving force is the dispersion in the agents' marginal utility growth, which is shaped by the idiosyncratic risk they face. This form of bubble is relevant for studying assets whose cash flows scale with the aggregate economy and thus serve as good hedges against idiosyncratic risk, such as government debt.

There is an interesting parallel between the second and our types of bubbles. The former relies on divergent growth rates across *sectors*, whereas ours depends on dispersion in *individual wealth accumulation*. However, our model does not require persistent divergence in wealth across agents; bubbles can arise even under a stationary wealth distribution.

Despite these similarities, our bubble is fundamentally different from the other two types in both mechanism and implications for asset pricing. Internet Appendix E presents stylized models for each bubble type to illustrate the following key differences:

(1) Cash Flow/Price Ratio: In the first two types of bubbles, this ratio is either zero or van-

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[1] Even though aggregate endowment is finite, household's valuation of aggregate endowment flow can be infinite. This is because a "safe" claim to aggregate endowment provides huge value in the states where the households fall in the far left tail of the wealth distribution and have extremely high marginal utility. Households' ex ante valuation on this claim thus depends on the probability of such states, i.e., the properties of the left tail of the wealth distribution. In contrast, households' valuation on their own wealth shrinks to zero as time goes to infinity in spite of those extremely high marginal utility states, as their individual wealth also shrinks to zero in those states. In other words, the idiosyncratic risk increases the discount rate for the individual wealth, which in turn guarantees the transversality condition and optimality.

ishing over time. In fact, when cash flows are non-negative, bubbles can arise only if the cash flow/price ratio is zero or vanishing (Montrucchio, 2004; Hirano and Toda, 2025). For instance, money (Type 1 bubble) has no cash flow, and rents diminish relative to land prices (Type 2 bubble) over time. In contrast, our bubble exhibits a non-zero, non-vanishing cash flow/price ratio, made possible by *negative* cash flows. In our setting, the government can run a persistent deficit with a stationary deficit/debt ratio.

(2) Transversality Condition: All three bubble types feature a positive present discounted value of the asset price in distant future, violating the standard TVC. However, this value is finite in Types 1 and 2 bubbles, but infinite in our bubble. Specifically, in the first type of bubble, since the asset has no cash flow, its value equals the current price of the bubble asset, which is finite. In the second type, its value measures the difference between the current asset price and the present value of cash flows, both of which are finite. In contrast, our bubble has an infinitely positive value, which is balanced by an infinitely negative present value of cash flows to result in a finite asset price.

(3) Short-Sale Constraint: The first two bubble types require short-sale constraints *on the bubble asset*, so that agents cannot profit from selling the overvalued asset to finance higher consumption (Kocherlakota, 1992). For instance, OLG models inherently impose such constraints since agents must liquidate positions at death. Our bubble, however, does not rely on such constraints. In our framework, agents can freely trade the bubble asset like government debt without violating equilibrium conditions. On the other hand, our agents cannot short-sell their own income stream to fully get rid of the idiosyncratic risk.

In sum, by providing a precise characterization of the conditions under which this new type of bubble emerges, our analysis broadens the range of asset pricing behaviors that can be rationalized by a bubbly economy, and offers a clear framework for future research to assess the empirical relevance of such bubbles.

Implications for Asset Valuation. Our results highlight that the left tail of the wealth distribution has important implications for asset valuation. For instance, viewed through the lens of our framework, the model proposed by Constantinides and Duffie (1996) corresponds to a case with permanent idiosyncratic risk, which leads to a diverging wealth distribution. When such risk is sufficiently large, the marginal utility of wealth in the lower tail becomes so steep that aggregate assets are infinitely valuable. In this case, the no-trade equilibrium analyzed by Constantinides and Duffie (1996) does not exist. Constantinides and Duffie (1996) also consider a model variant with a stationary wealth distribution in their appendix. Our framework implies that, even in this case, TVC violation can still occur if the wealth distribution is sufficiently left-skewed. This, too, precludes the existence of no-trade equilibrium.

These findings suggest that, when we characterize risk premia in heterogeneous-agent economies, the TVC cannot be taken for granted. Ignoring the possibility of TVC violation may risk an incomplete analysis of asset valuation. Conversely, our results clarify what is needed for the TVC to hold—agents need to have a finite expected growth rate in marginal utility, which requires mechanisms to induce mean-reversion in idiosyncratic risk and limit the left tail of the wealth distribution. Since these mechanisms are also central to determining the finite-period risk-return trade-off, it needs to be jointly considered with the infinite-horizon TVC.

Our characterization of the TVC also extends to Bewley-type models like Krueger and Lustig (2010), McKay, Nakamura, and Steinsson (2016) and Kaplan, Moll, and Violante (2018), which have occasionally binding borrowing constraints. These borrowing constraints introduce an important caveat, as the standard present value equation becomes an inequality: $P > PV(CF) + TVC$. Asset prices can exceed discounted cash flows, i.e., $P > PV(CF)$, even when TVC is not violated.

This inequality is a different mechanism which elevates asset value above discounted cash flows, which can arise even in models without idiosyncratic risk (Townsend, 1980; Woodford, 1990). Even in this case, our results still offer a valid characterization of TVC violation, which remains a sufficient, though no longer necessary condition for $P > PV(CF)$. For example, Krueger and Lustig (2010) consider a model in which households have a guaranteed labor income which generates a bounded wealth distribution, so that the TVC always holds. In workhorse HANK models like McKay, Nakamura, and Steinsson (2016); Kaplan, Moll, and Violante (2018), the wealth distribution is also bounded from below, and the TVC holds.

Literature Review. Our paper contributes to a large literature on heterogeneous-agent asset pricing (Grossman and Shiller, 1982; Constantinides and Duffie, 1996; Angeletos, 2007; Toda, 2014; Kogan, Papanikolaou, and Stoffman, 2020; Panageas, 2020; Schmidt, 2024; Gomez, 2025; Marin and Singh, 2025). While much of this literature focuses on the implications of idiosyncratic risk for risk premia, we investigate the implications for the TVC. We show that a large degree of wealth inequality, which is also the key ingredient to generate realistic risk premia, can lead to asset bubbles.

Our analysis parallels the literature on asset bubbles in overlapping-generation (OLG) models (Samuelson, 1958; Diamond, 1965; Blanchard and Watson, 1982), as well as in more general frameworks (Santos and Woodford, 1997; Hirano and Toda, 2024). Just as Tirole (1985) characterizes conditions under which bubbly equilibrium exists in OLG economies, we characterize conditions under which TVC fails in heterogeneous-agent economies. To our surprise, bubbles can arise even when wealth distribution is stationary. Our results can be applied to any aggregate assets including government debt, which has been studied by Blanchard (2019); Abel and Panageas (2022); Gârleanu and Panageas (2023); Panageas (2024) in OLG models, and by Jiang et al. (2020); Reis (2021); Mian et al. (2022); Chen et al. (2022); Kocherlakota (2025) in models with long-lived agents.

Moreover, Lucas (1992); Atkeson and Lucas (1992) study resource allocation under idiosyncratic shocks. Lucas (1992) notes that permanent idiosyncratic risk generates a diverging wealth distribution, which lowers the equilibrium interest rate. Our analysis of the permanent-shock case builds on this insight. Our contribution is to derive the valuation implications of this divergence, and characterize exactly when and how the discount rate is lower than the aggregate growth rate. We also show that this divergence in wealth distribution is neither sufficient nor necessary to generate TVC violation. What matters instead is sufficient dispersion in marginal utility growth, which can happen even when the wealth distribution is stationary.

Finally, our analysis complements the *dynamic trading perspective* in Brunnermeier et al. (2022), which prices the cash flows each individual receives from the bond market. These cash flows, due to individuals' trading and rebalancing, are not the same as the aggregate cash flows paid out by the government. This perspective leads to a well-defined TVC *for individuals' wealth and asset holdings*, which offers important insights about the value of government debt.

Our results do not contradict with their results. Instead, we focus on the *buy-and-hold perspective*, which prices the issuer's *aggregate cash flows*. While Brunnermeier et al. (2020, 2022) show that the issuer's TVC may fail, its precise behavior remains an open question. We clarify the precise conditions under which this issuer-level TVC fails, which is central to the present value approach commonly used in financial valuation, and offers a rigorous foundation to interpret the buy-and-hold valuation exercise in Jiang et al. (2024b).

Our paper proceeds as follows. Section 2 analyzes the baseline model with permanent idiosyncratic risk. Section 3 introduces wealth reset to generate transitory idiosyncratic risk, and discusses the model's empirical relevance. Section 4 studies models with mortality risk. Section 5 discusses our results' implications for workhorse heterogeneous-agent models. Section 6 concludes.

2 Transversality Condition under Permanent Idiosyncratic Risk

2.1 Model Set-Up

We build on the set-up of Brunnermeier et al. (2022), which provides a tractable example of heterogeneous-agent economies with idiosyncratic risk. Model details are presented in Internet Appendix A.1. The economy consists of a continuum of infinitely lived households indexed by $i \in [0, 1]$, each maximizing expected lifetime utility:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt \right],$$

where $\gamma > 0$ denotes the coefficient of relative risk aversion.

Each household manages its own capital, which evolves according to investment ι_t^i, depreciation δ, capital trading $d\Delta_t^{k,i}$, and an idiosyncratic shock $d\tilde{Z}_t^i$ with exogenously specified volatility $\tilde{\sigma}_t$:

$$\frac{dk^i}{k^i} = \left(\Phi(\iota_t^i) - \delta\right) dt + d\Delta_t^{k,i} + \tilde{\sigma}_t d\tilde{Z}_t^i. \tag{1}$$

To focus on the key intuition, we abstract from aggregate risk in the main text. In Appendix B, we extend our setting to introduce aggregate risk, and derive analytical solutions assuming log utility. We show that aggregate risk does not alter our main results.

Each household faces a skin-in-the-game constraint, which requires it to hold at least a fraction $\bar{\chi}$ of its own capital. The household can, and do, sell the remaining shares in exchange for aggregate bond and equity assets that are not exposed to its idiosyncratic risk. In equilibrium, the law of motion for individual household's wealth n_t^i is given by

$$\frac{dn_t^i}{n_t^i} = g_t dt + (1 - \vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i, \tag{2}$$

where the drift term g_t is the common growth rate of wealth, and the stochastic component captures the equilibrium exposure $(1 - \vartheta_t)\bar{\chi}\tilde{\sigma}_t$ to the idiosyncratic risk. This exposure captures the amount of idiosyncratic risk that the household has to bear in equilibrium, which is the product of its risk exposure $\bar{\chi}\tilde{\sigma}_t$ to its own capital, and the share $(1 - \vartheta_t)$ of this capital in the household's portfolio.

In equilibrium, the household has a constant consumption-wealth ratio $c_t^i/n_t^i = \hat{c}$, which can be solved by a non-linear equation (A.23) in the Internet Appendix. This ratio is increasing in the subjective discount factor ρ, since a more impatient household tends to consume more and save less. Using this ratio, we can express the marginal utility as

$$m_t^i = e^{-\rho t}(c_t^i)^{-\gamma} = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}. \tag{3}$$

Plugging in the law of motion for wealth into (3), we obtain the following dynamics for marginal utility:

$$\frac{dm_t^i}{m_t^i} = -r_t^f dt - \gamma(1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i, \tag{4}$$

which implies the following expression for the risk-free rate:

$$r_t^f = \rho + \gamma g_t - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 = \hat{c} + g_t - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2. \tag{5}$$

The risk-free rate r_t^f reflects the growth rate g_t and the idiosyncratic risk's variance. More volatile idiosyncratic risk strengthens the precautionary saving motive and lowers the risk-free rate.

2.2 Transversality Condition for Individual Wealth

Iterating forward the households' budget constraint, we obtain that the household's wealth is equal to the present discounted value of its future consumption plus a transversality term:

$$m_t^i n_t^i = \mathbb{E}_t\left[\int_0^\infty m_{t+j}^i c_{t+j}^i dj\right] + \lim_{T\to\infty}\mathbb{E}_t\left[m_T^i n_T^i\right]. \tag{6}$$

The transversality condition (TVC) for individual wealth is given by

$$\lim_{T\to\infty}\mathbb{E}_t\left[m_T^i n_T^i\right] = 0. \tag{7}$$

If this TVC holds, we can evaluate the household's wealth portfolio n_t^i in Eq. (6) based on the present discounted value of its consumption stream.

In this setting, the individual-level TVC always holds for all households. From (2) and (4), we obtain

$$\frac{d(m_t^i n_t^i)}{m_t^i n_t^i} = -[\underbrace{r_t^f}_{\text{risk-free rate}} + \underbrace{\gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2}_{\text{risk premium for idiosyncratic risk}} - \underbrace{g_t}_{\text{growth rate}}]dt, \tag{8}$$

which expresses the dynamics of the transversality term $m_t^i n_t^i$ in terms of the discount rate and the growth rate. Plugging in the risk-free rate in (5) yields

$$r_t^f + \gamma(1 - \vartheta_t)^2 \bar{\chi}^2 \tilde{\sigma}_t^2 - g_t = \hat{c} > 0, \tag{9}$$

which implies that the appropriate discount rate for individual wealth, which accounts for the risk premium for idiosyncratic risk, is higher than the growth rate: r is always above g. This guarantees that the discounted individual wealth $m_t^i n_t^i$ always tends to zero in the limit.

2.3 Transversality Condition for Aggregate Wealth

Next, we evaluate the TVC associated with the aggregate wealth $\bar{n}_t = \int_0^1 n_t^i di$. Its law of motion is given by

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt,$$

which, compared to the law of motion (2) for individual wealth, averages out the idiosyncratic risk.

Let us consider a claim that pays out the aggregate wealth $\bar{n}_T$ at time T in the distant future. If we evaluate this claim today using an individual household's marginal utility as the discount rate, we obtain

$$\lim_{T \to \infty} \mathbb{E}_0 \left[m_T^i \bar{n}_T \right], \tag{10}$$

which answers how much the household is willing to pay today in exchange for receiving the aggregate wealth in the distant future.

Why should the households care about this price? Consider, for example, how they price government debt. Just like the individual household's budget constraint (6), the government faces the following standard intertemporal budget condition:

$$b_t = \mathbb{E}_t \left[\int_0^\infty m^i_{t+j} s_{t+j} dj \right] + \lim_{T \to \infty} \mathbb{E}_t \left[m^i_T b_T \right], \tag{11}$$

where b_t denotes the market value of government debt and s_t denotes the primary surplus. In this set-up, each household allocates ϑ_t fraction of its wealth to government debt. So, in aggregate, $b_t = \vartheta_t \bar{n}_t$. The government issuer's transversality condition is given by

$$\lim_{T \to \infty} \mathbb{E}_t \left[m^i_T \vartheta_T \bar{n}_T \right].$$

Since $\vartheta_t \in (0,1)$, $0 < \mathbb{E}_t \left[m^i_T \vartheta_T \bar{n}_T \right] < \mathbb{E}_t \left[m^i_T \bar{n}_T \right]$ for all time T. So, the TVC (10) for aggregate wealth is a sufficient condition for the TVC for government debt to hold.[2] More broadly, for any asset whose cash flow is cointegrated with the size of the economy, the TVC (10) for aggregate wealth also sheds light on its TVC.

Equivalently, we can study this TVC from the perspective of the average household. We define the equal-weighted average marginal utility growth as $\bar{m}_t = \int_0^1 (m^i_t / m^i_0) di$, which has the following law of motion:

$$\frac{d\bar{m}_t}{\bar{m}_t} = -r^f_t dt.$$

Compared with the individual household's stochastic discount factor (SDF) m^i_t / m^i_0, the average SDF $\bar{m}_t$ has the same drift, but does not load on any idiosyncratic risk. Because the aggregate wealth $\bar{n}_T$ does not load on any idiosyncratic risk, either, the two TVCs are identical:

$$\lim_{T \to \infty} \mathbb{E}_0 \left[m^i_T \bar{n}_T \right] = 0 \quad \Leftrightarrow \quad \lim_{T \to \infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T \right] = 0.$$

For analytical convenience, we will focus on the TVC discounted by the average SDF $\bar{m}_t$.

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[2] As a special case, if the government maintains a constant debt/wealth ratio ϑ_t, then, Eq. (10) is also a necessary condition.

Unlike the TVC (7) for individual wealth n_t^i, the TVC for aggregate wealth $\bar{n}_t$ does not necessarily hold. Below, we provide two complementary perspectives to characterize this TVC, one focusing on the degree of inequality in the wealth distribution and the other focusing on the comparison between the discount rate (r) and the growth rate (g).

The Wealth Distribution Perspective. Let $\eta_t^i = n_t^i / \bar{n}_t$ denote the wealth share of individual i. By definition, the wealth share has an average of 1: $\int_0^1 \eta_t^i di = 1$. We have the following result.

Proposition 1. *The transversality condition for aggregate wealth can be expressed as*

$$\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} \hat{c}^{-\gamma} e^{-\rho T} \bar{n}_T^{1-\gamma} \mathbb{E}_0 \left[\int_0^1 (\eta_T^i)^{-\gamma} di \right]. \tag{12}$$

The proof is presented in Appendix A.2. Eq. (12) is the central equation of our paper: whether the aggregate TVC holds or not depends on the distribution of the wealth share as summarized by $\int_0^1 (\eta_T^i)^{-\gamma} di$. This is a measure of inequality: a more unequal wealth distribution implies that more households have lower values of wealth share η_T^i and hence higher values of marginal utility $(\eta_T^i)^{-\gamma}$. If inequality explodes over time, many households' wealth shares η_T^i will become infinitely small and their marginal utilities $(\eta_T^i)^{-\gamma}$ will become infinitely large. In this case, Eq. (12) may diverge to infinity. On the other hand, if the wealth share η_T^i is bounded from below, then, the TVC always holds.

Following Brunnermeier et al. (2022)'s set-up, the idiosyncratic risk is permanent. Each household's wealth share follows a random walk:

$$\frac{d\eta_t^i}{\eta_t^i} = (1 - \vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i.$$

As time passes, almost every household becomes relatively poor, and wealth is concentrated in a vanishing measure of rich households. As a result, the wealth distribution becomes increasingly skewed, leading to an exploding amount of inequality.

We illustrate this point by simulating the model under a simple calibration (see Appendix Table

A.1). We set the risk aversion parameter to $\gamma = 2$ and assume zero aggregate growth, i.e., $g_t = 0$. While this assumption is not necessary, it implies a constant aggregate wealth $\bar{n}_T$ which further simplifies Eq. (12), so that we can focus on the distribution term.[3]

We also make the following assumptions, which are not essential to our results, but simplify our analysis. The $r - g$ perspective below characterizes the TVC under more general assumptions.

Assumption 1. *(a) The idiosyncratic risk has a constant volatility:* $\tilde{\sigma}_t = \tilde{\sigma}$.

(b) The economy starts with a steady-state value of the portfolio share of equity $\vartheta_t = \vartheta$ *so that it remains constant over time in the absence of aggregate risk.*

(c) All households start with equal wealth shares: $\eta_0^i = 1$.

We illustrate the properties of the wealth distribution under these assumptions in Figure 1. Panel (a) plots the distribution of the wealth share η_t^i for different time t. As time passes, while the average wealth share remains at 1, its distribution becomes increasingly skewed. The median and the mode of the wealth distribution shift to the left, indicating that most households' wealth share η_t^i approaches zero, while a vanishing share of households become richer. Panel (b) plots the cumulative distribution of the wealth share, which makes a similar point.

Panel (c) plots the Gini coefficient of the wealth distribution, which measures the degree of inequality in the wealth distribution. As the wealth distribution diverges, the Gini coefficient approaches one.

As virtually all households will become vanishingly small relative to the size of aggregate wealth $\bar{n}_T$, they will impute an infinite valuation for claims on aggregate wealth or any aggregate asset. We can express the TVC as

$$\lim_{T\to\infty} \mathbb{E}_0 \left[m_T^i \bar{n}_T \right] = \lim_{T\to\infty} \hat{c}^{-\gamma} (\bar{n}_T)^{1-\gamma} \cdot \exp(-\rho T) \cdot \mathbb{E}_0 \left[(\eta_T^i)^{-\gamma} \right].$$

On the right-hand side, $\exp(-\rho T)$ represents the subjective discount factor, while $(\eta_T^i)^{-\gamma}$ captures the marginal utility growth. If the marginal utility grows to infinity faster than the subjective

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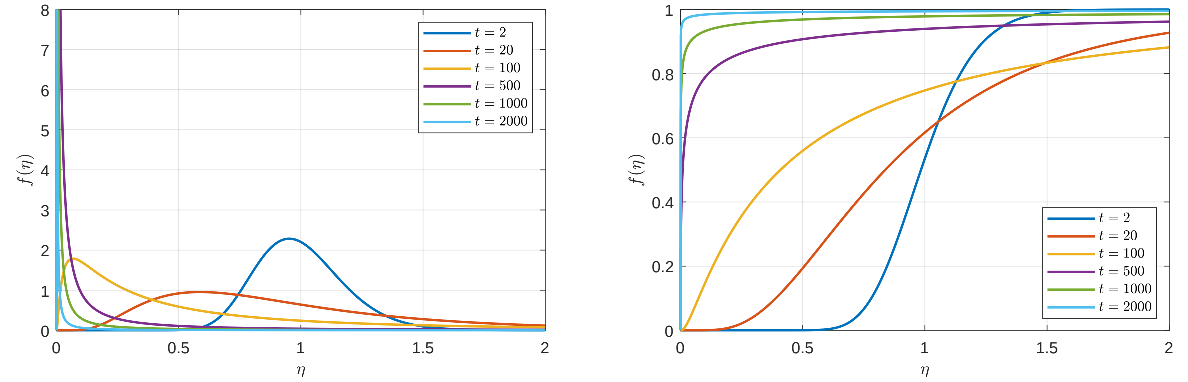
[3] A non-zero growth rate generates a time-varying $\bar{n}_T^{1-\gamma}$ term. While this term does not affect the distribution of household wealth, it influences the growth rate of marginal utility in the same way as the time discount factor ρ.

discount factor approaching zero, this transversality term diverges to infinity.

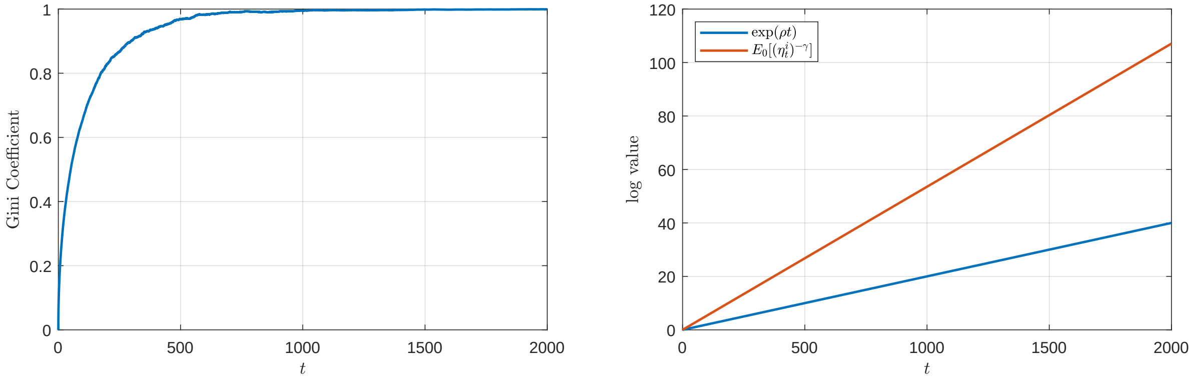
Panel (d) plots these two terms in the TVC. Both the integral of inverse wealth share $\int_0^1 (\eta_T^i)^{-\gamma} di$ and the inverse subjective discount factor $\exp(\rho T)$ approach infinity in the limit. Under our calibration, the magnitude of the idiosyncratic risk is sufficiently high, so that the rise in inequality—reflected in the inverse wealth share term—dominates the effect of discounting. This results in an infinite value of expected marginal utility growth in the limit.

It is worth noting that this result is not driven by a force which, as a household becomes poorer, increases the magnitude of its idiosyncratic risk and strengthens the precautionary saving motive. In our setting, poor and rich households hold the same portfolio, and their difference lies in the size of their wealth, which determines how large any aggregate asset appears to them. The TVC



(a) Distribution of Wealth Share.

(b) Cumulative Distribution of Wealth Share.



(c) Gini Coefficient.

(d) TVC for Aggregate Wealth.

Figure 1: Dynamics with Permanent Idiosyncratic Risk.

is violated when all but a vanishing share of the households become infinitesimally small relative to the size of the aggregate asset, which is different from a mechanism driven by increasing risk premium.

The r Minus g Perspective. We can also interpret our results in light of the "r minus g" condition. Note that the discounted aggregate wealth has the following dynamics:

$$\frac{d(\bar{m}_t\bar{n}_t)}{\bar{m}_t\bar{n}_t} = [-\underbrace{r_t^f}_{\text{risk-free rate}} + \underbrace{g_t}_{\text{growth rate}}]dt.$$

Compared to the discounted individual wealth in Eq. (8), the discounted aggregate wealth does not contain the risk premium for idiosyncratic risk. The relevant $r - g$ condition becomes

$$r_t^f - g_t = \hat{c} - \gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2,$$

which is no longer guaranteed to be positive. This condition determines the aggregate TVC, which holds if and only if

$$\lim_{T\to\infty}\mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] = \lim_{T\to\infty}\bar{n}_0^{1-\gamma}\mathbb{E}_0\left[\exp\left(-\int_0^T\left[\hat{c}-\gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2\right]dt\right)\right] = 0. \tag{13}$$

In other words, the aggregate TVC is violated when the magnitude of the idiosyncratic risk $\gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2$ is sufficiently large relative to the consumption-wealth ratio $\hat{c}$, which is increasing in the subjective discount rate ρ. In this case, because inequality explodes fast enough, households lower their discount rate on the aggregate wealth below its growth rate, leading to $r < g$.

On the other hand, if the variance of the permanent idiosyncratic risk is small enough, i.e., $\gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 < \hat{c}$, the discount rate r is above the growth rate g. In this case, the aggregate TVC holds: $\lim_{T\to\infty}\mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] = 0$. This result is consistent with the wealth distribution perspective above: in this case, while the wealth distribution is still diverging, its divergence is slow enough so that this effect is dominated by the household's subjective discount factor.

This discussion shows that the TVC for aggregate wealth is violated only when wealth inequality explodes sufficiently quickly, which drives the discount rate below the growth rate, i.e., $r < g$. Conversely, what r minus g captures in this setting is how fast permanent idiosyncratic risk drives the wealth inequality to explode.

What Does It Mean to Violate the TVC for Aggregate Wealth? When the TVC for aggregate wealth is violated, the government intertemporal budget condition, reproduced below,

$$b_t = \mathbb{E}_t \left[\int_0^\infty m^i_{t+j} s_{t+j} dj \right] + \lim_{T \to \infty} \mathbb{E}_t \left[m^i_T b_T \right],$$

has an infinite TVC term $\lim_{T\to\infty} \mathbb{E}_t \left[m^i_T b_T \right] = \infty$. Since b_t is well-defined and finite, it must be the case that the government runs deficits in enough states of the world to generate an infinitely negative present value of surpluses: $\mathbb{E}_t \left[\int_0^\infty m^i_{t+j} s_{t+j} dj \right] = -\infty$.

In other words, this asset bubble benefits the asset issuer, allowing it to run deficits in perpetuity. In contrast, when the TVC holds and there is no asset bubble, the government issuer must run surpluses to back up a positive value of debt. This result is consistent with Brunnermeier et al. (2022), who point out that the TVC may fail because it is not part of any households' optimality conditions, while our analysis offers a precise characterization of when the TVC fails.

3 Transversality Condition under Transitory Idiosyncratic Risk

So far, our analysis has focused on the case of *permanent* idiosyncratic risk, which can violate the TVC for aggregate assets. As shown by Proposition 1, the key driver of such violations is the behavior of the left tail of the wealth distribution. Permanent idiosyncratic risk induces an infinitely heavy left tail, which, in turn, causes households' expected marginal utility growth to diverge in the long run.

In practice, however, it is more realistic to consider settings where the wealth distribution remains stationary over time—an outcome that requires idiosyncratic risk to be *transitory*. In this section, we examine the implications of transitory idiosyncratic risk for the TVC. Surprisingly, we find that TVC violations can still arise. In fact, standard mechanisms that produce stationary wealth distributions can generate sufficiently heavy left tails that cause the expected marginal utility growth to diverge, despite the absence of permanent shocks.

3.1 Model Set-Up

We introduce a simple modification to our model in the previous section to generate transitory idiosyncratic risk. Specifically, each household's wealth n_t^i is reset at random intervals governed by a Poisson process with intensity ζ. Let $\tilde{J}_t^i$ denote this jump process, which is assumed to be i.i.d. across households.[4] For simplicity, we assume each household's wealth is always reset to the average level $\bar{n}_t$, which preserves the total amount of wealth in this economy and can thus be interpreted as a form of wealth redistribution. Later in the discussion of the $r-g$ condition, we will show that our key results are robust to more general specifications of post-reset wealth distribution.

We present detailed derivations of this model variant in Appendix A.1. In this set-up, individual wealth satisfies

$$\frac{dn_t^i}{n_t^i} = g_t dt + (1-\vartheta_t)\tilde{\sigma}_t\bar{\chi}d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i}d\tilde{J}_t^i,$$

which differs from Eq. (2) in the baseline model with no wealth reset in three ways. First, due to the household's exposure to the jump risk, the wealth n_t^i reverts to the mean level $\bar{n}_t$ when the jump $d\tilde{J}_t^i$ occurs.

Second, the household internalizes the jump risk in its consumption decision. Because the household may lose its savings when the jump occurs, it becomes effectively more impatient and consumes a higher share of its wealth: the consumption-wealth ratio $\hat{c} = c_t^i/n_t^i$ is increasing in ζ. Since the household consumes more and saves less, the growth rate in this economy can be

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[4] Modeling mean-reversion in this way allows us to generate a stationary wealth distribution in a tractable way. This is similar to Irie (2024), in which agents are subject to Poisson shocks that switch their types. In contrast, assuming proportional wealth tax leads to significantly more complex dynamics, even under log utility.

different from the baseline model under the same calibration.

Third, the risk-free rate is given by

$$r_t^f = \hat{c} + g - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

While this equation is identical to Eq. (5) in the baseline case with permanent idiosyncratic risk, the $\hat{c}$ term implicitly reflects the jump intensity ζ. A higher jump intensity implies a higher risk-free discount rate, again reflecting higher household impatience.

Throughout this section, we focus on parameter regions where the magnitude of idiosyncratic risk is sufficiently large such that the TVC fails in the absence of any jump risk. Otherwise, if the TVC holds when there are no resets, it will continue to hold for any positive level of jump intensity, since the reset mechanism can only dampen inequality and the dispersion in marginal utility growth.

3.2 Transversality Condition for Aggregate Wealth

Next, we revisit the TVC for aggregate assets in the presence of wealth reset. Similar to our analysis of the baseline case in Section 2, we can use either an individual household's SDF or the average SDF to characterize the TVC for aggregate wealth. We find it more convenient to use the average SDF $\bar{m}_t = \int_0^1 (m_t^i/m_0^i)di$. The dynamics of the average SDF is given by

$$d\bar{m}_t = -\left[\rho + \gamma g_t - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2\right]\bar{m}_t dt + \int_0^1 e^{-\rho t}\hat{c}^{-\gamma}\left(\bar{n}_t^{-\gamma} - (n_{t-}^i)^{-\gamma}\right)\frac{1}{m_0^i}d\tilde{J}_t^i di,$$

where the last term reflects the effect of the wealth reset shock. While this shock is purely redistributive and does not affect the aggregate wealth $\bar{n}_t$, it affects the average marginal utility $\bar{m}_t$ since individual marginal utility is non-linear in individual wealth.

Our analysis applies to any aggregate assets whose cash flows are unaffected by idiosyncratic risk. Here, we take the government debt as a concrete example. Let $b_t = B_t/P_t$ denote the aggregate market value of government debt in real terms. We obtain the following result which generalizes Eq. (11):

Proposition 2. *In the presence of wealth reset, the intertemporal government budget condition is given by*

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right] - \zeta \mathbb{E}_t\left[\int_0^T e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt\right].$$

The proof is presented in Appendix A.5. This proposition characterizes the intertemporal government budget condition from the perspective of the average household, which has the following interpretation: the government debt is backed by future surpluses (the first term on the right-hand side), the standard TVC term (the second term), and an additional term capturing the effect of wealth reset.

From an individual household's perspective, when wealth reset happens, its own wealth is reset to the average level regardless of how much and what assets it holds. This means that the household effectively loses any assets it holds when the jump happens. The last term reflects this loss of value, which is equal to the jump intensity ζ times the present value of the asset holding b_t. This loss of value lowers the household's valuation b_0 of the government debt below the present discounted value of government surpluses s_t plus the bond's terminal value b_T.

Since the market value of government debt is bounded by the aggregate wealth, i.e., $b_t < \bar{n}_t$, the last term is bounded. Proposition 2 implies

$$b_0 \leq \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right],$$

which shows that the discounted aggregate wealth $\bar{m}_t \bar{n}_t$ again arises as the key object determining the TVC for aggregate assets. Below, we use the same two perspectives to characterize its property.

The Wealth Distribution Perspective. For simplicity of exposition, we maintain the same assumption that the households begin with identical wealth: $\eta_0^i = 1$. Then, the TVC term (12) in

Proposition 1, reproduced below, also arises in this setting:

$$\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} \hat{c}^{-\gamma} e^{-\rho T} \bar{n}_T^{1-\gamma} \mathbb{E}_0 \left[\int_0^1 (\eta_T^i)^{-\gamma} di \right]. \tag{14}$$

Like the baseline case with permanent idiosyncratic risk, whether the TVC holds for the aggregate wealth depends on how the integral of inverse wealth share $\int_0^1 (\eta_T^i)^{-\gamma} di$ evolves over time and compares to the subjective discount rate $e^{-\rho T}$. The difference is that transitory idiosyncratic risk generates mean-reverting wealth dynamics, whereas permanent idiosyncratic risk leads to a diverging wealth distribution.

For a concrete example, we again consider the simple calibration under Assumption 1. We obtain the following result.

Proposition 3. *Starting from any initial distribution, the wealth distribution converges to the distribution with density function f that satisfies*

$$f(\eta) = \begin{cases} C\eta^{-\frac{3}{2} + \frac{\sqrt{1 + \frac{8\zeta}{\bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2}}}{2}}, & 0 < \eta < 1, \\ C\eta^{-\frac{3}{2} - \frac{\sqrt{1 + \frac{8\zeta}{\bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2}}}{2}}, & \eta \geq 1, \end{cases} \tag{15}$$

where C is a constant that ensures that the distribution integrates to one.

The proof is presented in the Appendix A.6. Even though this distribution is obtained by introducing wealth reset to an expanding log-normal wealth distribution, the stationary distribution in the limit is not log-normal—an important feature that we will revisit shortly.[5]

We examine this density function and find two distinct cases. First, if

$$\zeta > \frac{\gamma(1+\gamma)}{2} \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2, \tag{16}$$

i.e., the jump intensity is sufficiently high relative to the magnitude of the idiosyncratic risk, then,

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[5] This feature is consistent with the more general cases studied by Reed (2001); Toda (2014). The "power law on both tails" is a common feature in models with a constant probability of birth and death.

the left tail of the wealth distribution is light enough to ensure that the integral of inverse wealth share is well-defined:

$$\lim_{T\to\infty}\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]<\infty.$$

In this case, the TVC (14) for aggregate wealth always holds. Recognizing that the jump intensity is implicitly embedded in the consumption-wealth ratio $\hat{c}$, we can also rewrite condition (16) as $\hat{c}>\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2+\rho+(\gamma-1)g$, which we later use for our discussion of Table 1.

Conversely, if (16) does not hold, i.e., $\zeta\leq\frac{\gamma(1+\gamma)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$, the jump intensity is overshadowed by the magnitude of the idiosyncratic risk. While we still obtain a stationary wealth distribution in the limit, its left tail is heavy enough to generate an infinite $\lim_{T\to\infty}\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$, which, as we show below, may lead to TVC violation.

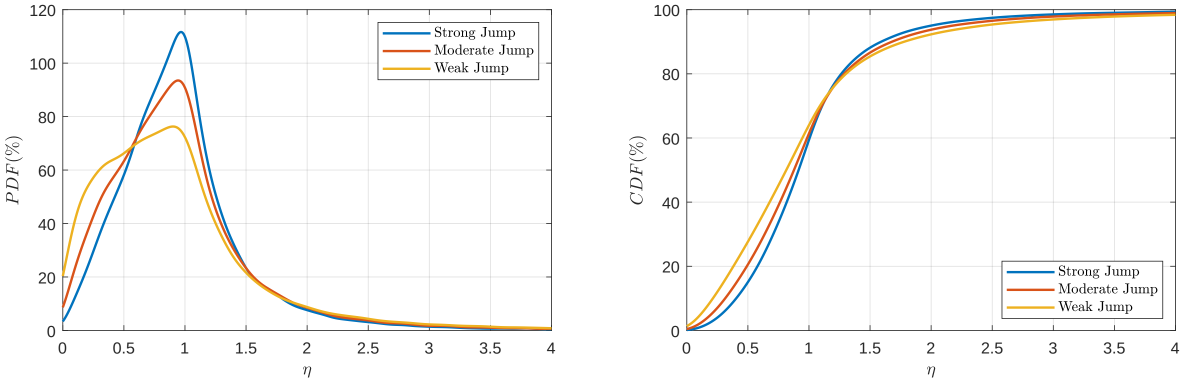
We consider three different calibrations to illustrate these cases: a case with a high jump intensity $\zeta=5.4\%$, a case with an intermediate jump intensity $\zeta=3.8\%$, and a case with a low jump intensity $\zeta=2.5\%$. The expected marginal utility growth $\lim_{T\to\infty}\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ is finite in the first case, and infinite in the second and third cases. All other variables are identical to the previous baseline case. If we set the jump intensity ζ_t to zero, we recover the case with permanent idiosyncratic risk in Figure 1.

In all three cases, we start with identical wealth shares for all households. The wealth distribution quickly converges to the stationary distribution given by (**??**). We illustrate the properties of these three cases in Figure 2. Panel (a) plots the stationary distribution of wealth share η_t^i described by Eq. (**??**). The case with a higher jump intensity has a greater mass around the mean and a thinner left tail, while the case with a lower jump intensity has a heavier left tail. In all three cases, the left tail of the wealth distribution declines with a power function. While it is not visually obvious, this tail is heavier than the one implied by log-normal distribution which guarantees a finite $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$.

Panel (b) plots the cumulative distribution of the wealth share η_t^i. Both panels show that there is no qualitative difference between the distributions obtained in these three cases, as they are from the same family of distributions with slightly different jump intensity ζ.

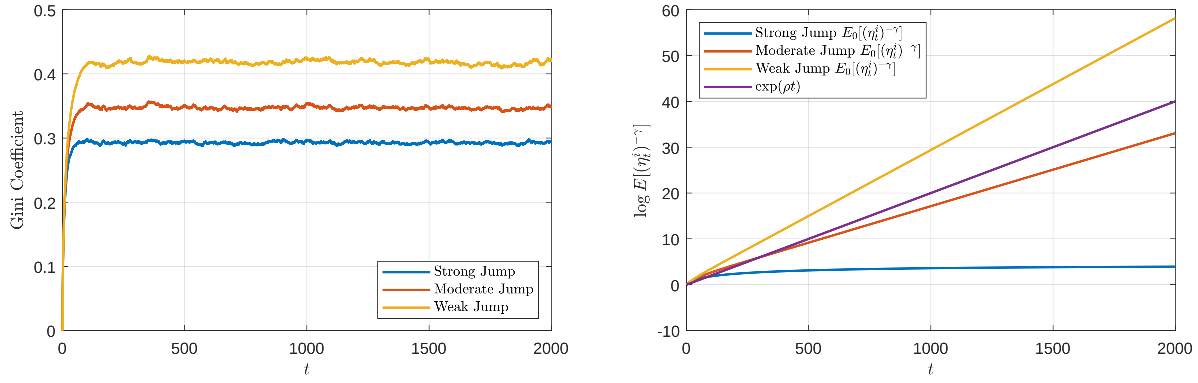
Panel (c) plots the evolution of Gini coefficient over time. Consistent with the convergent behavior of the wealth distribution, the Gini coefficient also stabilizes quickly. The Gini coefficients are similar across the three cases, which range from 0.3 to 0.4.

Finally, panel (d) reports the average path of marginal utility growth in Eq. (14), which determines the TVC for aggregate wealth. We break down this path into two components: the integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ and the subjective discount factor $\exp(\rho T)$. The expected marginal utility growth is the ratio between the two components. In this panel, we see meaningful differences between the three cases.



(a) Distribution of Wealth Share.

(b) Cumulative Distribution of Wealth Share.



(c) Gini Coefficient.

(d) TVC for Aggregate Wealth.

Figure 2: Dynamics with Transitory Idiosyncratic Risk

In the case with sufficiently high jump intensity (blue), the integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ converges to a finite value. In comparison, in the cases with lower (yellow) or intermediate (red) jump intensity, this integral diverges to infinity, as there is enough downside in each household's expected consumption path, leading to an infinite expected value of the marginal utility growth in the limit.

Comparing these values of expected marginal utility growth to the subjective discount factor $\exp(\rho T)$ leads to further differentiation. In the case with a high jump intensity (blue), the expected value of marginal utility growth is finite while the subjective discount factor is growing exponentially, leading to a finite TVC term. In the case with intermediate jump intensity (red), while the expected marginal utility growth diverges to infinity, it does so at a slower rate than the subjective discount factor. As a result, the TVC still holds. In contrast, in the case with a low jump intensity (yellow), the expected marginal utility growth diverges to infinity at a faster rate than the subjective discount factor, leading to a violation of the TVC for aggregate wealth.

Table 1(a) summarizes these cases. In the case with high jump intensity, the TVC holds and $\mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ converges to a finite value. In the case with low jump intensity, the TVC is violated and $\mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ diverges to infinity. In the case with an intermediate jump intensity, while $\mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ diverges to infinity, the TVC still holds.

This result shows that, despite the visual similarities of the wealth distribution in these three cases, different thickness in the left tail leads to different implications for how the expected marginal utility growth compares to the subjective discount factor. According to Eq. (14), the TVC for aggregate wealth is violated when the left tail is heavy enough to make the expected marginal utility growth diverge faster than the subjective discount rate.

Table 1(b) also summarizes the TVC in cases without wealth reset, which we studied in the previous section. In these cases, idiosyncratic risk is permanent and $\mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ always diverges to infinity. However, the TVC can still hold if the idiosyncratic risk is small enough, in which case the subjective discount factor dominates the expected marginal utility growth.

Table 1: Summary of the TVC for Aggregate Wealth.

(a) With Wealth Reset, Transitory Idiosyncratic Risk

Jump Intensity	Parameter Region	TVC holds?	$\mathbb{E}[\int(\eta^i_\infty)^{-\gamma}di] = \infty$?	$r > g$?
High	$\hat{c} > \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \rho + (\gamma-1)g$	Yes	No	Yes
Moderate	$\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 < \hat{c} \leq \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \rho + (\gamma-1)g$	Yes	Yes	Yes
Low	$0 < \hat{c} \leq \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	No	Yes	No

(b) Without Wealth Reset, Permanent Idiosyncratic Risk

Idiosyncratic Volatility	Parameter Region	TVC holds?	$\mathbb{E}[\int(\eta^i_\infty)^{-\gamma}di] = \infty$?	$r > g$?
Low	$\hat{c} > \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	Yes	Yes	Yes
High	$\hat{c} \leq \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	No	Yes	No

Comparison to U.S. Inequality Data. The results in this section show that a stationary wealth distribution does not guarantee the TVC for aggregate assets. TVC violation can still occur when the left tail of the wealth distribution is heavy enough, so that the households expect to experience a very high marginal utility upon negative realizations of idiosyncratic shock. In this case, the integral of inverse wealth share, which measures the degree of inequality in the wealth distribution, can diverge to infinity just like the case with permanent idiosyncratic risk.

A natural question is whether the wealth distribution implied by cases with TVC violation is consistent with the data. If the wealth distribution needs to be incredibly skewed to generate TVC violation, then TVC violation is just a theoretical curiosity.

Figure 3 compares our model variants with the U.S. data. We use the Gini coefficient based on income data to measure the degree of inequality in the U.S.[6] From 1963 to 2022, the Gini coefficient has remained stable, fluctuating around 0.4. In the baseline model with permanent idiosyncratic risk, if we start with the same degree of inequality, the wealth distribution will diverge over time. Under our calibration, which is motivated by the empirical moments considered in Brunnermeier et al. (2022), we expect the Gini coefficient to rise to 0.6 around 2020 and approach 0.7 around 2050.

In contrast, models with transitory idiosyncratic risk imply a stable Gini coefficient. We again consider the three cases with different values of jump intensity ζ. Both the case with moderate jump

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[6] The wealth in our model encompasses not only financial wealth but also human capital. Greenwald et al. (2021)'s estimates show that the Gini for financial wealth is approximately 0.74 in the U.S., whereas the adjusted total wealth Gini is 0.41, which aligns closely with the figures used here. 0.4 is also close to the Gini index for consumption, which is relevant in marginal utility dispersion.

intensity and the case with low jump intensity yield a Gini coefficient around 0.4, even though the TVC holds in the former case and fails in the latter.

Thus, we obtain a possibility result: we do not need an extremely skewed wealth distribution to generate TVC violation, as long as households have reasonably high risk aversion ($\gamma = 2$ in our calibration). In this case, the market value of long-lived assets may exceed the discounted present value of its cash flows.

That said, two caveats must be noted. First, the Gini coefficient is not the only relevant metric for evaluating the degree of inequality in the wealth distribution. TVC violation relies on households expecting sufficiently large dispersions in marginal utility growth, which makes it important to examine household beliefs about wealth dynamics. We leave this analysis to future work.

Second, and relatedly, TVC violation in our setting relies on households' marginal utility being unbounded—that is, the Inada condition plays an important role. If, for example, we assume that households derive a low but finite utility when their consumption and wealth approach zero, then marginal utility growth is always bounded and the TVC always holds, even when the wealth distribution has similar statistics.

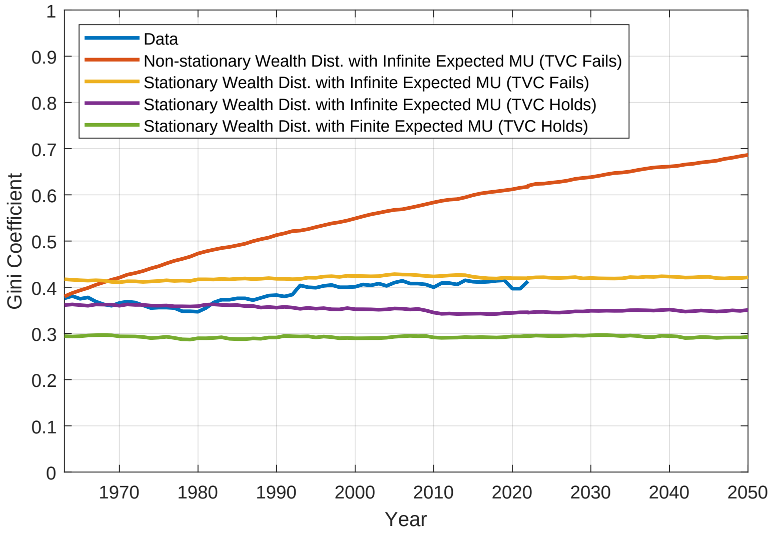

Figure 3: Gini Coefficient of Consumption Inequality.

The r Minus g Perspective. Next, we turn to the $r - g$ perspective, which allows us to obtain more general results characterizing the dynamics of the TVC term $\bar{m}_t\bar{n}_t$ without relying on Assumption 1 and the specific wealth distribution.

Proposition 4. *The TVC term for aggregate wealth has the following dynamics:*

$$d[\bar{m}_t\bar{n}_t] = -\left[\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right]\bar{m}_t\bar{n}_t dt + \zeta e^{-\rho t}\bar{n}_t^{1-\gamma}\bar{n}_0^{\gamma}dt.$$

The aggregate TVC holds if and only if

$$\hat{c} > \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2. \tag{17}$$

The proof is presented in Appendix A.7. This proposition shows that the sign of $\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$ plays a crucial role in determining the TVC for aggregate wealth. As

$$r_t^f - g_t = \hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2,$$

this proposition shows that the relevant condition for the TVC is again "$r - g > 0$." To satisfy the TVC for aggregate wealth, we need either a small enough volatility $\tilde{\sigma}_t$ of the idiosyncratic risk, or a sufficiently high jump risk ζ so that the households' wealth frequently reverts to the mean.

Table 1(a) also reports the $r - g$ condition in the last column. Eq. (17) is satisfied when the jump intensity is high or moderate, which ensures that $r > g$ and the TVC holds. In contrast, with a low jump intensity, the marginal utility growth explodes fast enough to dominate the subjective discount rate in (14), which leads to TVC violation.

The results we obtain from the $r-g$ perspective are more general than the previous results from the wealth distribution perspective, which assumes a specific wealth distribution. In the Internet Appendix A.8, we impose an arbitrary wealth distribution for households after experiencing the wealth reset, including a log-normal one. In this case, while the stationary wealth distribution no longer takes the form of Eq. (**??**), Proposition 4 from the $r - g$ perspective still holds. In particular, with a low enough jump intensity ζ, the TVC for aggregate wealth is still violated, which implies that the left tail of the stationary wealth distribution has to be heavier than the log-normal distribution, even when the stationary wealth distribution is obtained from introducing wealth reset to an expanding log-normal wealth distribution.

Summary. Our analysis of permanent and transitory idiosyncratic risk yields several theoretical insights. First, a non-stationary wealth distribution does not necessarily imply a violation of TVC for aggregate assets. The TVC can still hold if the wealth distribution diverges more slowly than the subjective discount factor. Only when the divergence is sufficiently rapid is the TVC violated.

Second, a stationary wealth distribution is not sufficient to ensure the aggregate TVC holds. The left tail is critical: if it is sufficiently heavy, the TVC can still be violated. In such cases, households anticipate their wealth shrinking to a negligible fraction of the economy for long enough time, perceiving aggregate assets to be infinitely valuable.

Third, consistent with OLG models, the condition $r - g > 0$ remains both necessary and sufficient for the TVC to hold. In our setting, this gap reflects the relative magnitude of the idiosyncratic risk and the subjective discount factor.

Fourth, the degree of inequality in post-war U.S. data does not rule out TVC violations. With a plausible level of risk aversion, the left-tail risk required for such violations may be consistent with the observed Gini coefficient. To evaluate the possibility of asset bubbles, we need to look further into the left tail of the wealth distribution in greater details.

4 Transversality Condition under Stochastic Death

Our characterizations of the TVC continue to hold in a model where households face a constant probability of death, denoted by ζ. For concreteness, let us first consider the following argument in discrete time. Let c_t^i denote the household's consumption at time t, conditional on survival, and assume utility is zero in all periods after death. Consider a long-lived asset with price p_t and dividend d_t at time t. The household alive at time t has the following first-order condition:

$$u'(c_t^i)p_t = (1-\zeta)\mathbb{E}_t\left[\delta u'(c_{t+1}^i)(p_{t+1}+d_{t+1})\right] + \zeta \times 0,$$

reflecting the fact that the asset is valuable only insofar as the household survives in the next period to consume the payoff.

Iterating this equation forward, we obtain

$$u'(c_t^i)p_t = \mathbb{E}_t\left[\sum_{k=1}^{\infty}((1-\zeta)\delta)^k u'(c_{t+k}^i)d_{t+k}\right] + \lim_{k\to\infty}\mathbb{E}_t\left[((1-\zeta)\delta)^k u'(c_{t+k}^i)p_{t+k}\right],$$

which is very similar to the case without death, except that the subjective discount rate δ is augmented with the survival probability $(1-\zeta)$. Intuitively, the risk of death increases the household's effective discount rate, making them behave more impatiently.

Aside from this adjustment, the valuation of the asset continues to depend on the distribution of marginal utility growth conditional on survival. If the wealth and consumption distributions are sufficiently left-skewed, the TVC can still fail. Thus, even in the presence of mortality risk, our main result holds: the validity of the aggregate TVC hinges on the thickness of the left tail of the wealth distribution.

In the Internet Appendix A.9, we formalize this argument in a continuous-time setting similar to our baseline model. Households face a constant death rate ζ, while a fraction ζ of new households are born and receive the wealth of the deceased households. This ensures that aggregate wealth and financial assets are preserved. We consider two plausible cases.

First, suppose each new household directly inherits the wealth of a deceased household. In this case, the model is equivalent to the baseline setting without jump risk, as discussed in Section 2. The wealth distribution has an explosive amount of inequality, and the aggregate TVC may be violated if idiosyncratic risk is sufficiently large. The only difference is that the subjective discount rate ρ needs to be augmented by the death rate ζ, mirroring the discrete-time example.

The necessary and sufficient condition for the TVC to hold is

$$\hat{c} - \gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 > 0, \tag{18}$$

which is identical to the condition in the permanent risk case, except that the wealth-consumption ratio now also reflects the death rate. For instance, under log utility, $\hat{c} = \rho + \zeta$, indicating that the mortality risk makes the households more impatient.

Second, suppose the wealth of deceased households is pooled and redistributed equally among the newborns. This setup corresponds to the model with wealth reset shocks described in Section 3. The wealth distribution becomes stationary, but the aggregate TVC remains sensitive to the left tail of the wealth distribution. As before, the death rate affects the consumption-wealth ratio $\hat{c}$, and the TVC condition is again given by Eq. (18). This condition ensures that the left tail of the stationary wealth distribution is not too heavy. Conversely, if this condition fails, TVC violation occurs even when the wealth distribution is stationary.

This discussion shows that introducing stochastic death is not a simple fix to TVC violation for long-lived assets. Whether the TVC holds or not continues to hinge on the degree of wealth inequality, or, more precisely, the balance between the effective subjective discount factor and the magnitude of idiosyncratic risk. While death increases impatience and thus makes TVC violations less likely to happen, it does not fundamentally alter the mechanism by which left-tail risk generates such violations.

5 Implications for Theories of Heterogeneous-Agent Economies

Our results highlight wealth inequality as the central determinant of TVC in heterogeneous-agent models. In this section, we apply this insight to several workhorse models in macroeconomics and finance, and examine its implications for asset valuation.

5.1 Constantinides-Duffie

Constantinides and Duffie (1996) study an economy with heterogeneous agents facing idiosyncratic risk, which admits a no-trade equilibrium. In their baseline model, each agent's labor income is subject to permanent idiosyncratic shocks. These shocks reduce some households' total wealth and consumption to zero in the limit. Specifically, household i's one-period pricing kernel is given by

$$m_{t+1}^{i} = e^{-\rho} \left(\frac{c_{t+1}^{i}}{c_t^{i}} \right)^{-\gamma},$$

with consumption growth specified as

$$\frac{c_{t+1}^{i}}{c_t^{i}} = \frac{c_{t+1}}{c_t} \exp\left(y_{t+1}\eta_{t+1}^{i} - \frac{y_{t+1}^{2}}{2} \right),$$

where c_{t+1}/c_t is the aggregate growth rate, and η_{t+1}^{i} is an i.i.d. idiosyncratic shock. Because these shocks are permanent, each household's consumption share follows a random walk, resulting in an exploding degree of inequality in the limit. The model details are presented in Appendix C.

This setting closely mirrors our baseline model in Section 2. If the magnitude of permanent idiosyncratic risk is sufficiently large, the asset valuation equation $P = PV(CF)$ may fail.[7] This possibility is overlooked in prior analysis, which typically assumes the TVC holds.

To illustrate our point, assume there is sufficiently high permanent idiosyncratic risk, and a long-lived asset like equity which pays a non-negative dividend d_t. While a household's SDF may be able to price the one-period return of this asset's return:

$$\mathbb{E}_t[m_{t,t+1}^{i} r_{t+1}] = 1,$$

---

[7] One detail is that Constantinides and Duffie (1996) impose an assumption that the cumulative SDF M_t satisfies $\lim_{t\to\infty} \mathbb{E}[M_t] = 0$ (see their Eq. (5)). This condition is not sufficient to rule out TVC violation. If the aggregate wealth grows at a constant rate $g > 0$, which is sufficiently large relative to the magnitude of the idiosyncratic risk, then, the cumulative SDF $\bar{m}_t$ in our baseline model will converge to zero. At the same time, the rate at which the SDF converges to zero can be dominated by the rate at which the cash flow grows, so that the discounted present value of the asset is still infinite.

the dividend stream can be infinitely valuable

$$\sum_{t=1}^{\infty} \mathbb{E}_0 \left[m_{0,t}^i d_t \right] = \infty.$$

In this case, there is no well-defined price process for this asset, and the no-trade equilibrium breaks down. This example shows that pricing one-period returns alone is not sufficient for asset valuation in heterogeneous-agent models. A complete characterization requires verifying whether the TVC holds in the presence of persistent heterogeneity and risk.

Moreover, Constantinides and Duffie (1996) also explore a variant of their model in their appendix that incorporates wealth reset, which resembles our model variant with a stationary wealth distribution. As we show in Section 3, even in this case, the TVC can still fail. Therefore, the TVC cannot be taken for granted in heterogeneous-agent economies.

5.2 Brunnermeier-Merkel-Sannikov

Our results offer a complementary view to Brunnermeier et al. (2022) on the intertemporal government budget constraint. As discussed in the literature review, their *dynamic trading perspective* focuses on individual wealth dynamics and time-varying Treasury holdings. Since the TVC for individual wealth always holds, their approach provides insights into how households value government debt. Our analysis does not contradict this view.

In contrast, we focus on the *buy-and-hold perspective*, which evaluates the aggregate government budget from the issuer's stand point. As Brunnermeier et al. (2022) point out, the TVC for aggregate government debt portfolio is not guaranteed to hold because it does not stem from an individual's optimization problem. Nevertheless, standard asset pricing approaches evaluate debt based on aggregate cash flows, making it essential to understand whether and when the TVC holds at the aggregate level.

Our results provide a general framework to consider this issue, as well as concrete examples in which the TVC fails. In particular, we show that the validity of the aggregate TVC hinges on the shape of the wealth distribution. When the left tail is sufficiently heavy, the TVC for aggregate assets can fail, and asset prices may diverge from the present value of expected cash flows.

5.3 Aiyagari-Bewley-Huggett Models

Compared to Brunnermeier et al. (2022); Constantinides and Duffie (1996) and the settings analyzed in our paper, classic Aiyagari-Bewley-Huggett models introduce a borrowing constraint, which restricts each household's asset holding b_t^i from falling below a certain level q_t^i:

$$b_t^i \geq q_t^i.$$

When $q_t^i = 0$, this is a no-short-selling constraint. More generally, $q_t^i > 0$ can reflect minimum asset holding requirements as in financial repression.

When the borrowing constraint binds, households would prefer to reallocate consumption from the future to the present, but they are unable to do so through borrowing. In the discrete-time framework typically used in this literature, the Euler equation becomes an inequality:

$$m_t^i \geq \mathbb{E}_t \left[m_{t+1}^i (1+r) \right],$$

with strict inequality when the constraint is binding.

As a result, the government's intertemporal budget condition (11) also becomes an inequality:

$$b_t \geq \lim_{T\to\infty} \mathbb{E}_t \left[\sum_{k=1}^{T} \frac{m_{t+k}^i}{m_t^i} s_{t+k} \right] + \lim_{T\to\infty} \mathbb{E}_t \left[\frac{m_T^i}{m_t^i} b_T \right]. \tag{19}$$

The proof is presented in the Internet Appendix D.1. This expression implies that the market value of government debt b_t can exceed the present value of expected future surpluses for two distinct reasons. First, the TVC fails, resulting in an infinite $\lim_{T\to\infty} \mathbb{E}_t \left[\frac{m_T^i}{m_t^i} b_T \right]$ term.

Second, the inequality in the Euler equation creates slack even if the TVC holds: the present value of surpluses is only a lower bound on the value of government debt. In this sense, TVC violation for long-lived assets provide a sufficient, though not necessary, condition for $P > PV(CF)$.[8]

Our characterizations of the TVC still apply to these settings. Consider Krueger and Lustig (2010) for example, who develop a heterogeneous-agent model in which a fraction of the aggregate endowment is distributed as equity dividends, while the remainder is allocated as labor income. On the equity side, all households hold the market index, albeit in different proportions. On the labor income side, households face idiosyncratic labor income risk governed by a Markov process.

In this environment, the distribution of the individual household's marginal utility growth, i.e., m_T^i/m_t^i, remains central to determining whether the TVC for aggregate assets holds. If labor income is bounded below by a strictly positive amount, then household consumption shares are likewise bounded from below. This, in turn, implies that marginal utility is bounded above, ruling out the possibility of explosive marginal utility growth. As we detail in the Internet Appendix D.2, this ensures that households do not expect to become infinitely poor relative to the population average, ruling out the possibility of TVC violation.

5.4 Townsend and Woodford

Townsend (1980) provides another illustration of how borrowing constraints affect asset valuation. We consider a simplified version of Woodford (1990) to highlight this mechanism.

The economy consists of two types of infinitely-lived households. Type A receives endowment e_1 in even periods and e_2 in odd periods, while type B receives endowment e_2 in even periods and e_1 in odd periods, with $e_1 > e_2 > 0$. This staggered income pattern generates a motive for intertemporal trade between the two types.

Households can invest in government debt with total supply d, which is financed by lump-sum taxes τ equally split between the two household types. The interest rate is r. Households face a

---

[8] Belief distortion can also lead to $P > PV(CF)$ under econometricians' information set, even when the TVC holds from the households' perspective. Jiang et al. (2024a) provides a concrete example in the context of government debt valuation.

borrowing constraint: $b_{i,t} \geq 0$, for $i \in \{A, B\}$. Each household's budget constraint is

$$c_{i,t} + \tau/2 + b_{i,t} = e_{i,t} + (1+r)b_{i,t-1},$$

where the tax equals the interest on the government debt, i.e., $\tau = rd$.

Households maximize lifetime utility $\sum_{t=0}^{\infty} \rho^t u(c_{i,t})$. We assume that the endowment level e_2 is sufficiently lower than e_1 so that the borrowing constraint is binding in the low-endowment period. By market clearing, the other household must hold all the government debt.

For example, in period 1, household A holds no government, whereas household B holds all the government debt:

$$\begin{aligned} c_{A,1} &= e_2 + (1+r)d - \tau/2 \overset{\text{def}}{=} \underline{c}, \\ c_{B,1} &= e_1 - d - \tau/2 \overset{\text{def}}{=} \bar{c}. \end{aligned}$$

Now, we are in a position to evaluate the government debt. Since the borrowing constraint always binds for the low-endowment type, Eq. (19) applies. For household A, we have

$$d > \lim_{T\to\infty} \mathbb{E}_0 \left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right] + \lim_{T\to\infty} \mathbb{E}_0 \left[\rho^T \frac{u'(c_{A,T})}{u'(c_{A,0})} d \right] = \lim_{T\to\infty} \mathbb{E}_0 \left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right].$$

The proof is presented in the Internet Appendix D.3. Because marginal utility is bounded, the transversality term vanishes. This example shows that even when the TVC holds, borrowing constraints can increase the value of government debt above the present value of surpluses.

5.5 HANK Models

HANK models also feature heterogeneous agents facing idiosyncratic risk. As a result, the TVC for long-lived assets also depends on the wealth distribution. For example, in McKay, Nakamura, and Steinsson (2016), households receive equal shares of firm dividends and face borrowing constraints. The present value relationship again becomes an inequality as in Eq. (19). Yet, as household wealth is bounded from below by the dividend-output ratio, which limits the degree of wealth inequality, the TVC holds in this model.

Similarly, in Kaplan, Moll, and Violante (2018), households receive lump-sum transfers and face borrowing constraints. Transfers minus borrowing costs ensure a lower bound on household income, which is positive under the paper's calibration. Thus, despite an extremely skewed financial asset distribution, the TVC for aggregate assets holds due to bounded left tails of the wealth distribution.

6 Conclusion

This paper characterizes the conditions under which asset bubbles arise and TVC fails for long-lived assets in heterogeneous-agent economies. We find that the degree of wealth inequality is the key to understanding this question. Both non-stationary wealth distributions with explosive inequality and stationary distributions with heavy left tails can lead to TVC violations. The r minus g condition remains a useful guide for interpreting the TVC in heterogeneous-agent models.

This new type of asset bubbles has important implications for asset valuation in heterogeneous-agent models. For example, the TVC may fail in settings considered by Constantinides and Duffie (1996) and Brunnermeier et al. (2022), where the wealth distribution explodes over time. In contrast, the TVC holds in settings considered by Krueger and Lustig (2010) and McKay et al. (2016); Kaplan et al. (2018), when the wealth distribution is bounded from below. These results offer a new perspective on the valuation of long-lived assets in heterogeneous-agent models.

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Appendix

Contents

A Model Details

In this Appendix, we provide derivation details and additional explanations for models in Section 3 and 4. Note that the permanent risk model in Section 2 can be viewed as a special case of the transitory risk model in Section 3 with wealth reset intensity $\zeta = 0$.

A.1 Model Derivation

A.1.1 Set-up

Households. Household's lifetime utility is given by

$$V_0^i = \mathbb{E}\left[\int_0^\infty e^{-\rho t}(c_t^i)^{-\gamma} dt\right].$$

The capital evolution follows

$$\frac{d\tilde{k}^i}{\tilde{k}^i} = \left(\Phi(\iota_t^i) - \delta\right) dt + \tilde{\sigma}_t d\tilde{Z}_t^i + d\Delta_t^{k,i},$$

where $d\Delta_t^{k,i}$ is the physical capital transactions between households, and

$$\Phi(\iota) = \frac{1}{\phi}\log(1 + \phi\iota).$$

The idiosyncratic risk processes $\tilde{Z}_t^i$ are independent across households. Besides, households are subject to wealth reset shock with intensity ζ. In the permanent shock case, $\zeta = 0$. Formally, let $\tilde{J}_t^i$ be a Poisson process with intensity ζ, the wealth dynamic for household i is given by

$$\frac{dn_t^i}{n_{t-}^i} = -\frac{c_t^i}{n_{t-}^i}dt + dr_t^B + \theta_t^{K,i}\left(dr_t^{K,i}(\iota_t^i) - dr_t^B\right) + \theta^{E,i}\left(dr_t^{E,i} - dr_t^B\right) + \theta_t^{\bar{E},i}\left(dr_t^{\bar{E},i} - dr_t^B\right) + \frac{\bar{n}_t - n_{t-}^i}{n_{t-}^i}d\tilde{J}_t^i,$$

where $\tilde{J}_t^i$ and $\tilde{J}_t^j$ are stochastically independent if $i \neq j$. Households face a skin-in-the-game constraint when issuing equity, i.e.,

$$-\theta_t^{E,i} \leq (1-\bar{\chi})\theta_t^{K,i}.$$

Government. The nominal growth of government debt follows

$$dB_t = \mu_t^B B_t dt.$$

Government budget constraint is given by

$$i_t B_t + P_t \mathfrak{g} K_t = \mu_t^B B_t + P_t \tau_t a_t K_t,$$

where nominal rate i_t, spending and tax rate $\mathfrak{g}$, productivity a_t are exogenous processes. Tax rate τ_t is endogenous. With q_t^B defined below, the budget constraint can be restated as

$$\breve{\mu}_t^B q_t^B + s_t = 0, \tag{A.1}$$

where $\breve{\mu}_t^B := \mu_t^B - i_t$ is the bond issuance (net of interest payment) rate. $s_t := \tau_t a_t - \mathfrak{g}$ is the real surplus-capital ratio.

A.1.2 Symmetric Equilibrium

Aggregation. Under symmetric equilibrium,

$$\forall i \in [0,1], \quad (\theta_t^{K,i}, \theta_t^{E,i}, \theta_t^{\bar{E},i}, \iota_t^i, \hat{c}_t^i) = (\theta_t^K, \theta_t^E, \theta_t^{\bar{E}}, \iota_t, \hat{c}_t),$$

where $\hat{c}_t^i = c_t^i / n_t^i$ is the consumption-wealth ratio.

Under symmetric equilibrium, the returns adjust such that no one trades physical capital with others, i.e., $\Delta_t^{k,i} = 0$. Aggregate capital evolves according to

$$dK_t = \left(\Phi(\iota_t) - \delta\right) K_t dt.$$

Define the real price of physical capital as q_t^K and the real bond-capital ratio as $q_t^B = \frac{B_t/P_t}{K_t}$. The state variable ϑ_t is defined as the share of wealth in government bond, i.e.,

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K}. \tag{A.2}$$

The difference of drift $\mu_t^{q,B} - \mu_t^{q,K}$ can be derived as follows,

$$\begin{aligned} \mu_t^{\vartheta}\vartheta_t &= \frac{q_t^B}{q_t^B + q_t^K}\left(\mu_t^{q,B} - \frac{q_t^B\mu_t^{q,B} + q_t^K\mu_t^{q,K}}{q_t^B + q_t^K}\right) \\ &= \frac{q_t^B}{q_t^B + q_t^K}(1 - \vartheta_t)\left(\mu_t^{q,B} - \mu_t^{q,K}\right), \end{aligned}$$

i.e.,

$$\frac{\mu_t^{\vartheta}}{1 - \vartheta_t} = \mu_t^{q,B} - \mu_t^{q,K}. \tag{A.3}$$

Return Processes. Here, we express the return process with the drift terms of q_t^B, q_t^K and ϑ_t.

Inflation:

$$\frac{dP_t}{P_t} = \frac{d(B_t/(q_t^B K_t))}{B_t/(q_t^B K_t)} = \left(\mu_t^B - \mu_t^{q,B} - \Phi(\iota_t) + \delta\right) dt.$$

Bond return:

$$dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^B K_t/B_t)}{q_t^B K_t/B_t} = \left(\Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B}\right) dt. \tag{A.4}$$

Capital return:

$$
\begin{aligned}
dr_t^K &= \frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K}dt + \frac{d(q_t^K \tilde{k}_t^i)}{q_t^K \tilde{k}_t^i} \\
&= \left(\frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i) - \delta) \right) dt + \tilde{\sigma}_t d\tilde{Z}_t^i. \qquad \text{(A.5)}
\end{aligned}
$$

Equity has the same risk loading as capital, but the expected return may be different when skin-in-the-game constraint is binding. Idiosyncratic risk loadings average out for aggregate equity.

Market Clearing. Goods market clearing:

$$\hat{c}_t(q_t^B + q_t^K) + \mathfrak{g} + \iota_t = a_t. \qquad \text{(A.6)}$$

Asset market clearing:

$$
\begin{aligned}
\theta_t^K &= \frac{q_t^K K_t}{(q_t^B + q_t^K)K_t} = 1 - \vartheta_t, \qquad \text{(A.7)} \\
\theta^E &= \theta^{\bar{E}}. \qquad \text{(A.8)}
\end{aligned}
$$

A.1.3 Solution

Households Problem. We use dynamic programming to solve the households' portfolio choice problem. Guess that the value function takes the form $V(n_t^i, t) = \frac{\omega_t}{\rho+\zeta}\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + v_t$, where ω_t and v_t depends on investment opportunity in aggregate level. Heuristically,

$$
\begin{aligned}
V(n_t^i, t) &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + e^{-\rho dt}(1_{\text{jump}} V(n_{t+dt}^i, t+dt) + 1_{\text{nojump}} V(n_{t+dt}^i, t+dt)) \right] \\
&= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + (1-\rho dt)(\zeta dt V(\bar{n}_t, t+dt) + (1-\zeta dt) V(n_{t+dt}^i, t+dt)) \right]
\end{aligned}
$$

The Hamilton-Jacobi-Bellman equation is

$$
\begin{aligned}
&(\rho+\zeta)V(n_t^i,t) = \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \zeta V(\bar{n}_t,t) + V_t(n_t^i,t) + V_n(n_t^i,t)dn_t^i + \frac{1}{2}V_{nn}(n_t^i,t)\langle dn_t^i, dn_t^i\rangle\right] \\
&\rho v_t + \omega_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} - \frac{\zeta}{\rho+\zeta}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma} \\
&\quad = \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}\left\{\frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \frac{\dot{w}_t}{\rho+\zeta}\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \dot{v}_t + \omega_t\frac{(n_t^i)^{1-\gamma}}{\rho+\zeta}\frac{dn_t^i}{n_t^i}\frac{1}{dt} - \omega_t\frac{\gamma}{2}\frac{(n_t^i)^{1-\gamma}}{\rho+\zeta}\left\langle\frac{dn_t^i}{n_t^i},\frac{dn_t^i}{n_t^i}\right\rangle\frac{1}{dt}\right\}.
\end{aligned}
$$

For simplicity, we denote $\hat{\rho} := \rho+\zeta$. The HJB equation can be restated as

$$
\begin{aligned}
0 = \max_{c_t^i,\iota_t^i,\theta_t^i} &\left\{-\rho v_t + \dot{v}_t + \frac{\zeta}{\hat{\rho}}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma} - \omega_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{1}{\hat{\rho}}\dot{\omega}_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{(c_t^i)^{1-\gamma}}{1-\gamma}\right. \\
&-\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\frac{c_t^i}{n_t^i} + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\left(\Phi(\iota_t)-\delta-\check{\mu}_t^B+\mu_t^{q,B}\right) \\
&+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{K,i}\left(\frac{(1-\tau_t)a_t-\iota_t^i}{q_t^K}+\mu_t^{q,K}+(\Phi(\iota_t^i)-\delta)\right) + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} \\
&\left.-\frac{\gamma}{2}\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(\theta_t^{K,i}+\theta_t^{E,i})^2\tilde{\sigma}_t^2\right\}. \qquad \text{(A.9)}
\end{aligned}
$$

s.t.

$$
-\theta_t^{E,i} \leq (1-\bar{\chi})\theta_t^{K,i}
$$

The lagrangian is hence given by

$$
\begin{aligned}
\mathcal{L}_t^i = &-\rho v_t + \dot{v}_t + \frac{\zeta}{\hat{\rho}}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma} - \omega_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{1}{\hat{\rho}}\dot{\omega}_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{(c_t^i)^{1-\gamma}}{1-\gamma} \\
&-\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\frac{c_t^i}{n_t^i} + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\left(\Phi(\iota_t)-\delta-\check{\mu}_t^B+\mu_t^{q,B}\right) \\
&+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{K,i}\left(\frac{(1-\tau_t)a_t-\iota_t^i}{q_t^K}+\mu_t^{q,K}+(\Phi(\iota_t^i)-\delta)\right) + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} \\
&-\omega_t\frac{\gamma}{2}\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(\theta_t^{K,i}+\theta_t^{E,i})^2\tilde{\sigma}_t^2 \\
&+\lambda_t^i(\theta_t^{E,i}+(1-\bar{\chi})\theta_t^{K,i}).
\end{aligned}
$$

The first order conditions w.r.t. c_t^i, ι_t^i, $\theta_t^{K,i}$, $\theta_t^{E,i}$ and $\theta_t^{\bar{E},i}$ are

$$-\omega_t \frac{(n_t^i)^{1-\gamma}}{\hat{\rho} n_t^i} + (c_t^i)^{-\gamma} = 0, \tag{A.10}$$

$$-\frac{1}{q_t^K} + \frac{1}{1+\phi\iota_t^i} = 0, \tag{A.11}$$

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \gamma(\theta_t^{K,i} + \theta_t^{E,i})\tilde{\sigma}_t^2 - (1-\bar{\chi})\lambda_t^i, \tag{A.12}$$

$$\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \gamma(\theta_t^{K,i} + \theta_t^{E,i})\tilde{\sigma}_t^2 - \lambda_t^i, \tag{A.13}$$

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = 0. \tag{A.14}$$

Characterizing Consumption, Investment q_t^B and q_t^K. The first order condition Eq. (A.10) implies the consumption-wealth ratio

$$c_t^i = \left(\frac{\hat{\rho}}{\omega_t}\right)^{1/\gamma} n_t^i =: \hat{c}_t n_t^i. \tag{A.15}$$

By goods market clearning condition, we obtain

$$\iota_t = \frac{(a_t - \mathfrak{g})(1-\vartheta_t) - \hat{c}_t}{1 + \hat{c}_t\phi - \vartheta_t}. \tag{A.16}$$

$$q_t^K = \frac{1 - \vartheta_t + \phi(a_t - \mathfrak{g})(1-\vartheta_t)}{1 + \hat{c}_t\phi - \vartheta_t} = (1-\vartheta_t)\frac{1+\phi(a_t - \mathfrak{g})}{1 + \hat{c}_t\phi - \vartheta_t}. \tag{A.17}$$

$$q_t^B = \vartheta_t \frac{1+\phi(a_t - \mathfrak{g})}{1 + \hat{c}_t\phi - \vartheta_t}. \tag{A.18}$$

Portfolio Weights. Again, under symmetric equilibrium, market clearing implies

$$\theta_t^K = 1 - \vartheta_t,$$

$$\theta_t^E = -(1-\bar{\chi})(1-\vartheta_t),$$

$$\theta_t^{\bar{E}} = (1-\bar{\chi})(1-\vartheta_t).$$

Return Processes. Under symmetric equilibrium, the outside equity has the same expected return as individual equity, i.e.,

$$\mathbb{E}_t[dr_t^{E,i}] = \mathbb{E}_t[dr_t^{\bar{E},i}], \tag{A.19}$$

with which we can pin down the lagrangian multiplier from Eq. (A.13) and (A.14)

$$\lambda_t^i = \gamma(\theta_t^{K,i} + \theta_t^{E,i})\tilde{\sigma}_t^2 = \gamma\bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t^2.$$

Note that $\theta_t^{E,i} + \theta_t^{K,i} \geq \chi\theta_t^{K,i}$, hence λ_t^i is always positive and the skin-in-the-game constraint is always binding. In the absence of aggregate risks, equity and real bond have the same expected return, which equals the risk-free rate r^f, i.e.,

$$\mathbb{E}_t[dr_t^{E,i}] = \mathbb{E}_t[dr_t^{\bar{E},i}] = \mathbb{E}_t[dr_t^{B}] = r_t^f dt. \tag{A.20}$$

The expected return of capital in excess of equity is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} = \chi\lambda_t^i = \gamma(1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2.$$

A.1.4 Drift of the State Variable and the Steady State

By Eq. (A.5) and (A.4), the expected return of capital in excess of real bond return is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{B}]}{dt} = \frac{(1-\tau_t)a_t - \iota_t}{q_t^K} + \mu_t^{q,K} + \check{\mu}_t^B - \mu_t^{q,B}.$$

Plugging Eq. (A.1) and (A.3) yields

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{B}]}{dt} = \frac{\check{\mu}_t^B - \mu_t^{\vartheta}}{1-\vartheta_t} + \frac{a_t - \mathfrak{g} - \iota_t}{q_t^K}.$$

By Eq. (A.12) and (A.16),

$$\gamma(1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2 = \frac{\breve{\mu}_t^B - \mu_t^\vartheta}{1-\vartheta_t} + \frac{a_t - \mathfrak{g} - \iota_t}{q_t^K}$$
$$\Rightarrow \mu_t^\vartheta = \breve{\mu}_t^B + \hat{c}_t - \gamma(1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2. \tag{A.21}$$

In the steady state, ϑ_t is constant, i.e., $\breve{\mu}^B + \hat{c} - \gamma(1-\vartheta^{SS})^2\bar{\chi}^2\tilde{\sigma}^2 = 0$.

Verifying Value Function. We verify the value function by plugging endogenous variables into the HJB Equation (A.9). The coefficient of $(n_t^i)^{1-\gamma}$ should be zero, i.e.,

$$\begin{aligned}0 = &-\omega_t\frac{1}{1-\gamma} + \frac{1}{\hat{\rho}}\dot{\omega}_t\frac{1}{1-\gamma} + \left(\frac{\omega_t}{\hat{\rho}}\right)^{-\frac{1-\gamma}{\gamma}}\frac{1}{1-\gamma} - \omega_t\frac{1}{\hat{\rho}}\left(\frac{\omega_t}{\hat{\rho}}\right)^{-\frac{1}{\gamma}}\\ &+ \omega_t\frac{1}{\hat{\rho}}\vartheta_t\mathbb{E}[r_t^B]/dt + \omega_t\frac{1}{\hat{\rho}}(1-\vartheta_t)\mathbb{E}[r_t^{K,i}]/dt - \frac{\gamma}{2}\omega_t\frac{1}{\hat{\rho}}\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2,\\ 0 = &-\rho v_t + \dot{v}_t + \frac{\zeta}{\hat{\rho}}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma}.\end{aligned}$$

which are ODEs for ω_t and v_t. Note that $\vartheta_t, \mu_t^\vartheta$ and $\bar{n}_t$ depend on aggregate level variables only, so do ω_t and v_t. The value function is verified. Besides, in the absence of aggregate shocks, $\omega_t = \omega$ is constant if the economy starts at the steady state. Plug in $\mathbb{E}[r_t^B]/dt = r^f$, $\mathbb{E}[r_t^{K,i}]/dt = r^f + \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$ and $\omega_t = \hat{\rho}\hat{c}^{-\gamma}$ to get

$$0 = -\hat{\rho}\hat{c}^{-\gamma}\frac{1}{1-\gamma} + \hat{c}^{1-\gamma}\frac{1}{1-\gamma} - \hat{c}^{1-\gamma} + \hat{c}^{-\gamma}r^f + \hat{c}^{-\gamma}\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \hat{c}^{-\gamma}\frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2,$$

i.e.,

$$r^f = \frac{-\gamma\hat{c} + \hat{\rho}}{1-\gamma} - \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{A.22}$$

In the mean time, according to Eq. (A.4), (A.20) and (A.21), r^f is given by

$$r^f = \Phi(\iota) - \delta + \hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

$\hat{c}$ can be solved numerically by

$$\hat{c} - (\gamma - 1)\Phi(\iota) = \hat{\rho} - (\gamma - 1)\delta - \frac{\gamma(\gamma - 1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 \tag{A.23}$$

where

$$\begin{aligned} \iota &= \frac{(a - \mathfrak{g})(1 - \vartheta) - \hat{c}}{1 + \hat{c}\phi - \vartheta} \\ \Phi(\iota) &= \frac{1}{\phi}\log(1 + \phi\iota). \end{aligned}$$

Since

$$\frac{d\iota}{d\hat{c}} = \frac{-(1 + \hat{c}\phi - \vartheta) - ((a - \mathfrak{g})(1 - \vartheta) - \hat{c})\phi}{(1 + \hat{c}\phi - \vartheta)^2} < 0,$$

ι and $\Phi(\iota)$ are decreasing in $\hat{c}$. The LHS is monotonically increasing in $\hat{c}$. There is a unique positive solution of $\hat{c}$ if

$$\frac{\gamma}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 - \frac{\hat{\rho}}{1 - \gamma} + \delta < \Phi(a - \mathfrak{g}).$$

The consumption rate $\hat{c}$ does not depend monotonically on γ. In Eq. (A.23), a higher γ decreases the right-hand side, but also decreases the $-(\gamma - 1)\Phi(\iota)$ term on the left-hand side. Either term could dominate, leading to an ambiguous effect of γ on $\hat{c}$.

Having said that, if γ is sufficiently high or $\tilde{\sigma}^2$ is sufficiently high, the effect on the right-hand side dominates, leading to a negative relationship between γ and the consumption response $\hat{c}$. Specifically, the derivative $dc/d\gamma$ is given by

$$d\hat{c}\left(1 - (\gamma - 1)\Phi'(\iota)\frac{d\iota}{dc}\right) = \left(\frac{\hat{c} - \hat{\rho}}{\gamma - 1} - \frac{\gamma - 1}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2\right)d\gamma,$$

where the LHS is always positive, while RHS tends to be negative with higher γ or $\tilde{\sigma}^2$. Moreover, note that

$$\frac{d\hat{c}}{d\hat{\rho}} = \left(1 - (\gamma - 1)\Phi'(\iota)\frac{d\iota}{dc}\right)^{-1} > 0.$$

Consumption rate increases when subjective discount rate ρ or wealth reset shock intensity ζ goes up.

A.2 Proof for Proposition 1

Recall that

$$m_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}.$$

We obtain

$$\begin{aligned}\lim_{T\to\infty}\mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] &= \lim_{T\to\infty}\mathbb{E}_0\left[\left(\int_0^1 e^{-\rho T}\left(\frac{n_T^i}{n_0^i}\right)^{-\gamma}di\right)\bar{n}_T\right]\\ &= \lim_{T\to\infty}\mathbb{E}_0\left[e^{-\rho T}\bar{n}_0\left(\frac{\bar{n}_T}{\bar{n}_0}\right)^{1-\gamma}\left(\int_0^1\left(\frac{\eta_T^i}{\eta_0^i}\right)^{-\gamma}di\right)\right].\end{aligned}$$

Normalizing $\eta_0^i = 1$ and $\bar{n}_0 = 1$ yields

$$\lim_{T\to\infty}\mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] = \lim_{T\to\infty}\mathbb{E}_0\left[e^{-\rho T}\bar{n}_T^{1-\gamma}\left(\int_0^1\left(\eta_T^i\right)^{-\gamma}di\right)\right].$$

A.3 Transversality Conditions with Permanent Idiosyncratic Risk

A.3.1 Individual Level TVC

The r Minus g Perspective. Individual TVC is satisfied iff $\mathbb{E}_t[d(m_t^i n_t^i)] < 0$, i.e.,

$$-\rho - \gamma g + \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + g - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 < 0,$$

i.e.,

$$\rho > (1-\gamma)g + \frac{\gamma}{2}(\gamma-1)\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2. \tag{A.24}$$

Note that

$$r^f = \rho + \gamma g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{A.25}$$

$$g = -\hat{c} + r^f + \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 \tag{A.26}$$

The first equation is from the drift term of the SDF. The second equation is obtained from the wealth dynamics. The equations jointly solve

$$g = \frac{-\hat{c}+\rho}{1-\gamma} + \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{A.27}$$

$$r^f = \frac{-\gamma\hat{c}+\rho}{1-\gamma} - \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2. \tag{A.28}$$

Plug Eq. (A.25) into (A.24), individual TVC is satisfied iff

$$r^f + \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 > g,$$

which together with Eq. (A.26) implies individual TVC holds if and only if $\hat{c} > 0$. As a result, individual TVC always hold. The intuition here is that the growth rate of individual wealth equals the return of individual wealth portfoli minus consumption rate. As long as consumption rate is positive, the growth rate of individual wealth is always smaller than the discount rate.

The Wealth Distribution Perspective. The individual level TVC can be stated as Wealth inequality view:

$$\lim_{T\to\infty}\mathbb{E}[m_T^i n_T^i] = \lim_{T\to\infty}\hat{c}^{-\gamma}\mathbb{E}\left[e^{-\rho T}\bar{n}_T^{1-\gamma}(\eta_T^i)^{1-\gamma}\right] = \lim_{T\to\infty}(\hat{c}^{-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma})\mathbb{E}\left[(\eta_T^i)^{1-\gamma}\right].$$

Recall that

$$\frac{d\eta_t^i}{\eta_t^i} = \bar{\chi}(1-\vartheta)\tilde{\sigma} d\tilde{Z}_t^i,$$

by Itô's lemma,

$$d(\eta_t^i)^{1-\gamma} = \frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(\eta_t^i)^{1-\gamma}dt + (1-\gamma)(\eta_t^i)^{1-\gamma}d\tilde{Z}_t^i,$$

hence

$$d\mathbb{E}[(\eta_t^i)^{1-\gamma}] = \frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\mathbb{E}[(\eta_t^i)^{1-\gamma}]dt,$$

which implies

$$\mathbb{E}[(\eta_T^i)^{1-\gamma}] = (\eta_0^i)^{1-\gamma}\exp\left(\frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 T\right).$$

The term can be further decomposed to

$$\begin{aligned}\mathbb{E}[(\eta_T^i)^{1-\gamma}] &= \mathbb{E}[(\eta_T^i)^{-\gamma}]\mathbb{E}[\eta_T^i] + Cov[(\eta_T^i)^{-\gamma}, \eta_T^i] \\ &= (\eta_0^i)^{1-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 T\right) \\ &+ (\eta_0^i)^{1-\gamma}\left[\exp\left(\frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 T\right) - \exp\left(\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 T\right)\right].\end{aligned}$$

The first term reflects how long-run idiosyncratic risk (the marginal utility inequality) increase the valuation of future wealth portfolio by shrinking the long-term risk-free rate through the precautionary saving channel, while the second term reflects how this force is largely offseted by the risk premium of the individual's own wealth portfolio in the long run. As risk-aversion γ goes up, while the preceived long-run idiosyncratic risk depresses the long-run risk-free rate by magnitude of $\exp((\gamma(\gamma+1)\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2/2)T)$, the associated long-term risk premium also explodes at the same speed. In the mean time, marginal utility decreases faster on average due to idiosyncratic risk premia. In equilibrium, the sum of the three forces ensures that the valuation of individual wealth portfolio in the long-run does not explode and individual TVC holds. Finally, recall that $\bar{n}_T$ grows at rate g, which is given in Eq. (A.27), i.e.,

$$\begin{aligned}(\bar{n}_T)^{1-\gamma} &= (\bar{n}_0)^{1-\gamma} \exp\left((1-\gamma)gT\right) \\ &= (\bar{n}_0)^{1-\gamma} \exp\left(\left(-\hat{c}+\rho-\frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right)T\right)\end{aligned}$$

The three terms $e^{-\rho T}$, $(\bar{n}_T)^{1-\gamma}$ and $\mathbb{E}[(\eta_T^i)^{1-\gamma}]$ echo the three terms in (A.24). Together, we obtain

$$\lim_{T\to\infty} \mathbb{E}[m_T^i n_T^i] = \lim_{T\to\infty} (\hat{c}^{-\gamma} e^{-\rho T} \bar{n}_T^{1-\gamma}) \mathbb{E}\left[(\eta_T^i)^{1-\gamma}\right] = \lim_{T\to\infty} \hat{c}^{-\gamma} (\bar{n}_0)^{1-\gamma} \exp(-\hat{c}T).$$

A.3.2 Aggregate Level TVC

The r Minus g Perspective. Similarly, individual TVC is satisfied iff $\mathbb{E}_t[d(m_t^i \bar{n}_t)] < 0$, i.e.,

$$-\rho - \gamma g + \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + g < 0,$$

i.e.,

$$\rho > (1-\gamma)g + \frac{\gamma}{2}(\gamma+1)\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

Plug in Eq. (A.25) and (A.26), we obtain that aggregate level TVC is satisfied if and only if

$$r^f > g.$$

Together with Eq. (A.28) and (A.27), the aggregate TVC holds if and only if

$$\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 < \hat{c}.$$

The Wealth Distribution Perspective. The transversality condition is given by

$$\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] = \lim_{T\to\infty}\hat{c}^{-\gamma}\mathbb{E}\left[e^{-\rho T}\bar{n}_T^{1-\gamma}(\eta_T^i)^{-\gamma}\right] = \lim_{T\to\infty}(\hat{c}^{-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma})\mathbb{E}\left[(\eta_T^i)^{-\gamma}\right].$$

Following the similar steps when we discussed the individual TVC, the wealth dynamics imply

$$\mathbb{E}[(\eta_T^i)^{-\gamma}] = (\eta_0^i)^{-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 T\right).$$

Hence,

$$\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] = \lim_{T\to\infty}\hat{c}^{-\gamma}(\eta_0^i)^{-\gamma}\bar{n}_0^{1-\gamma}\exp\left(\left(-\rho+(1-\gamma)g+\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right)T\right).$$

Plug in g from Eq. (A.27) to obtain

$$\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] = \lim_{T\to\infty}\hat{c}^{-\gamma}(\eta_0^i)^{-\gamma}\bar{n}_0^{1-\gamma}\exp\left(\left(\hat{c}-\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right)T\right).$$

The aggregate TVC holds if $\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 < \hat{c}$.

A.4 Transversality Conditions with Transitory Idiosyncratic Risk

A.4.1 Aggregate Level TVC

We show the wealth distribution perspective here only, and leave the r minus g perspective in Appendix A.7 which also serves as the proof for Proposition 4.

The Wealth Distribution Perspective. Consider the wealth reset shock with intensity ζ. The wealth dynamics become

$$d\eta_t^i = \bar{\chi}(1-\vartheta)\tilde{\sigma}\eta_{t-}^i d\tilde{Z}_t^i + (1-\eta_{t-}^i)d\tilde{J}_t^i.$$

By Itô's lemma,

$$d(\eta_t^i)^{-\gamma} = \left[\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(\eta_{t-}^i)^{-\gamma} + \zeta(1-(\eta_{t-}^i)^{-\gamma})\right]dt$$
$$-\gamma\bar{\chi}(1-\vartheta)\tilde{\sigma}(\eta_{t-}^i)^{-\gamma}d\tilde{Z}_t^i + (1-(\eta_{t-}^i)^{-\gamma})(d\tilde{J}_t^i - \zeta dt).$$

Take expectation on both sides yields an ODE w.r.t. $\mathbb{E}[(\eta_t^i)^{-\gamma}]$[9], i.e.,

$$d\mathbb{E}[(\eta_t^i)^{-\gamma}] = \left[\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\mathbb{E}[(\eta_t^i)^{-\gamma}] + \zeta(1-\mathbb{E}[(\eta_t^i)^{-\gamma}])\right]dt.$$

The solution to the ODE is

$$\mathbb{E}_0\left[(\eta_t^i)^{-\gamma}\right] = \left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta} + (\eta_0^i)^{-\gamma}\right)e^{\left[\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta\right]t} - \frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta}.$$

Recall that growth rate of aggregate wealth is

$$g = \frac{-\hat{c}+\rho+\zeta}{1-\gamma} + \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2,$$

Denote $\hat{c} = \hat{\mathfrak{c}}(\rho)$, which is given implicitly by

$$\hat{c} - (\gamma-1)\Phi(\iota) = \hat{\rho} - (\gamma-1)\delta - \frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2,$$

where

$$\iota = \frac{(a-\mathfrak{g})(1-\vartheta)-\hat{c}}{1+\hat{c}\phi-\vartheta}, \quad \Phi(\iota) = \frac{1}{\phi}\log(1+\phi\iota).$$

---

[9] Note that the difference between $\mathbb{E}[(\eta_t^i)^{-\gamma}]$ and $\mathbb{E}[(\eta_{t-}^i)^{-\gamma}]$ is of magnitude dt, and vanishes after multiplying dt outside the bracket.

The aggregate TVC condition can be stated as

$$\begin{aligned}
\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] &= \lim_{T\to\infty}(\hat{c}^{-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma})\mathbb{E}\left[(\eta_T^i)^{-\gamma}\right] \\
&= \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}e^{-\rho T}\exp\left(\left(-\hat{c}+\rho+\zeta-\frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right)T\right) \\
&\times\exp\left(\left(\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta\right)T\right)\left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta}+(\eta_0^i)^{-\gamma}\right) \\
&- \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma}\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta} \\
&= \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}\exp(-(\hat{c}-\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2)T)\left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta}+(\eta_0^i)^{-\gamma}\right) \\
&- \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma}\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta}.
\end{aligned}$$

Hence, the aggregate TVC holds if and only if

$$\hat{c} > \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$$

and

$$-\rho+(1-\gamma)g < 0.$$

The first condition states that the wealth reset intensity should be big enough such that the marginal utility dispersion does not explode too fast, hence risk free rate will not be lower than the growth rate. The second condition is imposed as we are evaluating long-lived assets with the realized marginal utility process, we want to rule out the case where the whole economy shrinks and marginal utilities explode on average. In other word, we assume transversality condition hold had the economy been populated by one representative agent instead of heterogeneous agents. In this way, we shed light on the effect of heterogeneity and idiosyncratic risk.

A.5 Proof for Proposition 2

Denote the nominal government bond outstanding as B_t and real government bond as b_t. We obtain

$$dB_t = i_t B_t dt - P_t s_t dt.$$

Hence,

$$\begin{aligned}
d(m_t^i b_t) &= b_t dm_t^i + m_{t-}^i / P_t dB_t + m_{t-}^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\
&= b_t \left(-(r_t^f - \zeta) m_{t-}^i dt - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_{t-}^i d\tilde{Z}_t^i + \left(e^{-\rho t} \hat{c}^{-\gamma} \bar{n}_t^{-\gamma} - m_{t-}^i \right) d\tilde{J}_t^i \right) \\
&+ i_t b_t m_{t-}^i dt - s_t m_t^i dt + m_{t-}^i b_t (dr_t^B - i_t dt) \\
&= -m_{t-}^i s_t dt + m_{t-}^i \zeta b_t dt - m_{t-}^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i + b_t \left(e^{-\rho t} \frac{1}{(\rho+\zeta)\bar{n}_t} - m_{t-}^i \right) d\tilde{J}_t^i \\
&= -m_{t-}^i s_t dt + \zeta e^{-\rho t} \hat{c}^{-\gamma} \bar{n}_t^{-\gamma} b_t dt - m_{t-}^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i \\
&+ b_t \left(e^{-\rho t} \hat{c}^{-\gamma} \bar{n}_t^{-\gamma} - m_{t-}^i \right) d(\tilde{J}_t^i - \zeta t),
\end{aligned}$$

where we plugged in the definition of bond return, i.e.,

$$dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^B K_t / B_t)}{q_t^B K_t / B_t} = r_t^f dt$$

Integrate $d(m_t^i b_t)$ from $t = 0$ to T and take expectation to obtain[10]

$$\mathbb{E}_0[m_T^i b_t] - m_0^i b_0 = -\mathbb{E}_0 \left[\int_0^T m_t^i s_t \, dt \right] + \zeta \mathbb{E}_0 \left[\int_0^T e^{-\rho t} \hat{c}^{-\gamma} \bar{n}_t^{-\gamma} \, dt \right].$$

Divide both sides by m_0^i to obtain

$$b_0 = \mathbb{E}_0 \left[\int_0^T \frac{m_t^i}{m_0^i} s_t \, dt \right] - \zeta \mathbb{E}_0 \left[\int_0^T e^{-\rho t} \left(\frac{\bar{n}_T}{\bar{n}_0} \right)^{-\gamma} b_t \, dt \right] + \mathbb{E}_0 \left[\frac{m_T^i}{m_0^i} b_T \right].$$

---

[10] Note that the set of jump time $\mathcal{T} = \{t | m_t^i \neq m_{t-}^i\}$ has zero measure, and we can replace $t-$ by t in the intergrals. Heuristically, $\int_0^T m_t^i s_t \, dt = \int_0^T m_{t-}^i s_t \, dt + \int_0^T (m_t^i - m_{t-}^i) s_t \, dt = \int_0^T m_{t-}^i s_t \, dt$.

To derive the bond valuation equation from the aggregate SDF, we start by dividing $d(m_t^i B_t/P_t)$ by m_0^i, i.e.,

$$d[(m_t^i/m_0^i)b_t] = -\frac{m_{t-}^i}{m_0^i}s_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt - \frac{m_{t-}^i}{m_0^i} b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i$$
$$+ b_t\left(e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} - \frac{m_{t-}^i}{m_0^i}\right) d(\tilde{J}_t^i - \zeta t).$$

Intergrate by i to obtain

$$d[\bar{m}_t b_t] = -\bar{m}_t s_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt.$$

Integrate by t and take expectation to obtain

$$\mathbb{E}_0[\bar{m}_T b_T] - b_0 = -\mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_t\left[\int_0^T \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt\right],$$

i.e.,

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] - \mathbb{E}_t\left[\int_0^T \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right].$$

A.6 Proof for Proposition 3

We use the same proof strategy in Toda (2014). Recall that the individual wealth satisfies

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t^m dZ_t + (1-\vartheta_t)\tilde{\sigma}_t \bar{\chi} d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i} d\tilde{J}_t^i.$$

For aggregate wealth,

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt + \sigma_t^m dZ_t.$$

Hence,

$$\frac{d\eta_t^i}{\eta_t^i} = \frac{d(n_t^i/\bar{n}_t)}{n_t^i/\bar{n}_t} = \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i + \frac{1-\eta_t^i}{\eta_t^i} d\tilde{J}_t^i. \tag{A.29}$$

In the absence of wealth reset, the log wealth share follows

$$d\log\eta_t^i = -\frac{1}{2}\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 dt + \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i. \tag{A.30}$$

Below, we use generation s to denote the collection of agents whose last wealth reset shock happened at time s. Specifically, generation 0 denotes the collection of agents who have never had any reset shock. We also use x to denote log wealth share, i.e., $x_t^i = \log\eta_t^i$.

At time t, denote the (within-generation) probability density function for the log wealth share of generation 0 as $g(x,t,0)$. Under Assumption 1, $g(x,t,0)$ is subject to the following Kolmogorov Forward Equation

$$\frac{\partial g(x,t,0)}{\partial t} = \frac{(\bar{\chi}(1-\vartheta)\tilde{\sigma})^2}{2}\left(\frac{\partial}{\partial x}g(x,t,0) + \frac{\partial^2}{\partial x^2}g(x,t,0)\right),$$

Note that $g(x,t,0)$ is always a valid pdf at any time t, i.e.,

$$\int_0^\infty g(x,t,0)\,dx = 1, \quad \forall t > 0.$$

Denote the p.d.f. of the initial wealth distribution as $f(x,0)$. Since g is the p.d.f. of the initial log wealth share, we have[11]

$$\frac{1}{\eta}g(\log\eta,0,0)=f(\eta,0).$$

Similarly, for every generation s such that $0<s<t$, we can find the within-generation p.d.f. of the log wealth share $g(x,t,s)$ by plugging the initial condition $\delta(x-1)$, which integrates to 1 but only has mass at $x=1$. By doing this, we obtain that

$$g(x,t,s)=\frac{1}{\bar{\chi}(1-\vartheta)\tilde{\sigma}\sqrt{2\pi(t-s)}}e^{-\frac{(x+\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(t-s)/2)^2}{2\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(t-s)}}$$

Note that the initial mass of generation s is ζds and only $\zeta e^{-\zeta(t-s)}ds$ are left at time t. Hence, the distribution of log wealth share of all agents is a mixture of the within-generation p.d.f.s, i.e.,

$$\begin{aligned}
g(x,t)&=e^{-\zeta t}g(x,t,0)+\int_0^t\zeta e^{-\zeta(t-s)}g(x,t,s)ds\\
&=e^{-\zeta t}g(x,t,0)\\
&+Ce^{-\frac{1}{2}x}\left[e^{-\frac{\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}|x|}{2}}\Phi\left(-\frac{|x|}{\bar{\chi}(1-\vartheta)\tilde{\sigma}\sqrt{t}}+\frac{\sqrt{8\zeta+\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}{2}\sqrt{t}\right)\right]\\
&-Ce^{-\frac{1}{2}x}\left[e^{-\frac{\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}|x|}{2}}\Phi\left(-\frac{|x|}{\bar{\chi}(1-\vartheta)\tilde{\sigma}\sqrt{t}}-\frac{\sqrt{8\zeta+\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}{2}\sqrt{t}\right)\right]
\end{aligned}$$

where $C=\frac{2\zeta}{\bar{\chi}(1-\vartheta)\tilde{\sigma}\sqrt{8\zeta+\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}$. As $t\to\infty$, the first and third term shrink to zero pointwise,

---

[11] Note that

$$\begin{aligned}
f(\eta,0)&=\frac{d}{d\eta}\mathbb{P}(\eta_0^i\leq\eta)\\
&=\frac{d}{d\eta}\mathbb{P}(\log\eta_0^i\leq\log\eta)\\
&=\frac{d}{d\eta}\int_{-\infty}^{\log\eta}g(x,0,0)dx\\
&=\frac{d}{d\log\eta}\int_{-\infty}^{\log\eta}g(x,0,0)dx\frac{d\log\eta}{d\eta}\\
&=\frac{1}{\eta}g(\log\eta,0,0).
\end{aligned}$$

while the second term converges to 1. Hence, $g(x,t)$ converges pointwise to

$$g(x,\infty) = Ce^{-\frac{1}{2}x - \frac{\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}|x|}{2}}.$$

Given the p.d.f. for log wealth share g, we obtain the limit wealth distribution as

$$\begin{aligned} f(\eta,\infty) &= \frac{1}{\eta} g(\log\eta,\infty) \\ &= \begin{cases} C\eta^{-\frac{3}{2}+\frac{\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}, & 0<\eta<1, \\ C\eta^{-\frac{3}{2}-\frac{\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}, & \eta\geq 1. \end{cases} \end{aligned}$$

A.7 Proof for Proposition 4

Note that

$$\begin{aligned} d(m_t^i\bar{n}_t) = &-\left[\rho-\gamma g+\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2+g\right]m_{t-}^i\bar{n}_t dt \\ &+\zeta[e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}-m_{t-}^i\bar{n}_t]dt \\ &-\gamma\bar{\chi}(1-\vartheta)\tilde{\sigma}m_{t-}^i\bar{n}_t d\tilde{Z}_t^i+[e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}-m_{t-}^i\bar{n}_t]d(\tilde{J}_t^i-\zeta t). \end{aligned}$$

By Eq. (A.22) and (A.26)

$$g=\frac{-\hat{c}+\rho+\zeta}{1-\gamma}+\frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

Plug in to obtain

$$\begin{aligned} d(m_t^i\bar{n}_t) = &-\left[\hat{c}-\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right]m_{t-}^i\bar{n}_t dt \\ &+\zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}dt \\ &-\gamma\bar{\chi}(1-\vartheta)\tilde{\sigma}m_{t-}^i\bar{n}_t d\tilde{Z}_t^i+[e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}-m_{t-}^i\bar{n}_t]d(\tilde{J}_t^i-\zeta t). \end{aligned}$$

Divide by m_0^i and integrate across i to obtain

$$d(\bar{m}_t\bar{n}_t) = -(\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2)\bar{m}_t\bar{n}_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{1-\gamma}\bar{n}_0 dt.$$

The ODE admits the following solution

$$\begin{aligned}\bar{m}_t\bar{n}_t = {} & \bar{n}_0\exp(-(\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2)t)\left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \zeta} + 1\right)\\ & - \bar{n}_0 e^{-\rho t}\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \zeta}.\end{aligned}$$

As a special case, when $t = 0$, we recover our initial condition $\bar{m}_0\bar{n}_0 = \bar{n}_0$.

Given $-\rho - (\gamma - 1)g < 0$, the aggregate TVC holds if and only if

$$\hat{c} > \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

A.8 Arbitrary Wealth Distribution After Wealth Reset

We show that the main results are preserved even if we impose arbitrary wealth distribution after wealth resets. Denote the pdf for wealth resets as $h(\tilde{\eta})$. We require that $\int_0^\infty \tilde{\eta}^{-\gamma}h(\tilde{\eta})\, d\tilde{\eta} < \infty$, i.e., the expected marginal utility immediately after the wealth reset is well-defined. The previous specification can be viewed as a special case where $h(\tilde{\eta}) = \delta(\tilde{\eta} - 1)$, where $\delta(.)$ is the dirac function. First, we show that this general case admits the same solution of aggregate variables in the reset-to-average case. Heuristically, household's HJB equation is given by

$$\begin{aligned}V(n_t^i, t) &= \max_{c_t^i,\iota_t^i,\theta_t^i}\mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + e^{-\rho dt}(1_{\text{jump}}V(n_{t+dt}^i, t+dt) + 1_{\text{nojump}}V(n_{t+dt}^i, t+dt))\right]\\ &= \max_{c_t^i,\iota_t^i,\theta_t^i}\mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + (1-\rho dt)(\zeta dt\mathbb{E}^\eta[V(h(\tilde{\eta})\bar{n}_t, t+dt)] + (1-\zeta dt)V(n_{t+dt}^i, t+dt))\right].\end{aligned}$$

$\mathbb{E}^\eta[.]$ denotes taking expectation w.r.t. $\tilde{\eta}$. The only difference is that the after-jump value function changed from $V(\bar{n}_t, t+dt)$ to $V(h(\tilde{\eta})\bar{n}_t, t+dt)$. Since the assignment of after-jump wealth is independent with agent's pre-jump wealth, the expectation depends on aggregate variables only. As a result, $h(\tilde{\eta})$ only affects v_t in the value function, which does not matter household's consumption or investment choices.

Next, we show that $h(\tilde{\eta})$ does not matter for the transversality condition either. Define the compound Poisson process

$$\tilde{Q}_t^i = \sum_{k=1}^{\tilde{J}_t^i} (\tilde{\eta}_k^i)^{-\gamma},$$

where $\tilde{\eta}_k^i$ is the post-reset wealth share in each reset shock, i.i.d. drawn from the probability density function $h(\tilde{\eta})$. The compensated compound Poisson process is

$$\tilde{Q}_t^i - \mathbb{E}[(\tilde{\eta})^{-\gamma}]\zeta t,$$

which is a martingale. The dynamics of SDF can be written as

$$\begin{aligned}
dm_t^i = & - \left[\rho + \gamma g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta\right] m_{t-}^i dt \\
& + \zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]dt \\
& - \gamma\bar{\chi}(1-\vartheta)\tilde{\sigma} m_{t-}^i d\tilde{Z}_t^i \\
& + e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma} d(\tilde{Q}_t^i - \mathbb{E}[(\tilde{\eta})^{-\gamma}]\zeta t) \\
& - m_{t-}^i (d\tilde{J}_t^i - \zeta t)
\end{aligned}$$

Hence,

$$
\begin{aligned}
dm_t^i\bar{n}_t = &-\left[\rho + (\gamma-1)g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta\right] m_{t-}^i\bar{n}_t dt \\
&+ \zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]dt \\
&- \gamma\bar{\chi}(1-\vartheta)\tilde{\sigma}m_{t-}^i\bar{n}_t d\tilde{Z}_t^i \\
&+ e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma}d(\tilde{Q}_t^i - \mathbb{E}[(\tilde{\eta})^{-\gamma}]\zeta t) \\
&- m_{t-}^i(d\tilde{J}_t^i - \zeta t).
\end{aligned}
$$

Take expectation to obtain

$$
\begin{aligned}
d\mathbb{E}\left[m_t^i n_t^i\right] = &-\left[\rho + (\gamma-1)g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta\right]\mathbb{E}[m_{t-}^i\bar{n}_t]dt \\
&+ \zeta e^{-\rho t}\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_t^{1-\gamma}dt.
\end{aligned}
$$

Note that $[\rho + (\gamma-1)g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta = r^f - g = \hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$. The solution is

$$
e^{(r^f-g)t}\mathbb{E}[m_t^i\bar{n}_t] - m_0^i\bar{n}_0 = \frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}(e^{[\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2]t} - 1),
$$

i.e.,

$$
\begin{aligned}
\mathbb{E}[m_t^i\bar{n}_t] = &\left(m_0^i\bar{n}_0 - \frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}\right)\exp(-(\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2)t) \\
&+ \frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}\exp(-(\rho + (\gamma-1)g)t).
\end{aligned}
$$

For all $h(\tilde{\eta})$ such that $\mathbb{E}[(\tilde{\eta})^{-\gamma}]$ exists, aggregate TVC depends on $\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$, i.e., $r^f - g$. In the first part we show that $h(\tilde{\eta})$ does not matter for any aggregate variables. Hence, $h(\tilde{\eta})$ does not matter for aggregate TVC.

A.9 Model with Stochastic Death

We model the stochastic death as follows. Define $\{d_t^i\}$ as a Poisson process with intensity ζ. Death events are independent across agents, i.e., $\{d_t^i\}$ and $\{d_t^j\}$ are stochastically independent for $i \neq j$. Let $d_0^i = 0$. The stochastic death time τ^i is defined as

$$\tau^i := \inf\{t > 0 \mid d_t^i \geq 1\}$$

The death process is hence defined by

$$death_t^i = d^i_{\{t \wedge \tau^i\}},$$

i.e., when agent i is alive, $death_t^i = 0$. Otherwise, $death_t^i = 1$.

Consider household's problem. Heuristically,

$$\begin{aligned} V(n_t^i, t) &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + e^{-\rho dt} (1_{\text{death}} V(n_{t+dt}^i, t+dt) + 1_{\text{nodeath}} V(n_{t+dt}^i, t+dt)) \right] \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + (1-\rho dt)(\zeta dt \times 0 + (1-\zeta dt) V(n_{t+dt}^i, t+dt)) \right] \end{aligned}$$

The Hamilton-Jacobi-Bellman equation is

$$\begin{aligned} &(\rho+\zeta) V(n_t^i, t) = \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \zeta \times 0 + V_t + V_n(n_t^i, t) dn_t^i + \frac{1}{2} V_{nn}(n_t^i, t) \langle dn_t^i, dn_t^i \rangle \right] \\ &\rho v_t + \omega_t \frac{(n_t^i)^{1-\gamma}}{1-\gamma} - 0 \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E} \left\{ \frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \frac{\dot{w}_t}{\rho+\zeta} \frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \dot{v}_t + \omega_t \frac{(n_t^i)^{1-\gamma}}{\rho+\zeta} \frac{dn_t^i}{n_t^i} \frac{1}{dt} - \omega_t \frac{\gamma}{2} \frac{(n_t^i)^{1-\gamma}}{\rho+\zeta} \left\langle \frac{dn_t^i}{n_t^i}, \frac{dn_t^i}{n_t^i} \right\rangle \frac{1}{dt} \right\}. \end{aligned}$$

Note that the HJB is essetianlly the same as the wealth reset model, except that the utility after death is zero while the utility after wealth reset is $V(\bar{n}_t, t)$. The first order conditions are the same, though the solution for v_t is different. Hence, we obtain the same equilibrium in the wealth reset model.

Below, we discuss household's decision and the relevant transversality conditions in a different perspective. Let n_t^i denote the wealth of agent i at time t. For $t > \tau^i$, i.e., after the agent's death, we assume that n_t^i denotes the wealth following the agent's consumption and investment plan if it were still alive. We denote $\check{m}_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}$ as the hypothetical marginal utility if the agent were still alive.

We denote m_t^i as the agent's actual marginal utility, which is zero after death:

$$m_t^i := \begin{cases} \check{m}_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}, & death_t^i = 0, \\ 0, & death_t^i = 1. \end{cases}$$

We obtain

$$dm_t^i = \begin{cases} -\left[\rho + \gamma g_t - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2\right] m_t^i dt - \gamma\bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i, & t < \tau^i, \\ -m_{\tau^i-}^i, & t = \tau, \\ 0 & t > \tau^i. \end{cases}$$

Aggregate TVC. For long-lived asset, e.g. government bond at $t < \tau^i$,

$$\begin{aligned} d(m_t^i b_t) &= b_t dm_t^i + m_t^i/P_t dB_t + m_t^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\ &= b_t\left(-(r_t^f - \zeta)m_t^i dt - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i\right) \\ &\quad + i_t b_t m_t^i dt - s_t m_t^i dt + m_t^i b_t(dr_t^B - i_t dt) \\ &= -m_t^i s_t dt + m_t^i \zeta b_t dt - m_t^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i. \end{aligned}$$

At $t = \tau^i$, $d(m_t^i b_t) = -m_{\tau^i\_}^i b_{\tau^i}$. At $t \geq \tau^i$, $d(m_t^i b_t) = 0$. Integrate from $t = 0$ to $t = T$ and take expectation. Recall that τ^i is independent of n_t^i and b_t, and

$$\mathbb{E}_0\left[1_{\{\tau^i \leq u\}}\right] = 1 - e^{-\zeta u}.$$

Hence,

$$\begin{aligned}
m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right] \\
&= (1-e^{-\zeta T})\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \le T\right] + e^{-\zeta T}\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right].
\end{aligned}$$

The first term is computed as

$$\begin{aligned}
&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \le T\right] \\
=&\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] - \mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] + \mathbb{E}_0\left[\check{m}_{\tau^i -}^i b_{\tau i} \middle| \tau^i \le T\right],
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] &= \mathbb{E}_0\left[\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t \, dt \middle| \tau^i\right] \middle| \tau^i \le T\right] \\
&= \frac{1}{1-e^{-\zeta T}}\int_0^T \zeta e^{-\zeta u}\mathbb{E}_0\left[\int_0^u \check{m}_t^i s_t \, dt\right] du \\
&= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \left(\int_0^u \zeta e^{-\zeta u}\check{m}_t^i s_t \, dt\right) du\right] \\
&= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \left(\int_t^T \zeta e^{-\zeta u}\check{m}_t^i s_t \, du\right) dt\right] \\
&= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i s_t dt\right],
\end{aligned}$$

and similarly

$$\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \zeta(e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i b_t dt\right], \\
\mathbb{E}_0\left[\check{m}_{\tau^i -}^i b_{\tau i} | \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta u}\check{m}_u^i b_u du\right].
\end{aligned}$$

The second term is computed as

$$\begin{aligned}
&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right] \\
=&\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - \mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + \mathbb{E}_0\left[\check{m}_T^i b_T\right].
\end{aligned}$$

Finally,

$$\begin{aligned}
m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right] \\
&= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i s_t dt\right] + \mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta T}\check{m}_t^i b_t dt\right] \\
&+ e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + e^{-\zeta T}\mathbb{E}_0\left[\check{m}_T^i b_T\right] \\
&= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t}\check{m}_t^i) s_t dt\right] + \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i) b_T\right].
\end{aligned}$$

Recall that $\check{m}_t^i$ is the marginal utility if the agent were alive at time t. The TVC term for this aggregate asset is the discounted asset value $m_T^i b_T$ times the probability of surviving until T, which is $e^{-\zeta T}$. The aggregate TVC holds if and only if $\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 > 0$.

Individual TVC. Similarly, for individual wealth, we obtain

$$\begin{aligned}
d(m_t^i n_t^i) &= n_t^i dm_t^i + m_t^i dn_t^i + dm_t^i dn_t^i \\
&= n_t^i(-(r_t^f - \zeta)m_t^i dt - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i) \\
&+ m_t^i(-c_t^i dt + (r_t^f + \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2)n_t^i dt + \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t n_t^i d\tilde{Z}_t^i) - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 m_t^i n_t^i dt \\
&= -c_t^i m_t^i dt + \zeta m_t^i n_t^i dt
\end{aligned}$$

for $t < \tau^i$, i.e., if the agent were still alive. Following the same steps above, we obtain

$$m_0^i n_0^i = \mathbb{E}_0\left[\int_0^T (e^{-\zeta t}\check{m}_t^i) c_t^i dt\right] + \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i)\check{n}_T^i\right].$$

The TVC for individual household's wealth is

$$\lim_{T\to\infty} \mathbb{E}_0 \left[(e^{-\zeta T} \check{m}_T^i \check{n}_T^i) \right] = 0.$$

Again, the analysis is the same as that in A.1 without wealth reset shock, but the subjective discount rate is augmented by ζ.

A.10 Calibration

Table A.1: Parameter Values.

Variable	Notation	Value	Notes
Baseline Model with Permanent Idiosyncratic Risk			
risk aversion	γ	2	
subjective discount rate	ρ	0.02	
capital adjustment cost	ϕ	8.1	
Depreciation rate	δ	0.055	
Government spending	$\mathfrak{g}$	0.15	
real growth rate	g	0%	Targeted by choosing a
idiosyncratic risk variance	$\tilde{\sigma}$	0.54	From Brunnermeier et al. (2022)
share of undiversifiable risk	$\bar{\chi}$	0.3	From Brunnermeier et al. (2022)
share of equity	ϑ	0.18	From Brunnermeier et al. (2022)
Model Variant with Transitory Idiosyncratic Risk			
weak wealth reset intensity	ζ	2.5%	
moderate wealth reset intensity	ζ	3.8%	
strong wealth reset intensity	ζ	5.4%	

B Model with Aggregate Risk

In this section, we consider a model with aggregate risk. We make two modifications to the model in the main text. First, following Brunnermeier et al. (2022), we assume that the production process is subject to an additional aggregate risk dZ_t that is common to all households. The law of motion for the wealth of household i is given by

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t^m dZ_t + (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i$$

which additionally loads on the aggregate risk dZ_t with loading σ_t^m. We can think of this aggregate risk as the shock to productivity level that is common to all households.

Second, for tractability, households have log preference, i.e., $\gamma = 1$, which allows us to solve the model in closed form.

Other assumptions are identical to those in the main text. We solve the model using the same steps. In this appendix, we report the key results in this setting, and confirm our claim that introducing aggregate risk does not change the main results of the paper.

Permanent Idiosyncratic Risk. First, we consider the case with permanent idiosyncratic risk. Under the wealth distribution perspective, the transversality condition for aggregate wealth can be expressed as

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T\right] = \bar{n}_0 \lim_{T\to\infty} \mathbb{E}_0 \left[e^{-\rho T} \int_0^1 \frac{\eta_0^i}{\eta_T^i} di\right].$$

Under the r minus g perspective, we can further write the discounted aggregate wealth as

$$\frac{d(\bar{m}_t\bar{n}_t)}{\bar{m}_t\bar{n}_t} = [-\underbrace{r_t^f}_{\text{risk-free rate}} - \underbrace{(\sigma_t^m)^2}_{\text{risk premium for aggregate risk}} + \underbrace{g_t}_{\text{growth rate}}]dt,$$

with the following $r-g$ condition

$$(r_t^f + (\sigma_t^m)^2) - g_t = \rho - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2.$$

Transitory Idiosyncratic Risk. Similarly, in the case with wealth reset shock, the TVC for aggregate wealth can be expressed as

$$\lim_{T\to\infty}\mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] = \lim_{T\to\infty} e^{-\rho T}\bar{n}_0\mathbb{E}_0\left[\int_0^1 \frac{1}{\eta_T^i}di\right].$$

The TVC term for aggregate wealth has the following dynamics:

$$d[\bar{m}_t\bar{n}_t] = -[\rho - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 + \zeta]\bar{m}_t\bar{n}_t dt + \zeta e^{-\rho t}\bar{n}_0 dt$$

which implies the following r minus g condition for the aggregate wealth:

$$(r_t^f + (\sigma_t^m)^2) - g_t = \rho - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 + \zeta.$$

All these results are consistent with our main results in the paper, which are obtained when we set $\gamma = 1$ and $\sigma_t^m = 0$.

C TVC in the Constantinides-Duffie Economy

The Economy. There is a continuum of households indexed by $i \in [0,1]$. Time is discrete. There is a single nondurable consumption good which serves as the numeraire. Households maximize lifetime expected utility with CRRA γ. The aggregate dividend is exogenous, denoted as d_t. The aggregate labor income process is exogenous, denoted as ℓ_t. The market clearing condition implies that the aggregate consumption process is $c_t = \ell_t + d_t$.

Uninsured Idiosyncratic Labor Income Risk. Consumer i has labor income ℓ_t^i defined by

$$\ell_t^i = \delta_t^i c_t - d_t,$$

where

$$\delta_t^i = \exp\left[\sum_{s=1}^{t}\left(\eta_s^i y_s - \frac{y_s^2}{2}\right)\right].$$

The shocks η_t^i are standard normal and i.i.d. across periods and individuals.

Equilibrium. We consider the no-trade equilibrium where each household holds one unit of equity. Household i's consumption is $c_t^i = \delta_t^i c_t$. Denote $\mathcal{F}_t$ as the information set available to consumers. With household i's SDF given by

$$\begin{aligned} m_{t,t+1}^i &= e^{-\rho}\left(\frac{c_{t+1}^i}{c_t^i}\right)^{-\gamma} \\ &= e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(-\gamma\left(\eta_{t+1}^i y_{t+1} - \frac{y_{t+1}^2}{2}\right)\right) \end{aligned}$$

The Euler equation for households i is

$$\begin{aligned} p_t &= \mathbb{E}\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(-\gamma\left(\eta_{i,t+1}y_{t+1} - \frac{y_{t+1}^2}{2}\right)\right)(p_{t+1}+d_{t+1})\middle|\mathcal{F}_t\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(-\gamma\left(\eta_{i,t+1}y_{t+1} - \frac{y_{t+1}^2}{2}\right)\right)(p_{t+1}+d_{t+1})\middle|\mathcal{F}_t \cup \{y_{t+1}\}\right]\middle|\mathcal{F}_t\right] \\ &= \mathbb{E}\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}y_{t+1}^2\right)(p_{t+1}+d_{t+1})\middle|\mathcal{F}_t\right]. \end{aligned}$$

It is verified that no-trade equilibrium is valid, as every household's valuation on the aggregate equity aligns.

The risk-free rate is given by

$$\exp(-r_t) = \mathbb{E}_t\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}y_{t+1}^2\right)\right]$$

We consider a special case with constant growth rate $c_{t+1}/c_t = \exp(g)$ and constant intensity of idiosyncratic risk $y_t = y$, which implies

$$r = \rho + \gamma g - \frac{\gamma(\gamma+1)}{2} y^2.$$

Pricing Long-Lived Asset with Positive Dividend. Assume that the aggregate dividend is positive and grows at the same speed as consumption. If y^2 is large enough, such that $\rho + \gamma g - \frac{\gamma(\gamma+1)}{2} y^2 < g$, i.e., $r < g$, there is no no-trade equilibrium. Indeed, suppose there were a no-trade equilibrium. The present value of dividend flow would be

$$PV_t(d) = \sum_{k=1}^{\infty} d_t \exp(gk - rk) = \infty.$$

The equity would be infinitely valuable. That is, $\forall\, p_t < \infty$, agent can buy and hold an arbitrarily small amount of the asset to obtain an (infinitely) larger life-time utility. Hence, there exists no well-defined price process and no no-trade equilibrium exists.

D Model Extension with Borrowing Constraints

In this appendix, we provide more details for the heterogeneous-agent models with borrowing constraints, which we discussed in Section 5.3.

D.1 Intertemporal Government Budget Condition

First, we show that with borrowing constraint, the Euler equation becomes an inequality:

$$\mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} \exp(r_t) \right] \leq 1.$$

Suppose the household maximizes lifetime utility:

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\exp(-\rho t) u(c_t) \right],$$

such that

$$y_t + b_{t-1}\exp(r_{t-1}) \geq c_t + b_t,$$
$$b_t \geq 0.$$

The Lagrangian is

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\exp(-\rho t) u(c_t) + \zeta_t (y_t + b_{t-1}\exp(r_{t-1}) - c_t - b_t) + \mu_t b_t \right].$$

The first-order conditions are

$$\exp(-\rho t) u'(c_t) = \zeta_t,$$
$$\zeta_t - \mu_t = \mathbb{E}_t[\zeta_{t+1}\exp(r_t)],$$

which implies

$$\mathbb{E}_t \left[\frac{\exp(-\rho) u'(c_{t+1})}{u'(c_t)} \exp(r_t) \right] = 1 - \frac{\mu_t}{u'(c_t)} \leq 1.$$

with $\mu_t > 0$ if the borrowing constraint is binding.

Next, We evaluate government bond in the presence of borrowing constraint. Recall that

$$b_t \exp(r_t) = s_{t+1} + b_{t+1}.$$

Multiplying both sides by m^i_{t+1}/m^i_t and take expectation to obtain

$$b_t \geq b_t \mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} \exp(r_t) \right] = \mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} s_{t+1} \right] + \mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} b_{t+1} \right].$$

The inequality follows $\mathbb{E}_t\left[\frac{m_{t+1}^i}{m_t^i}\exp(r_t)\right] \leq 1$, which we proved above. Similarly,

$$b_{t+1} \geq \mathbb{E}_{t+1}\left[\frac{m_{t+2}^i}{m_{t+1}^i}s_{t+2}\right] + \mathbb{E}_{t+1}\left[\frac{m_{t+2}^i}{m_{t+1}^i}b_{t+2}\right],$$

which implies

$$b_t \geq \sum_{k=1}^{2}\mathbb{E}_t\left[\frac{m_{t+k}^i}{m_t^i}s_{t+k}\right] + \mathbb{E}_t\left[\frac{m_{t+2}^i}{m_t^i}b_{t+2}\right].$$

In this way, we can iterate forward to obtain

$$b_t \geq \sum_{k=1}^{\infty}\mathbb{E}_t\left[\frac{m_{t+k}^i}{m_t^i}s_{t+k}\right] + \lim_{k\to\infty}\mathbb{E}_t\left[\frac{m_{t+k}^i}{m_t^i}b_{t+k}\right].$$

D.2 Consumption Lower Bound in Bewley Model

Here we feature a simple model without aggregate growth or aggregate uncertainty but with uninsured idiosyncratic risk. There is a continuum of agents on $[0,1]$. Agent i get endowment y_t^i each period, with $y_t^i \geq \underline{y} > 0$. Aggregate (after-tax) endowment is $y = \int_0^1 y_t^i \, di$. Agents can invest into risk-free bonds subject to borrowing constraints, i.e., $b_t^i \geq 0$. The supply of bond is fixed, $b = \int_0^1 b_t^i \, di$. Under steady state, interest rate is constant r. Agents maximize lifetime utility

$$\max \sum_{t=0}^{\infty}\mathbb{E}_0\left[\exp(-\rho t)u(c_t^i)\right].$$

We assume that the utility function satisfies the Inada condition, i.e., $\lim_{c\to 0} u'(c) = \infty$ and $u'(c)$ decreases in c.

In this setting, we show that consumption has a lower bound $\underline{c} > 0$. Denote $z_t^i = b_{t-1}^i \exp(r) + y_t^i$ as the resource available at time t. $z_t^i \geq \underline{y}$. The value function is

$$V(z_t^i, y_t^i) = \max_{c_t^i}\{u(c_t^i) + \exp(-\rho)\mathbb{E}_t[V(z_{t+1}^i, y_{t+1}^i)]\}.$$

If y_t^i is i.i.d., the expectation term on the right-hand side does not depend on y_t^i, and we can rewrite value function as $V(z_t^i)$, i.e., a function of available resources. In the case, Aiyagari (1994) proves that the consumption has a positive lower bound, at which household consumes all available resources. The Euler equation is

$$u'(c_t^i) \geq \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)], \tag{D.1}$$

with equality obtained when $b_t^i > 0$.

Next, we show that the consumption has a positive lower bound. Assume that the marginal value of wealth $V_z(z_t^i, y_t^i)$ is decreasing in wealth z_t^i for any given income y_t^i. Note that

$$V_z(z_{t+1}^i, y_{t+1}^i) \leq V_z(\underline{y}, y_{t+1}^i) \leq \overline{V}_z := \sup_{y_{t+1}^i} V_z(\underline{y}, y_{t+1}^i).$$

The first inequality follows that $z_{t+1}^i \geq \underline{y}$ no matter how much agent saves or consumes at time t. The second inequality assumes that given a positive level of resource available ($\underline{y}$), the marginal gain of extra resource $V_z(\underline{y}, y_{t+1}^i)$ is always finite across every possible y_{t+1}^i. Two special cases are (1) i.i.d. idiosyncratic shock case in Aiyagari (1994), where y_{t+1}^i is i.i.d., and (2) finite space Markov chain in Krueger and Lustig (2010), where y_{t+1}^i has finite numbers of value, and the upper bound of $V_z(\underline{y}, y_{t+1}^i)$ exists.

Now suppose to the contrary that c_t^i does not have a positive lower bound. On the one hand, there exists $c^* > 0$, such that

$$u'(c_t^i) > \exp(r-\rho)\overline{V}_z \geq \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)], \quad \forall c_t^i < c^*.$$

On the other hand, since c_t^i does not have a positive lower bound. There must be some states where $c_t^i < \underline{y}$. Note that since $c_t^i + b_t^i \geq \underline{y}$, $c_t^i < \underline{y}$ must imply $b_t^i > 0$ and equality in the Euler equation

(D.1), i.e.,

$$u'(c_t^i) = \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)].$$

By contradiction, c_t^i must have a positive lower bound $\underline{c}$. Moreover, if $\underline{c} < \underline{y}$, since the agent must be saving, the Euler equation holds. Assume the inverse of marginal utility u'^{-1} exists, we obtain

$$\underline{c} \geq u'^{-1}(\bar{V}_z).$$

Hence, $\underline{c} \geq \min\{u'^{-1}(\bar{V}_z), \underline{y}\}$. Following the same steps, it is straightforward to extend the result to an economy with constant growth rate $\exp(g)$. The consumption over aggregate endowment share is no smaller than $\underline{c}/y$.

Finally, we show that the TVC holds for aggregate debt with CRRA $\gamma > 1$ and positive growth rate. Denote the aggregate endowment as $y_t = y\exp(gt)$ and aggregate debt as $b_t = b\exp(gt)$ at time t. Note that

$$\lim_{T\to\infty}[e^{-\rho T} y_T^{-\gamma} b_T] = \lim_{T\to\infty} \exp((-\rho + (1-\gamma)g)T) = 0,$$

i.e., the TVC holds if the economy were populated by a hypothetical representative agent. With bounded left tail in consumption distribution, the TVC also holds with heterogeneous agents. Indeed,

$$\lim_{T\to\infty}\mathbb{E}_0[e^{-\rho T}(c_T^i)^{-\gamma} b_T] = \lim_{T\to\infty}\mathbb{E}_0[e^{-\rho T} y_T^{-\gamma} b_T]\mathbb{E}_0\left[\left(\frac{c_T^i}{y_T}\right)^{-\gamma}\right] \leq \lim_{T\to\infty}\mathbb{E}_0[e^{-\rho T} y_T^{-\gamma} b_T]\underline{c}^{-\gamma} = 0.$$

D.3 Townsend and Woodford Model

The government budget constraint is

$$d(1+r) = \tau + d,$$

which implies

$$d = \frac{\tau}{r}$$

for $r > 0$ cases. Since household purchase government bonds only at good times, we obtain the following Euler equations:

$$\rho \frac{u'(\underline{c})}{u'(\bar{c})} = (1+r)^{-1},$$
$$\rho \frac{u'(\bar{c})}{u'(\underline{c})} \leq (1+r)^{-1},$$

which imply

$$\mathbb{E}_0 \left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right] \leq \mathbb{E}_0 \left[\sum_{t=1}^{T} (1+r)^{-t} \tau \right] = \frac{\tau}{r}.$$

We have $\mathbb{E}_0 \left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right] < d$ if borrowing constraint binds infinitely often and $\rho \frac{u'(\bar{c})}{u'(\underline{c})} \leq (1+r)^{-1}$. In the meantime, the transversality condition holds.

$$0 \leq \lim_{T\to\infty} \mathbb{E}_0 \left[\rho^T \frac{u'(c_{A,T})}{u'(c_{A,0})} d \right] \leq \lim_{T\to\infty} \mathbb{E}_0 \left[\rho^T \frac{u'(\underline{c})}{u'(\bar{c})} d \right] = \lim_{T\to\infty} \rho^{T-1} (1+r)^{-1} d = 0$$

E Comparing Different Types of Asset Bubbles

In this section we provide two simple models to illustrate the first and second types of bubbles, which follow Hirano and Toda (2024). For simplicity, we focus on deterministic economies. Denote q_t as the price of the Arrow-Debreu security that delivers one unit of consumption at time t. Denote the price and cash flow as p_t and d_t. No arbitrage implies

$$q_t p_t = q_{t+1}(p_{t+1} + d_{t+1}).$$

Iterate forward to obtain

$$q_t p_t = \sum_{k=1}^{\infty} q_{t+k} d_{t+k} + \lim_{T \to \infty} q_T p_T.$$

We define the first term as fundamental value, and the second term (the TVC term) as the bubble term.

E.1 Classic Rational Bubbles

Consider a model of classic rational bubble in Hirano and Toda (2024), which is based on Samuelson (1958). In period t, a new agent is born and lives for two periods. Its utility function is given by

$$u(c_t(t), c_{t+1}(t)) = \log c_t(t) + \delta \log c_{t+1}(t).$$

We use (t) to denote the agent's cohort, which is based on when it is born. In any given period, there is a young agent who is just born and an old agent who was born in the previous period.

Each agent receives an endowment of a when young and b when old. There is also an intrinsically useless asset (i.e., cash) that has a unit supply and is initially owned by the old agent in period 0. Let p_t denote the asset's price and let $x_t(t)$ denote the amount of the asset that is purchase by the young agent. The agent's budget constraints are given by

$$\begin{aligned} c_t(t) + p_t x_t(t) &= a, \\ c_{t+1}(t) &= b + p_{t+1} x_t(t). \end{aligned}$$

We are interested in competitive equilibria in which all agents maximize their utility functions subject to their budget constraints, and the following asset and goods market clearing conditions hold:

$$a + b = c_t(t) + c_t(t-1),$$

$$x_t(t) + 0 = 1.$$

The young agent's Euler equation with respect to the asset is given by

$$\frac{1}{c_t(t)} p_t = \delta \frac{1}{c_{t+1}(t)} p_{t+1}.$$

Unlike earlier models we studied, there are multiple equilibria in this economy. First, there is a *fundamental equilibrium* in which the asset price is zero, i.e., $p_t = 0$ for all t. The asset is fundamentally worthless, and it does not affect agents' autarky consumption patterns.

In addition, there may exist multiple *bubbly equilibria* in which the asset price is positive, i.e., $p_t > 0$ for all t. In this case, the Euler equation requires

$$\frac{1}{p_{t+1}} = \frac{a\delta}{b} \frac{1}{p_t} - \frac{\delta + 1}{b},$$

If $a\delta \leq b$, the fundamental equilibrium is the only equilibrium. Otherwise, given $a\delta > b$, there exist a continuum of bubbly equilibria indexed by initial asset price $0 < p_0 \leq \frac{a\delta - b}{\delta + 1}$, such that

$$\frac{1}{p_t} = \left(\frac{a\delta}{b}\right)^t \left(\frac{1}{p_0} - \frac{\delta + 1}{a\delta - b}\right) + \frac{\delta + 1}{a\delta - b}.$$

The bubble is stationary if $p_0 = \frac{a\delta - b}{1 + \delta}$.

The bubble asset offers a saving vehicle that allows the agents to smooth their consumption across periods. The young agent can purchase the bubble asset from the old agent, with the expectation that it can sell the asset to the next generation when it becomes old. In this way, the agent delays some of its consumption until it becomes old. Note that since the bubble is intrinsically worthless, the fundamental value is zero. The (finite) value of TVC term equals the price of the bubble asset.

E.2 Unbalanced Growth

Consider another type of bubble of which the cash flow/price ratio is vanishing from Hirano and Toda (2025). The model features similar OLG settings. In period t, a new agent is born and lives for two periods. The agent has a unit supply of labor when young and zero when old.

There is only one type of goods and two production sectors in this economy. The two sectors are indexed by $i = 1, 2$. The first sector produces goods using labor ℓ, while the second sector produces goods y using labor ℓ and land x. The production functions are given by

$$\begin{aligned} f_{1,t}(h) &= \exp(g_1 t)\ell, \\ f_{2,t}(h) &= \exp(g_2 t)\ell^{\alpha} x^{1-\alpha}, \end{aligned}$$

where the productivity growth in the two sectors satisfies $g_1 > g_2$. Let w_t denote the wage rate, and h_t denote the rental rate of land. In equilibrium, the rental rate of land is

$$h_t = (1-\alpha)\alpha^{\alpha/(1-\alpha)} \exp\left(\left(g_2 + \frac{\alpha}{(1-\alpha)}(g_2 - g_1)\right) t\right),$$

and the land price is

$$p_t = \frac{\delta}{1+\delta}\exp(g_1 t).$$

The cash flow/price ratio is

$$\frac{h_t}{p_t} = \delta^{-1}(1+\delta)(1-\alpha)\alpha^{\alpha/(1-\alpha)} \exp\left(\frac{g_2 - g_1}{1-\alpha} t\right),$$

which decays geometrically, hence

$$\sum_{t=1}^{\infty}\frac{h_t}{p_t}=\delta^{-1}(1+\delta)(1-\alpha)\alpha^{\alpha/(1-\alpha)}\frac{\exp\left(\frac{g_2-g_1}{1-\alpha}\right)}{1-\exp\left(\frac{g_2-g_1}{1-\alpha}\right)}<\infty$$

By the bubble characterization lemma (Hirano and Toda, 2025; Montrucchio, 2004), the land price exhibits a bubble.

Over time, the cash flow of the land is going to be a vanishing share of its price, generating a classical behavior of asset bubbles. Intuitively, think of the two sectors as tech and traditional industries. The tech industry is growing faster, and its growth ignites a fast growth rate of wage. A fraction of wage income becomes saving and flows into the land, the only saving vehicle in the economy. Since the land supply is fixed, the price have to go up. The gap between the land price and its fundamental value, i.e., the present value of rents, can be viewed as its premium for being the store of value. As the rent vanishes relative to land price and the tech industry overwhelm the traditional sector, land loses its function as production factor and becomes a pure bubble in the limit.

In this example, there is a unique bubbly equilibrium, and no fundamental equilibrium with no asset bubble. More generally, Hirano and Toda (2025) show that asset bubbles have to arise when two sectors' growth rates diverge while the counterfactual long-run autarky interest rate is below dividend growth, which rules out fundamental equilibria with no asset bubble.