Heterogeneous Beliefs, Asset Prices, and Business Cycles*

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March 3, 2026

Abstract

We develop a complete-market production economy in which heterogeneous beliefs about productivity growth affect both asset prices and real activity. Firms must hire workers before productivity is realized, so operating leverage increases the exposure of profits to productivity risk. Because the firm's discount factor depends on a wealth-weighted aggregation of investor beliefs, shifts in optimism compress risk premia and raise employment, while reversals trigger contractions. This mechanism generates an equilibrium feedback between asset prices, hiring, and risk exposure. A taxonomy of belief systems shows that only extrapolative beliefs produce amplification and endogenous risk build-up. Disciplined by survey expectations, the model jointly matches key asset-pricing, trading-volume, and business-cycle moments.

KEYWORDS: Heterogeneous Beliefs; Business Cycles; Asset Prices; Speculation.

JEL CLASSIFICATION: D84, E32, E44, E71, G41.

*We are grateful for helpful conversations with Andrew Atkeson, Sebastian Di Tella, Hamilton Gallardo, Fatih Guvenen, Fernando Mendo, Stavros Panageos, and Alp Simsek. We also thank three anonymous referees, the Central Bank of Chile, SED, and PUC-Rio participants, and the Santiago Macro Workshop 2019 for helpful comments. Bigio thanks the hospitality of the Federal Reserve Bank of San Francisco. We are indebted to Renata Avila for outstanding research assistance. All errors are ours.

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1 Introduction

One of the oldest and most prominent economic narratives holds that swings in investor beliefs drive asset prices and business cycles. From Sprague (1910) and Fisher (1933) to Keynes (1936), Minsky (1986), Kindleberger (1996), and Shiller (2000), a common theme emerges: waves of optimism fuel asset-price booms and real expansion, while subsequent reversals trigger financial collapse and economic contraction. In this view, exuberance on Wall Street ultimately spills over onto Main Street.

Despite the prominence of this narrative, it remains unclear whether empirically plausible belief fluctuations can simultaneously account for observed patterns in asset prices, trading behavior, and business cycles. In this paper, we formalize this narrative in a tractable production-based asset-pricing framework in which heterogeneous beliefs generate fluctuations in both financial markets and real activity.

In our setting, the connection between investor beliefs and output operates through firms' hiring decisions. Reminiscent of the search-and-matching literature, employment is predetermined: firms must choose labor one period in advance. Because productivity is realized only after hiring decisions are made, employment exposes firms to productivity risk. Firms cannot instantaneously adjust labor when expectations about productivity growth are disappointed.

The timing of hiring renders the firm's problem inherently dynamic. As a result, the economy's stochastic discount factor (SDF) directly shapes labor demand. The SDF depends on a wealth-weighted aggregation of investor beliefs. When optimistic investors accumulate wealth, risk premia compress and firms discount future payoffs less aggressively, raising employment. When optimism recedes, risk premia rise and labor demand contracts. Belief-driven movements in asset prices therefore propagate directly into the real economy.

The environment is otherwise frictionless: markets are complete, prices are flexible, and there are no financial constraints. Despite the absence of additional amplification mechanisms, the model quantitatively matches a wide range of empirical facts.

First, it reproduces standard asset-pricing moments, including the equity premium, return volatility, and return predictability—moments that are notoriously difficult to match in production economies. Second, it generates realistic labor volatility at business-cycle frequencies, suggesting that belief-driven fluctuations in risk premia may help resolve the labor volatility puzzle of Shimer (2010). Third, it delivers a state-dependent relationship between belief dispersion and trading volume, with stronger effects in recessions. We show that extrapolative belief dynamics are crucial for this asymmetry and find strong empirical support for this prediction. Fourth, it matches observed forecast biases in productivity-growth expectations and the empirical link between survey expectations and objective risk premia. Fifth, the model generates objective and subjective risk premia that evolve in line with the evidence in Nagel and Xu (2023): objective risk premia vary countercyclically with business-cycle and valuation measures, whereas subjective risk premia are essentially acyclical. Finally, it replicates the documented association between labor income and expected returns (Santos and Veronesi, 2006).

Taken together, these results show that even in a parsimonious environment, empirically disciplined belief fluctuations can jointly account for observed patterns in asset prices, trading behavior, and business-cycle dynamics.

Theoretical results. Households differ in their beliefs about the evolution of total factor productivity (TFP). Distortions in beliefs about TFP growth propagate to expectations of dividends, returns, and other variables. Households have Epstein–Zin (EZ) preferences over *net consumption*—the difference between consumption and labor disutility—and choose portfolios and labor supply accordingly.

Despite incorporating EZ preferences and endogenous labor supply, the model remains tractable due to a convenient "as-if" representation: equilibrium allocations can be characterized as if households solved a standard portfolio problem without labor.

Because firms must choose labor one period in advance, hiring decisions depend on the *risk-neutral* expectation of productivity growth. When expected productivity growth is high, firms optimally expand employment. If these expectations are subsequently disappointed, profits are disproportionately affected because the wage bill is predetermined. This *operating leverage* channel endogenously generates dividends that are more volatile than consumption, consistent with the evidence in Campbell (2003).

The risk-neutral expectation of productivity growth depends crucially on *market beliefs*—a wealth-weighted average of household beliefs. When investors are relatively optimistic, their demand for risky assets rises, compressing risk premia. Lower risk premia induce firms to hire more workers, which—through the operating-leverage channel—increases return volatility. Higher volatility in turn affects the demand for risky assets and the risk premium. As a result, hiring decisions and asset prices are jointly determined in equilibrium.

In the special case of log utility, the model admits an analytic solution that permits a transparent taxonomy of belief systems. We classify beliefs into two broad categories. The first consists of *rank-preserving* beliefs, under which investors remain more optimistic or pessimistic across states. The second consists of *rank-alternating* beliefs, under which optimism switches across states, as in extrapolative and contrarian belief specifications.

This taxonomy yields several general results. First, only extrapolative beliefs generate amplification of business-cycle fluctuations in all states relative to rational expectations. Amplification arises because extrapolation increases the firm's effective discounting of bad states during booms, while reversing sharply in downturns.

Second, only extrapolative beliefs generate endogenous *risk build-up*: longer expansions lead to deeper contractions. During booms, extrapolative investors accumulate wealth. When the economy turns, these now-wealthy investors simultaneously become the most pessimistic agents, magnifying the downturn through the wealth-weighted SDF.

Third, belief heterogeneity generates trading volume through two first-order channels: passive rebalancing driven by differential wealth accumulation and active portfolio reallocation when relative beliefs change across states. Under rank-alternating beliefs, turnover rises most sharply when the economy enters a recession—a prediction we confirm empirically.

The taxonomy highlights the role of belief heterogeneity in macro-finance dynamics. Empirical evidence shows that risk premia are often compressed during credit booms that precede severe downturns (López-Salido, Stein and Zakrajšek, 2017; Krishnamurthy and Muir, 2017). In contrast, standard macro-finance models without belief heterogeneity predict rising risk premia ahead of downturns (e.g., Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2013). Moreover, our framework helps distinguish heterogeneous-belief models from models driven solely by heterogeneous risk aversion (e.g., Panageas, 2020). Heterogeneity in risk aversion cannot generate the endogenous risk build-up characteristic of extrapolative belief systems.

Quantitative Results. To complement the theoretical analysis, we conduct a quantitative evaluation of the model. To discipline beliefs, we use survey data on earnings expectations from I/B/E/S to estimate a belief process that captures heterogeneity across households and reflects empirically plausible belief fluctuations. We model both actual productivity growth and subjective beliefs as continuous Markov processes. Actual productivity growth follows an i.i.d. process, while subjective beliefs allow households to over- or under-react to productivity shocks through an overreaction parameter calibrated to match the degree of extrapolation observed in survey expectations. In addition, we introduce a non-fundamental *sentiment shock* that generates fluctuations in beliefs. The volatility of this shock is chosen to match the empirical ratio between the volatility of survey-based dividend-growth expectations and that of realized dividend growth.

In the model, belief distortions in all variables originate from distortions in expectations of productivity growth. To assess whether the model captures these distortions in the data, we estimate Coibion–Gorodnichenko (CG) regressions using productivity-growth expectations from the Survey of Professional Forecasters (SPF). We then run the same regression on simulated data from the calibrated model. The model reproduces the key bias pattern observed in the data, even though it is not directly calibrated to match productivity expectations.

Our quantitative analysis highlights the role of belief heterogeneity through a sequence of increasingly rich environments. We begin with a stripped-down economy featuring a representative rational investor in the endowment limit of the model. As is well known, this benchmark fails to generate realistic equity premia or stock market volatility. Introducing operating leverage—where firms hire before productivity is realized—matches the observed volatility of dividend growth. However, under rational expectations labor remains constant, and the model generates no employment volatility.

Next, we introduce subjective beliefs calibrated to match the empirical targets. Extrapolative beliefs alone generate realistic fluctuations in labor demand but still fail to produce a sufficiently large equity premium or adequate asset-price volatility. When belief heterogeneity is introduced— allowing rational and extrapolative investors to interact—wealth dynamics amplify fluctuations in risk premia and asset prices. In this environment, the model simultaneously matches the magnitude and volatility of equity returns as well as substantial volatility in employment. Hence, heterogeneous beliefs are crucial for simultaneously matching asset-pricing and business-cycle moments.

Beyond matching unconditional moments, the model also reproduces several important conditional patterns in the data. In particular, it is consistent with the Campbell–Shiller decomposition of returns, the limited predictability of subjective expectations, and the time-varying relationship between labor income and expected returns. Overall, the quantitative results show that belief fluctuations disciplined by survey data can jointly account for key asset-pricing and business-cycle regularities.

Literature Review. This paper connects to the literature initiated by Kydland and Prescott (1982) and Mehra and Prescott (1985), which uses complete-market benchmarks to study business cycle fluctuations and asset prices in environments where these two dimensions do not interact. A more recent strand emphasizes the role of time-varying risk premia in driving real cycles, particularly through the riskiness of labor hiring (e.g., Hall, 2017; Borovička and Borovičková, 2019; Di Tella and Hall, 2019; Kehoe, Lopez, Midrigan and Pastorino, 2019). We bridge this channel with the literature on heterogeneous beliefs and asset pricing.

Harrison and Kreps (1978) and Scheinkman and Xiong (2003) formalized how heterogeneous beliefs generate asset-market trade and price volatility through leverage. A key feature of these frameworks is their subtle departure from strict rational expectations: agents fully understand the structure of the economy and the risks it entails, yet disagree about the distribution of future fundamentals. Our framework adopts this approach.

Building on these foundations, a large literature studies how belief heterogeneity interacts with financial constraints to generate speculative bubbles (e.g., Geanakoplos, 2003; Fostel and Geanakoplos, 2008; Geanakoplos, 2010; Simsek, 2013a; Iachan, Nenov and Simsek, 2019; Barlevy, 2014, 2022). Closely related work examines how wealth redistribution across investors with different beliefs affects asset-price dynamics (Detemple and Murthy, 1994; Xiong and Yan, 2009; Kubler and Schmedders, 2012; Martin and Papadimitriou, 2022). This earlier literature largely abstracts from the real economy. Our paper can be viewed as a natural extension of this line of work, linking belief heterogeneity to real activity through the labor market.

A growing macro-finance literature embeds belief differences in production economies with market imperfections such as incomplete markets, financial frictions, or demand externalities. Some models link speculative bubbles to real activity through capital investment,[1] while others emphasize how heterogeneous beliefs interact with demand externalities, highlighting how wealth redistribution can amplify demand-driven recessions (Caballero and Simsek, 2020a,b; Guerreiro, 2022; Caramp and Silva, 2024). Simsek (2021) provides a comprehensive review.

Our paper fills a gap between these two literatures: models in which belief heterogeneity drives asset prices but has no real effects, and models in which beliefs affect output only through market imperfections. To our knowledge, there is no frictionless benchmark economy where belief heterogeneity itself generates business-cycle fluctuations. Developing such a benchmark is useful because it isolates the direct effects of beliefs on the business cycle, and provides a natural reference point for evaluating policies that address market imperfections.

Our theoretical results highlight the role of extrapolative beliefs, which align closely with the narratives and empirical patterns discussed above. This connects our work to models featuring diagnostic expectations (Gennaioli and Shleifer, 2010; Bianchi, Ilut and Saijo, 2024) and, more broadly, to the literature on subjective beliefs and macroeconomic dynamics (e.g., Eusepi and Preston, 2011; Angeletos, Collard and Dellas, 2018; Bordalo, Gennaioli and Shleifer, 2018a; Bhandari, Borovička and Ho, 2019; Falato, Kaboski and

[1] See Gilchrist, Himmelberg and Huberman (2005), Bolton, Scheinkman and Xiong (2006), Panageas (2005), and Buss, Dumas, Uppal and Vilkov (2016).

Xiao, 2024). Most closely related, Adam and Merkel (2019) show how extrapolative beliefs can jointly explain stock market and business cycle fluctuations, but in a setting with homogeneous beliefs. Our taxonomy clarifies that heterogeneity is essential for generating risk build-up and the turnover dynamics central to optimism-wave narratives.

Finally, our model generates sharp predictions for trading volume, a dimension that receives relatively little attention in equilibrium asset-pricing models.[2] Empirically, van der Beck, Bretscher and Fu (2025) show that the ratio of return volatility to portfolio turnover provides a lower bound on price impact that tightens with disagreement, highlighting the importance of trading activity for asset prices. Thesmar and Verner (2025) and McCarthy and Hillenbrand (2025) document that disagreement between sophisticated and extrapolative investors is associated with higher trading volume and explains a large share of stock price variation. Our model provides a structural setting linking disagreement, trading volume, asset prices, and employment within a unified framework.

Roadmap. Section 2 presents three empirical facts about investor beliefs. Section 3 lays out the model environment. Section 4 characterizes equilibrium allocations and asset prices. Section 5 derives the analytic solution under log utility and develops the taxonomy of belief systems. Section 6 presents the quantitative evaluation. Section 7 concludes.

2 Three motivating facts

This section discusses three facts about subjective expectations that motivate our model.

Fact 1. Expectations are volatile. As Shiller (1981) famously observed, stock dividends are too stable for rational movements in expectations to explain the large swings in asset prices. In contrast, recent survey evidence shows that *subjective expectations* of future cash flows are highly volatile, offering a potential explanation for the stock market's excess volatility (Bordalo, Gennaioli, La Porta and Shleifer, 2020a; De La O and Myers, 2021).

We illustrate the magnitude of the difference in the volatility of subjective and objective expectations by constructing an "objective" forecast—based on the statistical properties of dividends—against a survey-based "subjective" forecast. Specifically, we estimate an AR(1) process for quarterly aggregate dividend growth. The predicted one-year-ahead dividend growth from this AR(1) serves as our measure of objective expectations,

[2] Alvarez and Atkeson (2018) develop a general equilibrium model where trading volume becomes a pricing factor through idiosyncratic shocks to risk tolerance.

$\mathbb{E}^{obj}[\Delta d_{t,t+4}]$. We then compute the fraction of the total variance in realized dividend growth expectations relative to the variance of realized dividend growth.[3] We construct an analogous measure for subjective expectations, $\mathbb{E}^{sub}[\Delta d_{t,t+4}]$, by using the survey of analyst expectations from I/B/E/S. The differences in volatility are significant:

$$\frac{\text{Var}[\mathbb{E}^{obj}[\Delta d_{t,t+4}]]}{\text{Var}[\Delta d_{t,t+4}]} = 0.05, \qquad \frac{\text{Var}[\mathbb{E}^{sub}[\Delta d_{t,t+4}]]}{\text{Var}[\Delta d_{t,t+4}]} = 0.83.$$

Whereas objective expectations represent a negligible fraction of the total variation in dividend growth, subjective expectations are an order of magnitude more volatile.[4]

Fact 2. Expectations are heterogeneous. Fact 1 provides evidence of deviations from full-information rational expectations (FIRE). Our business cycle model emphasizes how heterogeneity in these expectations leads to business cycle amplification through asset markets. Critical to our theory is the presence of heterogeneity in beliefs. Concrete evidence is surveyed in Nagel and Xu (2023) reporting subjective return expectations across individual investors, CFOs, and professional forecasters. Table 1 summarizes their findings: The first column shows that realized future returns are positively predicted by the dividend-price ratio (a standard return predictor). In contrast, past returns do not appear significantly. In contrast, regressions for subjective expected returns reveal heterogeneous beliefs: individual investors place a positive and significant weight on past returns (consistent with extrapolation), professional forecasters behave in a contrarian way (a negative coefficient on past returns indicates interpolation), and CFOs lie somewhere in between. These results underscore the importance of heterogeneity in expectations among different agents.[5]

Belief heterogeneity is also reflected in trading behavior. Greenwood and Shleifer (2014) shows that subjective expectations correlate with mutual fund flows, and Giglio, Maggiori, Stroebel and Utkus (2021) documents a relationship between beliefs and portfolio choices. In Section 5.2, we construct an analyst-disagreement measure and show that it correlates with stock-market turnover, especially during recessions.

[3]See Appendix O1.1 for robustness to alternative objective-forecast specifications (including a VAR; Table O.2).

[4]The small fraction of variation explained by objective expectations is consistent with the estimated autocorrelation of cash flows in, for example, Bansal and Yaron (2004), and the findings in Cochrane (2008).

[5]See also Atmaz, Gulen, Cassella and Ruan (2023) for evidence of both extrapolative and contrarian behavior. Greenwood and Shleifer (2014) provides further evidence of extrapolative expectations from past stock return, also associated with the cyclicality of credit and leverage (López-Salido et al., 2017).

Table 1: Subjective return expectations regressions by type of investor.

Dependent var.:	**Objective**	**Individual**	**CFO**	**Professional**
Predictors				
D/P	5.83	-0.00	-0.41	0.42
(p-value)	(0.00)	(1.00)	(0.31)	(0.73)
R^e_{past}	0.36	0.87	0.30	-2.78
(p-value)	(0.71)	(0.02)	(0.29)	(0.01)

Fact 3. Expectations correlate with hiring decisions. The previous two facts focus on *subjective expectations* among groups of investors. Toward a business cycle theory, a natural follow-up question is whether these expectations are reflected in *real* economic outcomes. Indeed, Gennaioli, Ma and Shleifer (2016) shows that CFO expectations predict corporate capital investment, and Armona, Fuster and Zafar (2019) finds that expectations about home price appreciations influence construction. In this spirit, we present related evidence: investor expectations also predict firm-level *employment* decisions. To demonstrate this, we use two labor-related measures from Compustat Annual: the realized growth in staff expenses (payroll) and the employee count (workers). These two measures are standardized using the same procedure we apply to our earnings data.

We follow the approach of Gennaioli et al. (2016): we regress realized employment growth on the firm-level earnings growth expectations drawn from I/B/E/S. We control for past 12-month stock returns (constructed using individual stock prices from CRSP) and contemporaneous returns, clustering standard errors at the firm level. By including past returns, we aim to capture systematic shocks that might otherwise confound the relationship between expectations and subsequent employment growth.

Table 2 shows that earnings growth expectations predict realized employment growth in both total payroll and the number of workers. In light of Facts 1 and 2—namely, that earnings growth expectations exhibit greater volatility than actual outcomes and that CFOs behave differently from individual investors—these results provide reassuring evidence that firm employment decisions correlate with investor expectations.

Motivated by the three facts, we proceed to the model.

Table 2: I/B/E/S Expectations and Labor

Dependent Variable:	Payroll			Number of workers		
Model:	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	0.491***	0.456***	0.467***	0.177***	0.129***	0.158***
	(0.035)	(0.037)	(0.038)	(0.033)	(0.036)	(0.033)
Lag earning growth expectation	0.230***	0.187***	0.228***	0.277***	0.219***	0.276***
	(0.037)	(0.038)	(0.036)	(0.070)	(0.078)	(0.070)
12-month lag return		0.326***			0.443***	
		(0.084)			(0.101)	
12-month return			0.178**			0.134
			(0.079)			(0.097)
Fit statistics						
Observations	1,746	1,746	1,746	1,746	1,746	1,746
R^2	0.036	0.044	0.039	0.036	0.047	0.037

Standard errors clustered at the firm level in parentheses.
Significance: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

3 Model

We consider a two-state complete-markets economy with time indexed by $t \in \{0, 1, ..., \}$. The economy is populated by heterogeneous households that differ in their beliefs regarding TFP growth. Households hold (or issue) risk-free bonds and hold (or short-sell) shares of a single representative firm. Differences in beliefs induce a desire to lever up. The firm hires labor one period in advance before TFP is realized. Hiring in advance links labor demand with asset pricing.

The exogenous state. Total factor productivity A_t grows according to a two-state Markov process:

$$\frac{A_{t+1}}{A_t} = x_{t+1}, \tag{1}$$

where $x_{t+1} \in \{x_L, x_H\}, 0 < x_L < x_H$. Transition probabilities from state s to s' are denoted by $\{p_{ss'}\}$.

The firm. The representative firm produces a final good according to $A_{t+1}h_{t+1}^{\alpha}$, where labor h_{t+1} is hired in period t, prior to the realization of x_{t+1}. While firms hire and contract the wage W_{t+1} one period ahead, the wage bill is paid when production is finished.

The firm takes hours at the initial date h_0 as given and hires labor in subsequent periods to maximize its value using a stochastic discount factor (SDF), $\Lambda_{t,t+1}$:

$$Q_t = \max_{h_{t+1}} \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\pi_{t+1} + Q_{t+1}\right)\right]. \tag{2}$$

Q_t denotes the firm value and $\pi_{t+1} \equiv A_{t+1}h_{t+1}^{\alpha} - W_{t+1}h_{t+1}$ denotes the profit (dividend). Expectations are taken with respect to the transition probabilities $\{p_{ss'}\}$. Because markets are complete, there is a unique SDF for any fixed set of beliefs. Hence, there is unanimity regarding the firm's objective among shareholders, as discussed in Section 4. Beliefs affect employment decisions through their impact on the SDF.

Households. There is a finite number of infinite-lived households, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$ with masses $\{\mu_i\}$, $\sum_i \mu_i = 1$. Household i derives utility from consumption $C_{i,t}$ and disutility from working $h_{i,t}$. They have Epstein-Zin preferences over a GHH consumption-labor composite:

$$V_{i,t} = (1-\beta)U\left(C_{i,t} - \xi_t \frac{h_{i,t}^{1+\nu}}{1+\nu}\right) + \beta U\left(\mathcal{V}_{i,t}\right), \tag{3}$$

where $V_{i,t}$ denotes the utility level, β the discount factor, and ξ_t controls the labor disutility. $\mathcal{V}_{i,t}$ is the certainty-equivalent of future utility, $\mathcal{V}_{i,t} = \Psi^{-1}\left(\mathbb{E}_{i,t}\left[\Psi\left(U^{-1}(V_{i,t+1})\right]\right)$.

The labor disutility coefficient is indexed by *lagged* productivity, $\xi_t = \xi A_{t-1}$ and acts as a long-run wealth effect—as in Jaimovich and Rebelo (2009), this ensures that hours are stationary. We adopt the functional forms: $U(C) = \frac{C^{1-1/\psi}-1}{1-1/\psi}$ and $\Psi(Z) = \frac{Z^{1-\gamma}-1}{1-\gamma}$, where γ controls risk aversion and ψ the elasticity of intertemporal substitution (EIS).[6] As $U(\cdot)$ is defined over positive values, *net consumption*, $C_{i,t} - \xi_t \frac{h_{i,t}^{1+\nu}}{1+\nu}$, must be positive.

Household i has beliefs $\{p_{ss'}^i\}$ regarding TFP growth x_{t+1} from state s to s' and forms an expectation $\mathbb{E}_{i,t}$ accordingly. Households are dogmatic, as in Chen, Joslin and Tran (2012) and Simsek (2013b): they *agree to disagree* and do not learn from the views of others. Beliefs about productivity translate into beliefs about earnings and asset prices.

Household i chooses consumption $C_{i,t}$, hours $h_{i,t}$, firm shares $S_{i,t}$, and risk-free bonds $B_{i,t}$ to maximize (3) subject to a flow budget constraint

$$C_{i,t} + Q_t S_{i,t} + B_{i,t} = R_{r,t}Q_{t-1}S_{i,t-1} + R_{f,t-1}B_{i,t-1} + W_t h_{i,t}. \tag{4}$$

[6]CRRA preferences correspond to $\psi = \gamma^{-1}$. Given the endogenous labor supply, γ controls but does not coincide with the risk aversion for lotteries on financial wealth (see, e.g., Swanson 2018).

We denote human wealth by:

$$\mathcal{H}_{i,t} = \mathbb{E}_t \left[\sum_{k=1}^{\infty} \Lambda_{t,t+k} \left(W_{t+k} h_{i,t+k} - \xi_{t+k} \frac{h_{i,t+k}^{1+\nu}}{1+\nu} \right) \right]. \tag{5}$$

Human wealth is the present discounted value of future *net labor income*. Net labor income equals the labor earnings minus labor disutility. The present value is discounted using the SDF $\Lambda_{t,t+k} = \prod_{j=1}^{k} \Lambda_{t+j-1,t+j}$. The SDF is the same as the one used to value firms.

Households face a natural borrowing limit: $R_{r,t} Q_{t-1} S_{i,t-1} + R_{f,t-1} B_{i,t-1} + W_t h_{i,t} - \xi_t \frac{h_{i,t}^{1+\nu}}{1+\nu} \geq -\mathcal{H}_{i,t}$, given $S_{i,-1}$ and $B_{i,-1}$, where $R_{f,t}$ denotes the return on the risk-free bond and $R_{r,t} = \frac{Q_t + \pi_t}{Q_{t-1}}$ the return on stocks, the risky asset in this economy.[7]

Example I: Diagnostic expectations. We do not impose restrictions on beliefs. In particular, our formulation is flexible enough to capture different belief dynamics, including extrapolation or underreaction. For example, consider the case of *heterogeneous diagnostic expectations*. Diagnostic expectations correspond to the following belief structure:[8]

$$p_{sH}^{i} = \underbrace{p_{sH}}_{\text{rational beliefs}} \times \underbrace{\left(\frac{p_{sH}}{p_{-sH}} \right)^{\theta_i}}_{\text{belief distortion}} \times C,$$

where C is a normalizing constant, p_{sH} and p_{-sH} denote the probability of being in the high state next period if the current state is s and not s, respectively, and θ_i is a parameter controlling diagnosticity. If productivity growth is persistent under rational beliefs, i.e., $p_{HH} > p_{LH}$, then under diagnostic expectations, households overreact to news: households believe it is more likely they would remain in the high (low) state after switching to the high (low) state. If $p_{ss} = \frac{1+\rho}{2}$, as in Mehra and Prescott (1985), then $p_{ss}^{i} = \frac{1+\rho_i}{2}$, where $\rho_i > \rho$, and the (endogenous) subjective persistence parameter ρ_i is a function of θ_i.

Example II: Optimism and pessimism. Diagnostic expectations capture a form of extrapolation: households are optimistic in the boom and pessimistic in the bust. Alternatively, we could have a persistent degree of optimism/pessimism: $p_{sH}^{i} = p_{sH} + \Delta_i$ where households with $\Delta_i > 0$ would be optimistic at all states.

[7] This borrowing limit corresponds to the maximum households can borrow without violating the non-negativity of net consumption and is therefore never binding in equilibrium.

[8] Diagnostic expectations are defined using distorted probabilities of the form $\Pr^{i}(T = t \mid G) \propto \Pr(T = t \mid G) \left(\frac{\Pr(T=t|G)}{\Pr(T=t|-G)} \right)^{\theta_i}$ (see, e.g., Bordalo, Gennaioli and Shleifer 2018b). In our setting, the trait T is next period's state s', the group G is the current state s, and the reference group $-G$ is the alternative state $-s$.

Both the diagnostic expectations and persistent optimism formulations are one-parameter belief specifications. In general, beliefs can depend on two parameters, capturing a combination of these two cases: $p^i_{HH} = (1+\rho_i)/2 + \Delta_i$ and $p^i_{LL} = (1+\rho_i)/2 - \Delta_i$.[9]

SDF and equilibrium. The SDF can be inferred from the process of asset returns through no-arbitrage conditions:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\pi_{t+1} + Q_{t+1}}{Q_t} \right], \qquad 1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R_{f,t} \right]. \tag{6}$$

A competitive equilibrium is defined next.

Definition 1 (Competitive equilibrium)**.** *Given initial bond holdings and shares $\{B_{i,-1}, S_{i,-1}\}_{i=1}^{I}$ and hours h_0, a competitive equilibrium is a set of stochastic process for quantities $\{\{C_{i,t}, h_{i,t}, B_{i,t}, S_{i,t}\}_{i=1}^{I}, h_t\}$ and prices $\{W_t, R_{f,t}, Q_t\}$ such that*

(i) $\{h_{t+1}\}$ maximizes (2) given wages W_{t+1} and the SDF $\Lambda_{t,t+1}$.

(ii) $\{C_{i,t}, h_{i,t}, B_{i,t}, S_{i,t}\}$ maximizes (3) subject to (4) given prices, for $i \in \mathcal{I}$.

(iii) Markets for goods, labor, bonds, and shares clear

$$\sum_{i=1}^{I} \mu_i C_{i,t} = A_t h_t^{\alpha}, \qquad \sum_{i=1}^{I} \mu_i h_{i,t} = h_t, \qquad \sum_{i=1}^{I} \mu_i B_{i,t} = 0, \qquad \sum_{i=1}^{I} \mu_i S_{i,t} = 1.$$

We proceed to a characterization.

4 Characterization

We now present a recursive characterization of the equilibrium. The economy evolves with an exogenous state s and an aggregate endogenous state X. In our setting, X consists of the wealth distribution (i.e., the wealth shares of investors) and, given the timing of labor decisions, an aggregate variable that pins down firms' hiring choices. As we show below, this variable corresponds to the risk-neutral expectation of productivity growth.

All aggregate quantities and prices depend on (X, s). In particular, asset returns depend on the current state and the realization of next period's shock; for example, the return on the risky asset satisfies $R_{r,t+1} = R_r(X_t, s_t, s_{t+1})$.

[9] Notice that diagnostic beliefs coincide with rational beliefs in the case productivity growth is iid, $p_{sH} = p_{-sH}$. Our formulation allows for an arbitrary persistence under subjective beliefs, even in this case.

Households choose both portfolios and labor supply. In general, this joint choice makes closed-form characterizations difficult. However, under complete markets and GHH preferences, an *as-if* result applies: the household problem can be recast as a pure portfolio problem in an endowment economy, abstracting from labor decisions.[10]

To simplify exposition and notation, we focus on the case of linear labor disutility ($\nu = 0$), which is analogous to models with an extensive margin of labor supply—see, e.g., Rogerson (1988) and Ljungqvist and Sargent (2007). We return to the general case $\nu > 0$ in the quantitative analysis in Section 6.

Recursive representation. From the labor-supply condition and the assumed labor-supply elasticity, the wage rate satisfies

$$W_t = \xi_t. \tag{7}$$

Given $\xi_t = \xi A_{t-1}$, the wage is predetermined, introducing a form of real rigidity.

We now recast the household problem as a consumption–savings problem without labor. Using (7), the household flow budget constraint can be written in terms of *net consumption*, $\tilde{C}_{i,t} \equiv C_{i,t} - \xi_t h_{i,t}$, as

$$\tilde{C}_{i,t} + Q_t S_{i,t} + B_{i,t} = R_{r,t} Q_{t-1} S_{i,t-1} + R_{f,t-1} B_{i,t-1} \equiv N_{i,t}, \tag{8}$$

where $N_{i,t}$ denotes the household's *net worth*.

We can express the household problem as choosing net consumption $\tilde{C}_{i,t}$ and the share of post-consumption wealth invested in the risky asset, $\omega_{i,t} \equiv \frac{Q_t S_{i,t}}{N_{i,t} - \tilde{C}_{i,t}}$.

Problem 1. *The household's recursive problem is*

$$V_i(N, X, s) = \max_{\tilde{C}_i, \omega_i} (1-\beta) U(\tilde{C}_i) + \beta\, U(\mathcal{V}_i(N, X, s)), \tag{9}$$

subject to

$$N' = R_i(X, s, s')\,(N - \tilde{C}_i), \tag{10}$$

and $N' \geq 0$, *where the portfolio return is*

$$R_i(X, s, s') \equiv (1-\omega_i) R_f(X, s) + \omega_i R_r(X, s, s'), \tag{11}$$

[10] In Appendix B, we show our results also hold in an environment where labor is supplied by hand-to-mouth workers rather than by investors.

and

$$\mathcal{V}_i(N,X,s) = \Psi^{-1}\Big(\mathbb{E}_i\Big[\Psi\Big(U^{-1}(V_i(N',X',s'))\Big) \mid X,s\Big]\Big).$$

Although the original problem features endogenous labor supply, under complete markets and GHH preferences it admits an *as-if* representation as a standard portfolio problem. The case $\nu = 0$ simplifies the exposition by eliminating human wealth. In the case $\nu > 0$, the household's problem can still be recast in the form of Problem (1), but N_t has to be interpreted as *total wealth*, the sum of financial and human wealth, and the relevant risky asset becomes a claim on total surplus, i.e., output net of labor disutility. Appendix A.1 discusses the case $\nu > 0$ in detail.

Value function and consumption–wealth ratio. Given the homothetic structure of preferences, the value function admits the representation

$$V_i(N,X,s) = \frac{(v_i(X,s)\,N)^{1-1/\psi}}{1-1/\psi}, \tag{12}$$

where $v_i(X,s)$ denotes the *wealth multiplier*.

Define the consumption–wealth ratio as $c_{i,t} \equiv \frac{\tilde{C}_{i,t}}{N_{i,t}}$. The optimal consumption–wealth ratio is then given by

$$c_i(X,s) = (1-\beta)^{\psi}\, v_i(X,s)^{1-\psi}. \tag{13}$$

When the EIS is unity ($\psi = 1$), the consumption–wealth ratio is constant. In the general case, it varies with aggregate conditions through the wealth multiplier $v_i(X,s)$.

Euler equations and portfolio choice. The characterization of the modified household problem is standard and is provided in Appendix A.2. The Euler equations are given by:

$$1 = \mathbb{E}_i\left[\Lambda_i(X,s,s')\, R_j(X,s,s')\right], \qquad \text{for } j \in \{r,f\}, \tag{14}$$

where $\Lambda_i(X,s,s') = \beta^{\theta}\left(\frac{c_i(X',s')N'}{c_i(X,s)N}\right)^{-\frac{\theta}{\psi}} R_i(X,s,s')^{-(1-\theta)}$, given $\theta \equiv \frac{1-\gamma}{1-\psi^{-1}}$, denotes investor i's stochastic discount factor. In the case of separable preferences ($\theta = 1$), the SDF simplifies to the usual expression for CRRA preferences.

The Euler equations are the counterpart, under subjective beliefs, of the no-arbitrage conditions in Equation (6). In particular, individual SDFs satisfy the condition:

$$\Lambda_i(X,s,s') = \frac{p_{ss'}}{p^i_{ss'}}\,\Lambda(X,s,s'),$$

that is, each investor's SDF equals the common SDF scaled by the ratio of objective to subjective probabilities. This condition implies that all investors agree on the value of a unit of consumption in each state, despite disagreeing about the likelihood of those states.

Given objective probabilities, the economy-wide stochastic discount factor can be recovered from asset prices by inverting the no-arbitrage conditions:

$$\Lambda(X,s,L) = \frac{1}{p_{sL}} \frac{R_r^e(X,s,H)}{\Delta R_r(X,s)}, \qquad \Lambda(X,s,H) = -\frac{1}{p_{sH}} \frac{R_r^e(X,s,L)}{\Delta R_r(X,s)}, \tag{15}$$

where $R_r^e(X,s,s') \equiv R_r(X,s,s')/R_f(X,s) - 1$ denotes the excess return on the risky asset and $\Delta R_r(X,s) \equiv R_r(X,s,H) - R_r(X,s,L)$ is the difference in realized returns across states.

Although investors agree on state prices, disagreement about probabilities implies that they choose different portfolios. For instance, we show in Appendix A.3 that, under log utility, the portfolio share is given by:

$$\omega_i(X,s) = \frac{\mathbb{E}_i[R_r^e(X,s,s')]}{\text{Var}_{RN,s}(R_r^e(X,s,s'))} = \frac{p_{sH}^i}{|R_r^e(X,s,L)|} - \frac{p_{sL}^i}{R_r^e(X,s,H)}. \tag{16}$$

where $\text{Var}_{RN,s}(R_r^e(X,s,s'))$ is the excess returns variance under risk-neutral probabilities. The first equality shows that the portfolio share follows a version of the standard Merton (1969) formula. The second equality shows that more optimistic investors hold larger positions in the risky asset. Heterogeneity in beliefs therefore translates into heterogeneity in portfolio exposures and, in turn, into fluctuations in the wealth distribution. These fluctuations feedback into the economy-wide stochastic discount factor. We next link this stochastic discount factor to firms' hiring decisions.

Firm's problem. The firm's first-order condition for labor is

$$\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\alpha A_{t+1} h_{t+1}^{\alpha-1} - W_{t+1}\right)\right] = 0. \tag{17}$$

Given the timing of hiring, the marginal product of labor generally differs ex post from the realized wage, so that the labor wedge is non-zero state by state.

The firm's labor demand can be written as

$$\alpha \mathcal{L}_t\, h_{t+1}^{\alpha-1} = w_{t+1}, \tag{18}$$

where $w_{t+1} \equiv W_{t+1}/A_t$ denotes the detrended wage, and $\mathcal{L}_t$ is a shifter of labor demand.

The term $\mathcal{L}_t$ corresponds to the *risk-neutral expectation of productivity growth*:

$$\mathcal{L}_t = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\mathbb{E}_t[\Lambda_{t,t+1}]} \, x_{t+1} \right].$$

Productivity growth is evaluated under probabilities proportional to $p_{ss'}\Lambda(X, s, s')$, that is, under risk-neutral probabilities. Hence, the firm's optimality condition implies a zero expected labor wedge under these risk-adjusted expectations.

This measure of risk-neutral productivity growth plays a central role in the model. Because firms choose employment one period in advance, current labor depends on the lagged value $\mathcal{L}_{t-1}$, making it an endogenous aggregate state variable.

Its law of motion is given by

$$\mathcal{L}'(X,s) = \frac{p_{sL}\Lambda(X,s,L)}{p_{sL}\Lambda(X,s,L) + p_{sH}\Lambda(X,s,H)} \, x_L + \frac{p_{sH}\Lambda(X,s,H)}{p_{sL}\Lambda(X,s,L) + p_{sH}\Lambda(X,s,H)} \, x_H, \quad (19)$$

where $\mathcal{L}_t = \mathcal{L}'(X_t, s_t)$ is the risk-neutral expectation of productivity growth at time t.

Labor market equilibrium. Combining labor supply and labor demand yields equilibrium hours and detrended wages:

$$h(X,s) = \left(\frac{\alpha \mathcal{L}}{\xi} \right)^{\frac{1}{1-\alpha}}, \qquad w(X,s) = \xi, \quad (20)$$

where $\mathcal{L}$ denotes the lagged value of risk-neutral productivity growth in recursive notation, and is included as a component of the aggregate state vector X.

Given hours, realized profits depend on both risk-neutral productivity growth and realized productivity growth. Firm profits, detrended by lagged TFP, are given by

$$\pi(X,s) = x_s \left(\frac{\alpha \mathcal{L}}{\xi} \right)^{\frac{\alpha}{1-\alpha}} \left[1 - \alpha \frac{\mathcal{L}}{x_s} \right],$$

where x_s denotes the realization of productivity growth in state s.

Because labor is chosen in advance, profits may in principle be negative for some realizations of productivity. We assume $x_L > \alpha x_H$ to ensure that profits are positive.[11]

[11] Since the largest possible value of risk-neutral productivity growth is x_H, profits are positive whenever $x_s > \alpha \mathcal{L}$. The sufficient condition $x_L > \alpha x_H$ guarantees this holds in both states.

Labor decisions and the SDF. Let $\mathbb{E}_s[z_{s'}]$ and $\sigma_s[z_{s'}]$ denote the mean and standard deviation of a random variable $z_{s'}$ conditional on the state s, computed under the objective probabilities $p_{ss'}$. The next proposition provides a convenient representation of the risk-neutral expectation of productivity growth. All proofs are provided in Appendix A.

Proposition 1. *The risk-neutral expectation of next period productivity growth satisfies*

$$\mathcal{L}'(X,s) = \mathbb{E}_s[x_{s'}] - \underbrace{\frac{\mathbb{E}_s[R_r^e(X,s,s')]}{\sigma_s[R_r^e(X,s,s')]}}_{\text{price of risk}} \underbrace{\sigma_s[x_{s'}]}_{\text{quantity of risk}}. \tag{21}$$

Moreover, the Sharpe ratio of the risky asset admits the representation

$$\frac{\mathbb{E}_s[R_r^e(X,s,s')]}{\sigma_s[R_r^e(X,s,s')]} = \frac{\sigma_s[\Lambda(X,s,s')]}{\mathbb{E}_s[\Lambda(X,s,s')]}.$$

Proposition 1 shows that deviations of the risk-neutral expectation of productivity growth from the objective expectation are governed by two components. The first is a *quantity of risk*, given by the volatility of productivity growth, $\sigma_s[x_{s'}]$. The second is a *price of risk*, which measures the compensation required by financial markets per unit of aggregate risk and is proportional to the volatility of the stochastic discount factor.

Intuitively, when investors are more willing to bear risk, the price of risk and expected excess returns are low. Risk-neutral probabilities then place greater weight on high productivity-growth states, increasing the risk-neutral expectation of productivity growth. As a result, firms optimally take on more risk by hiring more workers.

The labor market equilibrium links fluctuations in employment directly to movements in the risk-neutral expectation of productivity growth. Moreover, Proposition 1 implies that if productivity growth is iid under the objective measure, so that both $\mathbb{E}_s[x_{s'}]$ and $\sigma_s[x_{s'}]$ are constant, all fluctuations in $\mathcal{L}'$ are driven by fluctuations in the price of risk.

This observation connects the labor volatility puzzle to asset pricing. Shimer (2010), for example, studies a search model in which the risk-neutral expectation of productivity growth is constant, implying no fluctuations in employment and giving rise to the labor (or unemployment) volatility puzzle.[12] In contrast, asset-pricing evidence shows that the price of risk is highly volatile (e.g., Hansen and Jagannathan, 1991; Cochrane, 2011). In our setting, generating large fluctuations in employment therefore requires generating a large and volatile equity premium: the labor volatility and equity volatility puzzles are two manifestations of the same underlying phenomenon in our setting.

[12] Appendix C provides a formal mapping between our labor-timing assumption and a generalized search-and-matching environment.

Connection with search models. Our setting is closely related to search-and-matching models. Appendix C shows that both our baseline environment and the model in Shimer (2010) can be embedded in a more general search-and-matching framework. In that framework, firms must engage in recruitment before hiring takes place, which naturally implies that labor is predetermined. Our model corresponds to the limit in which recruiting costs vanish and firms have all bargaining power. Abstracting from recruiting costs delivers a particularly tractable benchmark, while preserving the key feature that firms cannot adjust labor contemporaneously to shocks.

The operating leverage channel. If labor could be adjusted after productivity is realized, the labor share would be constant. Firms would then scale labor input one-for-one with productivity, so that revenues, wages, and profits would move proportionally. In contrast, the timing of hiring plays a central role in shaping the dividend process and the connection between asset prices and labor volatility in our setting.

With labor chosen in advance, dividend (or profit) growth satisfies

$$\frac{x_s\,\pi(X',s')}{\pi(X,s)} = x_s \frac{x_{s'} - \alpha\mathcal{L}'(X,s)}{x_s - \alpha\mathcal{L}} \left(\frac{\mathcal{L}'(X,s)}{\mathcal{L}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{22}$$

In the endowment limit, obtained by setting $\alpha = 0$, dividend growth coincides with productivity growth, and dividend volatility equals aggregate consumption volatility. By contrast, when labor is a quasi-fixed factor, dividends are endogenously riskier than aggregate consumption.

The amplification of volatility due to quasi-fixed factors is known as the *operating leverage channel*.[13] Formally, the conditional volatility of dividend growth can be written as

$$\underbrace{\sigma_s\left[\frac{x_s\,\pi(X',s')}{\pi(X,s)}\right]}_{\substack{\text{dividend growth}\\ \text{volatility}}} = \underbrace{\frac{x_s}{x_s - \alpha\mathcal{L}}}_{\substack{\text{operating}\\ \text{leverage}}} \times \underbrace{\sigma_s\left[\frac{x_{s'}\,h(X',s')^{\alpha}}{h(X,s)^{\alpha}}\right]}_{\substack{\text{consumption growth}\\ \text{volatility}}}. \tag{23}$$

The literature defines operating leverage as revenues minus current variable costs, divided by profits. In our setting, this corresponds to

$$\frac{x_s h(X,s)^{\alpha}}{x_s h(X,s)^{\alpha} - w(X,s)h(X,s)} = \frac{x_s}{x_s - \alpha\mathcal{L}} > 1.$$

Equation (23) shows that operating leverage endogenously makes dividends more

[13] The relationship between risk and operating leverage was first emphasized by Lev (1974).

volatile than consumption, a feature that is absent in the endowment limit but consistent with the data (e.g., Campbell, 2003).

Operating leverage also induces mean reversion in dividends. Even if productivity growth is iid and the risk-neutral expectation of productivity growth is constant, expected dividend growth varies with the current state:

$$\mathbb{E}_s\left[\frac{x_s\,\pi(X',s')}{\pi(X,s)}\right] = x_s\frac{\mathbb{E}[x_{s'}]-\alpha\mathcal{L}}{x_s-\alpha\mathcal{L}}. \tag{24}$$

Following a negative productivity shock, dividends are therefore expected to recover.

Because operating leverage increases with the risk-neutral expectation of productivity growth, this channel links dividend volatility to movements in $\mathcal{L}$, which in turn respond to fluctuations in beliefs. As a result, waves of optimism can generate an endogenous buildup of risk.

The timing of hiring provides a parsimonious way of capturing the operating leverage channel. It delivers labor costs that are smoother than productivity and a countercyclical labor share, two features that are central to operating leverage.[14] Donangelo, Gourio, Kehrig and Palacios (2019) provide direct evidence that these conditions hold in the data and show that the sensitivity of profits to productivity shocks increases with the labor share, consistent with the mechanism in Equation (23).

Discussion: the firm's objective. When labor is chosen in advance, hiring is inherently risky. As a result, evaluating the firm's hiring decision requires specifying the stochastic discount factor used to value future payoffs. This raises the question of which stochastic discount factor is relevant for firms.

Under complete markets, this question has a simple answer. Any pair of beliefs and stochastic discount factor that correctly prices traded assets delivers the same firm value. Consequently, the firm's value-maximizing hiring decision can be characterized using the economy-wide stochastic discount factor derived from asset prices. Importantly, this decision is independent of shareholders' beliefs: all investors agree on the hiring choice that maximizes firm value.

For expositional convenience, we assume firms compute expectations using objective probabilities. However, any alternative managerial belief would lead to the same choice, provided the firm's objective is to maximize shareholder value under complete markets.[15]

[14] Quasi-fixed labor is not necessary for operating leverage to arise. Similar mechanisms operate in models with implicit contracts (Danthine and Donaldson, 2002), labor adjustment costs (Belo, Lin and Bazdresch, 2014), and wage rigidities (Favilukis and Lin, 2016).

[15] How firms' objectives should be specified away from complete markets remains an open question; see,

Markov equilibrium. In addition to $\mathcal{L}$, the distribution of wealth across investors is an endogenous state variable. Denote the share of wealth held by investor $i \in \mathcal{I}$ as

$$\eta_{i,t} \equiv \frac{\mu_i N_{i,t}}{\sum_{j=1}^{I} \mu_j N_{j,t}}$$

The wealth shares evolve according to

$$\eta_i'(X,s,s') = \frac{\tilde{\eta}_i(X,s)\, R_i(X,s,s')}{\sum_{j=1}^{I} \tilde{\eta}_j(X,s)\, R_j(X,s,s')}, \tag{25}$$

where $\tilde{\eta}_i(X,s) \equiv \frac{\eta_i \left(1-c_i(X,s)\right)}{\sum_{j=1}^{I} \eta_j \left(1-c_j(X,s)\right)}$.

Given $\mathcal{L}$, the wealth shares $\{\eta_i\}_{i=1}^{I-1}$, and the realization of productivity growth s, all aggregate variables are determined. We stack the endogenous state variables into

$$X \equiv \left(\mathcal{L}, \{\eta_i\}_{i=1}^{I-1}\right)$$

and study equilibria that are Markov in (X,s).

Definition 2 (Markov equilibrium)**.** *A Markov equilibrium in (X,s) consists of functions for the price–dividend ratio $q(X,s)$, the risk-free rate $R_f(X,s)$, hours $h(X,s)$, wages $w(X,s)$, wealth multipliers $\{v_i(X,s)\}_{i=1}^{I}$, and policy functions $\{c_i(X,s), \omega_i(X,s)\}_{i=1}^{I}$, such that:*

(i) Given prices, the value functions and policy functions solve the household problem (9). The consumption–wealth ratio and portfolio shares are given in Appendix A.2 and A.3.

(ii) Labor hours and wages satisfy the labor market equilibrium conditions (20).

(iii) Goods and asset markets clear:

$$\sum_{i=1}^{I} \eta_i\, c_i(X,s) = \frac{1}{q(X,s)+1}, \qquad \sum_{i=1}^{I} \tilde{\eta}_i(X,s)\, \omega_i(X,s) = 1, \tag{26}$$

where $q(X,s)$ is the price-dividend ratio.

The law of motion for the state vector X is given by (21) and (25).

For the remainder of the paper, we work directly with this Markovian representation.

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for example, Geanakoplos, Magill, Quinzii and Dreze (1990).

5 Analytic Solution: Log Utility Case

In this section, we consider the case of log utility, $\psi = \gamma = 1$. This special case allows us to characterize in closed form how beliefs affect asset prices and labor demand in equilibrium. We use this analytic solution to highlight the mechanisms through which belief heterogeneity amplifies business-cycle fluctuations and how it affects trading patterns. We return to more general preferences in the quantitative analysis.

5.1 Beliefs, asset prices, and labor demand

Asset prices. Under log utility, both the consumption–wealth ratio and the price–dividend ratio are constant:

$$c_i(X,s) = 1-\beta, \qquad q(X,s) = \frac{\beta}{1-\beta}, \tag{27}$$

which follows directly from Equations (13) and (26). As a result, the return on the risky asset is proportional to dividend growth:

$$R_{r,t+1} = \frac{1+q_{t+1}}{q_t}\frac{\pi_{t+1}}{\pi_t} = \beta^{-1}\frac{\pi_{t+1}}{\pi_t}.$$

Using the expression for dividend growth derived in Equation (22), the gross return on the risky asset can be written as

$$R_r(X,s,s') = \beta^{-1}\, x_s \frac{x_{s'} - \alpha\mathcal{L}'(X,s)}{x_s - \alpha\mathcal{L}} \left(\frac{\mathcal{L}'(X,s)}{\mathcal{L}}\right)^{\frac{\alpha}{1-\alpha}}.$$

The risk-free rate equals the risk-neutral expectation of the return on the risky asset. Using the fact that $\mathcal{L}'(X,s)$ is the risk-neutral expectation of $x_{s'}$, we obtain

$$R_f(X,s) = \beta^{-1}\, x_s \frac{(1-\alpha)\mathcal{L}'(X,s)}{x_s - \alpha\mathcal{L}} \left(\frac{\mathcal{L}'(X,s)}{\mathcal{L}}\right)^{\frac{\alpha}{1-\alpha}}.$$

The next proposition summarizes the relationship between the risk-free rate, the risk premium, and the risk-neutral expectation of productivity growth.

Proposition 2 (Risk-free rate and risk premium)**.** *Given $\mathcal{L}'(X,s)$:*

(i) The risk-free rate satisfies

$$R_f(X,s) = (1-\alpha)\frac{x_s}{\beta}\frac{\mathcal{L}'(X,s)^{\frac{1}{1-\alpha}}}{x_s\mathcal{L}^{\frac{\alpha}{1-\alpha}} - \alpha\mathcal{L}^{\frac{1}{1-\alpha}}}. \tag{28}$$

(ii) The conditional risk premium is

$$\mathbb{E}_s\left[R_r^e(X,s,s')\right] = \frac{1}{1-\alpha}\frac{\mathbb{E}_s[x_{s'}] - \mathcal{L}'(X,s)}{\mathcal{L}'(X,s)}, \tag{29}$$

which is decreasing in $\mathcal{L}'(X,s)$.

Proposition 2 shows that asset prices are fully pinned down by the current state (X,s) and the risk-neutral expectation of productivity growth, $\mathcal{L}'(X,s)$. Intuitively, a higher $\mathcal{L}'(X,s)$ reflects either higher expected productivity growth or lower aggregate risk, both of which raise the risk-free rate.

At the same time, interest rates are decreasing in current productivity growth x_s. As discussed in Section 4, the timing of hiring induces mean reversion in dividends: following a negative productivity shock, expected dividend growth is high, raising equilibrium interest rates.

Finally, the risk premium is increasing in the wedge between physical and risk-neutral expectations of productivity growth. Conditional on $\mathbb{E}_s[x_{s'}]$, a lower $\mathcal{L}'(X,s)$ corresponds to a larger risk adjustment and therefore a higher expected excess return.

A demand–supply representation. Proposition 3 implies that firm's labor demand, as summarized by risk-neutral expected growth, and asset prices are tightly connected. Next, we solve for the equilibrium $\mathcal{L}'(X,s)$ by translating the market clearing conditions into a demand and supply system.

This demand and supply of risk representation is a convenient transformation of the asset-market clearing condition (26): multiply both sides of the condition for risky assets by $\sigma_s[R_r(X,s,s')]$, and define $\overline{\sigma}_s(X,s) \equiv \sum_{i=1}^{I}\eta_i\omega_i(X,s)\sigma_s[R_r(X,s,s')]$, to obtain:

$$\underbrace{\overline{\sigma}_s(X,s)}_{\text{demand for risk}} = \underbrace{\sigma_s[R_r(X,s,s')]}_{\text{supply of risk}}.$$

The left-hand side of the expression is the *demand for risk*, corresponding to the households' risk exposure. The right-hand side represents the *supply of risk*, that is, the volatility of assets in positive net supply. The following proposition expresses the demand and supply of risk as a function of $\mathcal{L}'(X,s)$.

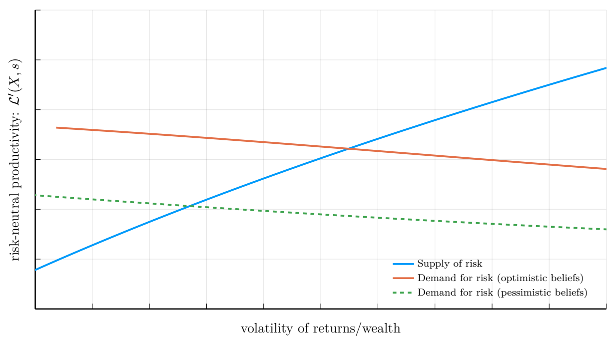


Figure 1: Market for risky assets

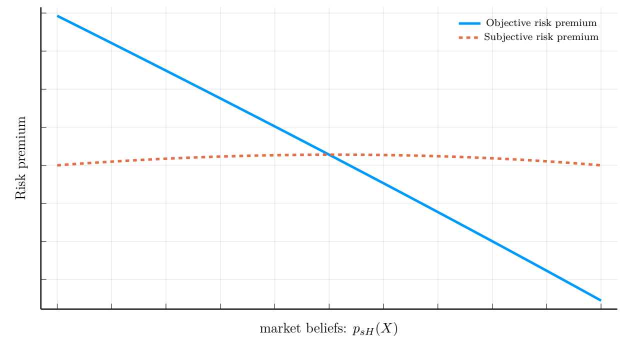


Figure 2: Beliefs and risk premia

Proposition 3 (The demand and supply of risk)**.** *Suppose* $x_L > \alpha x_H$. *Then,*

(i) The supply of risk is

$$\sigma_s[R_r(X,s,s')] = \frac{\sigma_s[x_{s'}]}{\beta} \frac{x_s}{x_s - \alpha \mathcal{L}} \frac{\mathcal{L}'(X,s)^{\frac{\alpha}{1-\alpha}}}{\mathcal{L}^{\frac{\alpha}{1-\alpha}}}. \tag{30}$$

(ii) The demand for risk is

$$\overline{\sigma}_s(X,s) = \sigma_s[x_{s'}] R_f(X,s) \left[\frac{\overline{p}^m_{sH}(X,s)}{\mathcal{L}'(X,s) - x_L} - \frac{\overline{p}^m_{sL}(X,s)}{x_H - \mathcal{L}'(X,s)} \right], \tag{31}$$

where $\overline{p}^m_{ss'}(X,s) \equiv \sum_{i=1}^{I} \eta_i p^i_{ss'}$.

Proposition 3 implies that the supply of risk increases while the demand for risk decreases with the $\mathcal{L}'(X,s)$, as shown in Figure 1. Recall return volatility is proportional to dividend volatility. The supply of risk increases with $\mathcal{L}'(X,s)$ because dividend volatility increases with $\mathcal{L}'(X,s)$ due to the operating leverage channel. As firms hire more labor, profits are more exposed to productivity shocks, leading to higher volatility.

The demand for risk is decreasing in $\mathcal{L}'(X,s)$. As shown in Proposition 2, the risk premium is inversely related to $\mathcal{L}'(X,S)$, so investors are more willing to hold risky assets when the risk premium is high. The demand for risk is also a function of *market beliefs*, a weighted average of investors' beliefs, $\overline{p}^m_{ss'}(X,s)$. As market beliefs become more optimistic, investors are willing to hold more risky assets for a given risk premium.

Figure 1 illustrates how we can exploit this demand system representation to explain the effects of changes in market beliefs: When market beliefs become more pessimistic, there is a decline in the demand for risk, which leads to a decrease in the risk-neutral expected growth and, ultimately, a drop in employment.

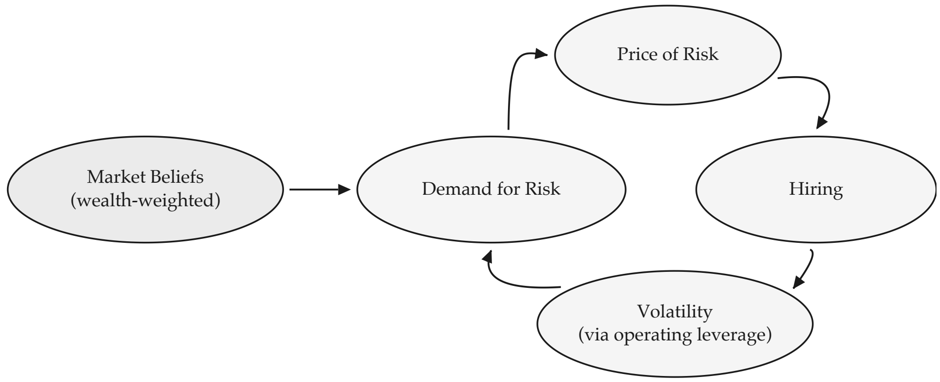


Figure 3: Belief-driven equilibrium feedback between asset prices and hiring

The following corollary formally demonstrates this by presenting the solution to the equilibrium risk-neutral expected growth as a function of market beliefs.

Corollary 1 (Labor demand factor)**.** *The risk-neutral expectation of productivity growth* $\mathcal{L}'(X,s)$ *is given by:*

$$\mathcal{L}'(X,s) = \Gamma(X,s) - \sqrt{\Gamma(X,s)^2 - \frac{1}{\alpha}x_L x_H}.$$

where $\Gamma(X,s)$ *is a function of market beliefs:*

$$\Gamma(X,s) \equiv \frac{\overline{p}^m_{sH}(X,s)}{2}\left(\frac{x_L}{\alpha} + x_H\right) + \frac{\overline{p}^m_{sL}(X,s)}{2}\left(x_L + \frac{x_H}{\alpha}\right).$$

The key implication of Corollary 1 is that the risk-neutral expectation of productivity growth is *entirely* determined by market beliefs. As a result, fluctuations in market beliefs translate directly into fluctuations in labor demand through firms' hiring decisions. Moreover, because market beliefs are a wealth-weighted average of individual beliefs, they depend on the distribution of wealth across investors. Changes in the wealth distribution therefore affect production through their impact on aggregate beliefs, providing a direct link between asset prices, labor demand, and the evolution of wealth inequality.

The feedback between asset prices and hiring decisions. Figure 3 summarizes the feedback loop linking market beliefs, risk pricing, hiring, and volatility. A shift in the wealth distribution that increases aggregate optimism raises the demand for risky assets and compresses the price of risk. Proposition 1 shows that a lower price of risk increases labor demand and stimulates hiring. Through the operating leverage channel, higher hiring makes dividends more exposed to aggregate shocks, increasing volatility. Higher volatility in turn affects the demand for risk and thus the price of risk, closing the loop.

This mechanism highlights how asset prices and hiring decisions are jointly determined in equilibrium and how belief-driven fluctuations propagate between the financial and real sides of the economy.

The real effects of market beliefs. The joint determination of asset prices and labor demand hinges critically on the timing of hiring decisions. To make this point explicit, the next proposition shows that market beliefs have no real effects when firms are allowed to adjust labor after productivity is realized.

Proposition 4 (Real effects of market beliefs)**.** *Suppose firms choose hours after observing current productivity. Then hours, detrended profits, and the return on the risky asset are independent of market beliefs:*

$$h(X,s) = \left(\frac{\alpha x_s}{\xi}\right)^{\frac{1}{1-\alpha}}, \qquad \pi(X,s) = (1-\alpha)\left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} x_s^{\frac{1}{1-\alpha}}, \qquad R_r(X,s,s') = \frac{x_s}{\beta}\left(\frac{x_{s'}}{x_s}\right)^{\frac{1}{1-\alpha}}. \tag{32}$$

Fluctuations in market beliefs affect the risk-free rate and the risk premium, but they do not affect real variables or the expected return and volatility of the risky asset under the objective measure.

Proposition 4 establishes a form of *macro–finance separation* (Tallarini, 2000): absent a hiring friction, belief-driven fluctuations in risk premia have asset-pricing implications but no real effects on employment or output. In terms of Figure 3, the link from the price of risk to hiring—and thus from hiring to volatility—is severed.

The timing assumption on hiring breaks this separation. When labor must be chosen in advance, hiring becomes risky from the firm's perspective. As discussed in the context of Figure 3, an increase in optimism compresses the risk premium and raises the valuation of future payoffs, which—through the firm's optimality condition—induces higher employment. Belief-driven fluctuations in risk premia therefore have real effects.

Absent fluctuations in market beliefs, employment would be constant in this economy, consistent with the labor volatility puzzle documented by Shimer (2010). Section 6 explores the quantitative implications of this mechanism by assessing how much labor volatility the model can generate under empirically realistic fluctuations in beliefs.

Beliefs and risk premia. Fluctuations in market beliefs generate movements in both risk premia and labor demand. Why does optimism lead to changes in risk premia?

To answer this question, it is useful to distinguish between *subjective* and *objective* risk premia. With a representative investor, the subjective risk premium is simply given by:

$$\mathbb{E}_{i,s}\left[R^e_r(X,s,s')\right] \approx \text{Var}_{i,s}\left[R^e_r(X,s,s')\right]. \tag{33}$$

Thus, absent changes in perceived volatility, shifts in beliefs about average productivity growth have little effect on the subjective risk premium in this case.

In contrast, the objective risk premium responds to differences in first moments. Using the equilibrium expressions derived above, it can be written as

$$\mathbb{E}_s\left[R^e_r(X,s,s')\right] = \mathbb{E}_{i,s}\left[R^e_r(X,s,s')\right] - \frac{\mathbb{E}_{i,s}[x_{s'}] - \mathbb{E}_s[x_{s'}]}{(1-\alpha)\mathcal{L}'(X,s)}. \tag{34}$$

When investors become optimistic, they expect higher future dividends and bid up asset prices. From the perspective of a rational investor, higher prices imply lower expected returns going forward, compressing the objective risk premium.

Figure 2 illustrates this mechanism. An increase in optimism, captured by a rise in market beliefs $\overline{p}^m_{sH}(X,s)$, substantially reduces the objective risk premium, while having a more muted effect on the subjective risk premium. This pattern is consistent with empirical evidence documenting large movements in expected returns with relatively stable subjective risk perceptions (see, e.g., De La O and Myers 2021 and Nagel and Xu 2023).

The evolution of market beliefs. We have seen how market beliefs are determined by the wealth distribution. Next, we show how the wealth distribution evolves over time:

Proposition 5 (Wealth dynamics)**.** *The wealth share of investor $i \in \mathcal{I}$ evolves as:*

$$\eta'_i(X,s,s') = \eta_i \frac{p^i_{ss'}}{\overline{p}^m_{ss'}(X,s)}. \tag{35}$$

Proposition 5 shows that the wealth of household i increases when she assigns a greater likelihood to the realized state than market beliefs.

Next, we present a taxonomy of belief types. In particular, we discuss how different belief structures classified according to our taxonomy impact business cycles.

5.2 Belief taxonomy and business-cycle properties

Although a large literature documents departures from full-information rational expectations (Greenwood and Shleifer, 2014; Coibion and Gorodnichenko, 2015), there is no consensus on how precisely belief dynamics deviate from rational expectations.[16] This section classifies belief structures and shows how each maps into distinct business-cycle implications. We organize the analysis around the following taxonomy.

Definition 3 (Taxonomy of beliefs)**.** *Household i is* optimistic *(*pessimistic*) relative to a benchmark belief o in state s if $p^i_{sH} > p^o_{sH}$ ($p^i_{sH} < p^o_{sH}$). We further distinguish two classes of belief dynamics:*

(i) ***Rank-preserving beliefs****: household i is optimistic or pessimistic relative to o in all states.*

(ii) ***Rank-alternating beliefs****: household i is optimistic relative to o in one state but pessimistic in the other.*

The benchmark belief o can represent either rational expectations or the beliefs of another household. This taxonomy allows us to characterize how different belief structures affect both the amplitude and the phase of business-cycle fluctuations.

Homogeneous beliefs. With homogeneous beliefs, the risk-neutral expectation of productivity growth, $\mathcal{L}'(X, s)$, depends only on current productivity. Therefore, the models lacks an internal propagation mechanism, as there is no variation in hours in the absence of changes in productivity growth. Moreover, if investors believe productivity growth is i.i.d., so that $p^i_{LH} = p^i_{HH}$, then $\mathcal{L}'(X, s)$ and, consequently, hours are constant over time.

Away from i.i.d. beliefs, homogeneous beliefs can either amplify or dampen fluctuations. Figure 4 illustrates this point by simulating four periods of recession within a twenty-period interval for five belief specifications: rational expectations ($p^i_{ss'} = p_{ss'}$ for all s, s'); two rank-preserving cases—always optimistic ($p^i_{sH} > p_{sH}$ for all s) and always pessimistic ($p^i_{sH} < p_{sH}$ for all s); and two rank-alternating cases—*extrapolative* beliefs, defined as optimistic in booms and pessimistic in recessions, and *contrarian* beliefs, defined as pessimistic in booms and optimistic in recessions.

Optimistic rank-preserving beliefs amplify expansions but dampen recessions relative to rational expectations, while pessimistic rank-preserving beliefs have the opposite effect. Thus, rank-preserving beliefs generate state-dependent amplification.

Rank-alternating beliefs operate differently. Under extrapolative beliefs, optimism in booms and pessimism in recessions magnify the amplitude of the cycle. By contrast, contrarian beliefs dampen fluctuations by reducing labor demand in booms and supporting

[16] Some studies emphasize over-extrapolation (Bordalo, Gennaioli, Ma and Shleifer, 2020b; Fuster, Laibson and Mendel, 2010), as in models with diagnostic expectations (Bordalo et al., 2018b) or fading memory (Nagel and Xu, 2022). Others emphasize under-reaction, such as sticky information (Mankiw and Reis, 2010), cognitive discounting (Gabaix, 2019), and level-k thinking (Farhi and Werning, 2019).

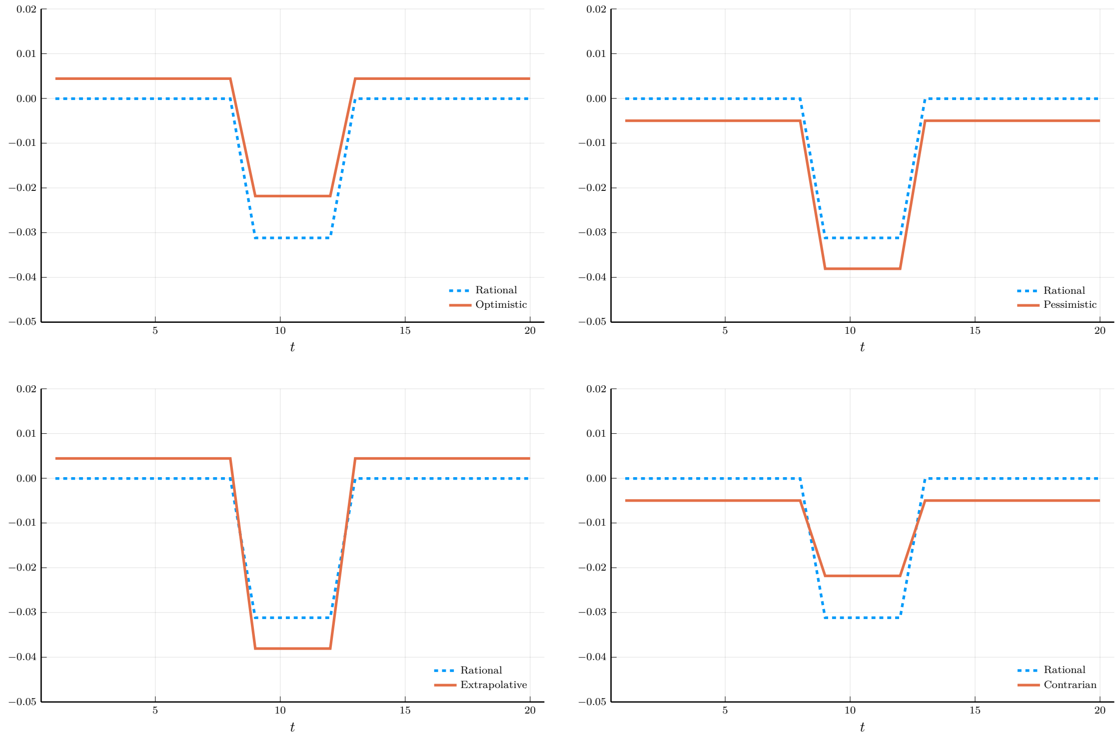


Figure 4: Homogeneous beliefs

Note: examples of cycles (four periods of bad shocks with sixteen periods of expansions) for log labor hours with homogeneous beliefs: rational, always optimistic (top-left panel), always pessimistic (top-right panel), extrapolative (bottom-left panel), and contrarian (bottom-right panel) beliefs.

it in recessions. Overall, extrapolative beliefs are the belief structure that most strongly amplifies business-cycle fluctuations relative to rational expectations.

Heterogeneous beliefs. Next, we investigate the role of belief heterogeneity. It is well known that RBC models typically have very weak internal propagation mechanisms (Cogley and Nason, 1995). With heterogeneous beliefs and labor chosen one period in advance, the model features an important internal propagation mechanism: the wealth distribution, and market beliefs, evolve over time even without changes in the productivity state. From Proposition 5, optimists accumulate wealth during booms and lose wealth during downturns. This observation is key to understanding the model dynamics.

Corollary 2. *If beliefs are heterogeneous, then:*

- *Conditional on remaining in the same state, the risk-neutral expectation of productivity growth $\mathcal{L}$ increases (decreases) over time in the high-growth (low-growth) state.*
- *Consider an initial state (X, H) and a transition from H to L at some future date. Then:*

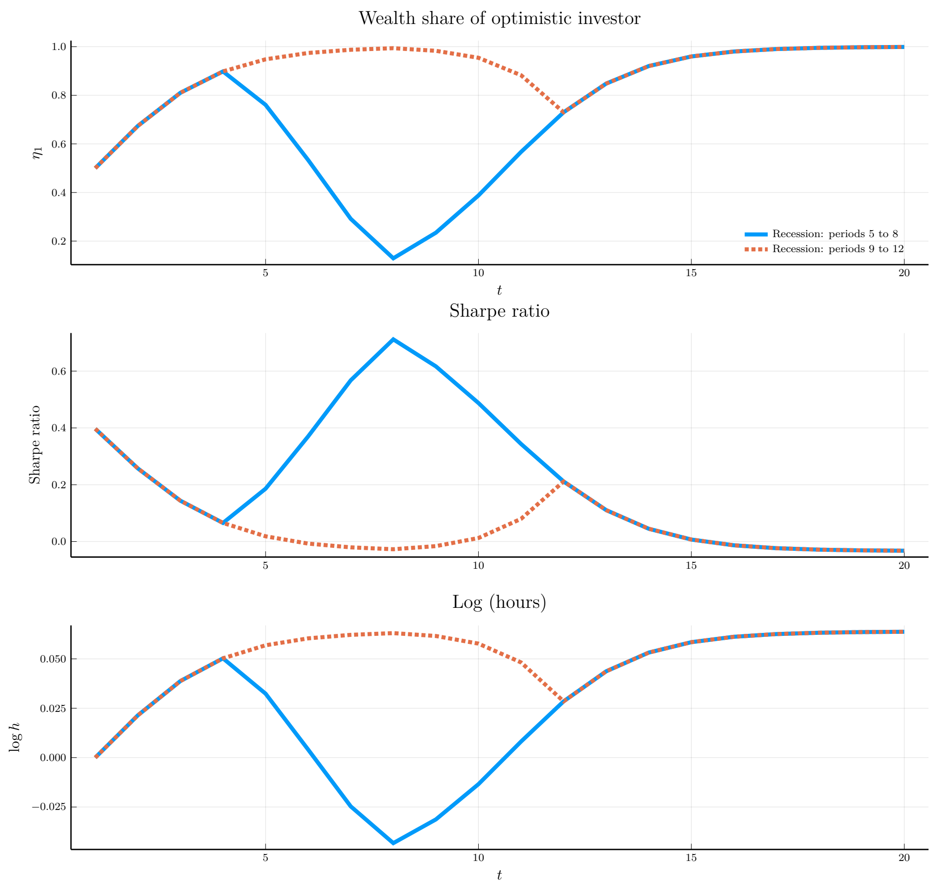


Figure 5: Heterogeneous beliefs: optimistic vs pessimistic

Note: examples of cycles (four periods of bad shocks with sixteen periods of expansions) with heterogeneous beliefs: $I = 2$ investors, always optimistic and always pessimistic. The top panel shows the wealth share of the optimistic investor, the middle panel shows the Sharpe ratio (under objective beliefs), and the bottom panel shows log labor hours.

- *If beliefs are* rank-preserving, *a longer boom leads to a smaller subsequent decline in output.*
- *If beliefs are* rank-alternating, *a longer boom leads to a larger subsequent decline in output.*

With belief heterogeneity, the economy evolves even in the absence of changes in productivity growth. During booms, optimistic investors accumulate wealth, causing market beliefs to become more optimistic relative to rational expectations and increasing labor demand. The opposite occurs during prolonged low-growth phases, when pessimistic investors gain wealth and market beliefs tilt toward pessimism.

Figure 5 illustrates this mechanism for the case of rank-preserving beliefs. The figure shows simulations with two investors, one optimistic and one pessimistic in all states. The top panel displays the optimist's wealth share, the middle panel the objective Sharpe

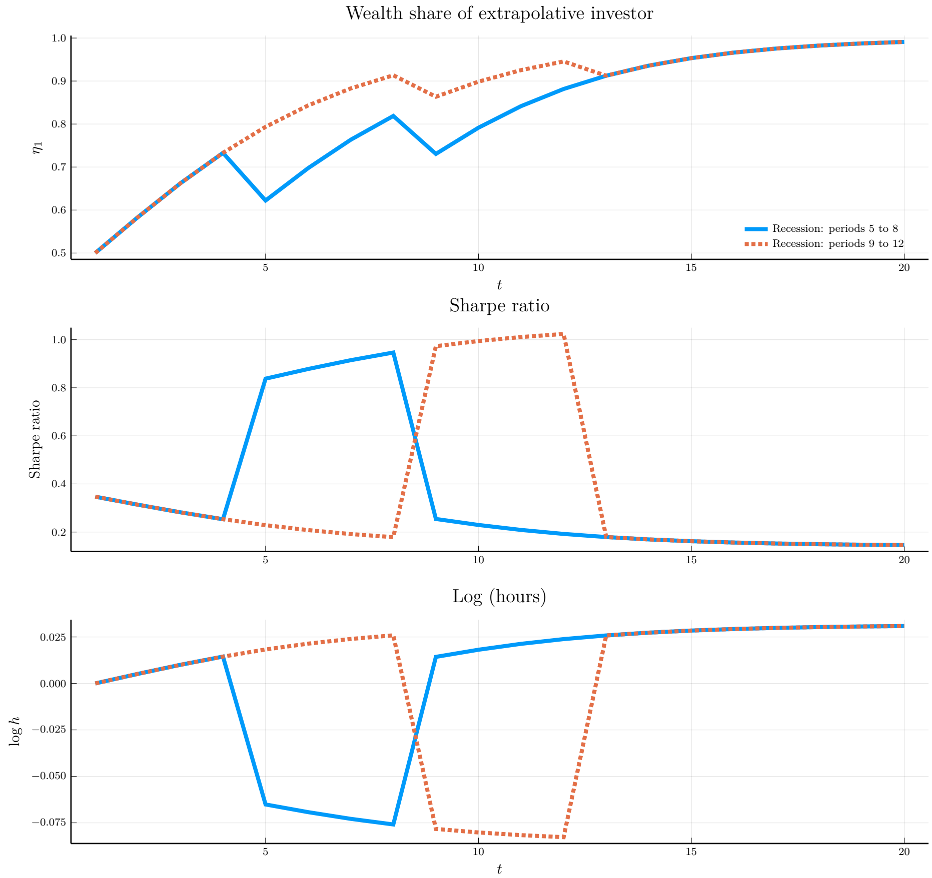


Figure 6: Heterogeneous beliefs: rational vs extrapolative

Note: examples of cycles (four periods of bad shocks with sixteen periods of expansions) with heterogeneous beliefs: $I = 2$ investors, rational and extrapolative. The top panel shows the wealth share of the extrapolative investor, the middle panel shows the Sharpe ratio (under objective beliefs), and the bottom panel shows log labor hours.

ratio, and the bottom panel log labor hours. As the economy remains longer in the high-growth state, the optimist accumulates more wealth. This reduces the Sharpe ratio (i.e., the price of risk), and increases labor demand. When the economy eventually transitions to the low-growth state, the greater wealth share of optimistic investors mitigates the increase in risk premia and attenuates the decline in hours.

Figure 6 presents the corresponding case of rank-alternating beliefs, with one rational and one extrapolative investor. During booms, extrapolative investors accumulate wealth, but they become pessimistic when the economy transitions to the low-growth state. As a result, market beliefs turn sharply pessimistic during downturns, leading to a larger increase in risk premia and a more pronounced contraction in labor demand. Hence, the longer the boom, the greater the bust.

The role of risk aversion. The results above rely on two key features of the wealth dynamics. First, the wealth share of optimistic investors increases during booms and decreases during recessions. Second, as implied by Proposition 5, the future wealth share of any investor is increasing in her current wealth share:

$$\frac{\partial \eta_i'(X,s,s')}{\partial \eta_i} = \frac{p^i_{ss'}}{\overline{p}^m_{ss'}(X,s)} - \eta_i \frac{p^i_{ss'}\left(p^i_{ss'} - p^I_{ss'}\right)}{\left(\overline{p}^m_{ss'}(X,s)\right)^2} = \frac{p^i_{ss'}}{\left(\overline{p}^m_{ss'}(X,s)\right)^2}\left(\sum_{j\neq i}\eta_j p^j_{ss'} + \eta_i p^I_{ss'}\right) \geq 0,$$

and it is strictly positive if, e.g., $p^j_{ss'} > 0$ for all j.

This property contrasts with models featuring risk-neutral investors, in which optimistic agents are driven to concentrate their wealth in the good state and are wiped out in the bad state (Geanakoplos, 2010). With risk-averse investors, by contrast, agents choose to allocate wealth across both states of the world. As a result, optimistic investors retain wealth in downturns, allowing the belief-driven amplification mechanism to operate.

The strength of this mechanism depends on the degree of risk aversion. When optimistic investors are close to risk-neutral, additional wealth accumulated during a boom is largely tilted toward the good state, attenuating the amplification of fluctuations in bad times. Higher risk aversion induces investors to distribute wealth more evenly across states, increasing the influence of optimistic investors during downturns and thereby strengthening the amplification of adverse shocks.

Frothy markets and risk build-ups. An implication of Corollary 2 is that the risk premium declines with the length of economic expansions. To the extent that risk premia are reflected in credit spreads, the model is consistent with the discussion in López-Salido et al. (2017) and Krishnamurthy and Muir (2017) that economic booms are characterized by "frothy" market conditions driven by credit-market sentiment.

Importantly, the model delivers a joint prediction: as optimism builds during prolonged expansions, objective risk premia decline while dividend volatility rises. The compression in spreads reflects the greater risk-bearing capacity of optimistic investors. At the same time, lower risk premia raise labor demand, increase the labor share, and amplify operating leverage. Because labor is chosen in advance, profits, and therefore dividends, become more sensitive to productivity shocks. This combination implies an endogenous *risk build-up* during booms: fragility increases precisely when spreads are low.

As emphasized by Krishnamurthy and Li (2020), generating simultaneously low spreads and rising risk exposure is challenging in standard macro-finance models, such as He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014). In contrast, heterogeneous extrapolative beliefs naturally generate this pattern in our framework.

Each link in this mechanism is supported by existing evidence. First, survey-based measures of optimism are associated with lower expected returns, consistent with compressed risk premia (e.g., De La O and Myers 2021). Second, measures of labor demand and labor share predict lower future stock returns (Santos and Veronesi 2006; Belo, Donangelo, Lin and Luo 2023), consistent with the link between labor demand and risk premia. Third, higher labor share is associated with stronger operating leverage and greater sensitivity of profits to shocks (Donangelo et al. 2019). Taken together, these findings support the interpretation that periods of elevated optimism are characterized by compressed spreads and increased fragility.

Discussion: belief heterogeneity vs. risk aversion. As discussed above, rank-alternating beliefs amplify inherent economic fluctuations. The key distinction is that heterogeneity in beliefs can induce changes in the *ranking* of investors' effective risk-taking across states. In models with heterogeneous risk aversion, differences in risk tolerance can generate portfolio dispersion and time variation in expected returns. However, the ranking of agents in terms of risk-taking is fixed: the more risk-tolerant agent always holds more risky assets in every state of the world. This property is analogous to the case of rank-preserving beliefs shown in Figure 5. In contrast, under rank-alternating beliefs, the identity of the relatively risk-taking investor depends on the state. An agent who takes more risk in booms may take less risk in busts. There is no counterpart to this mechanism in models with heterogeneous risk aversion alone.

5.3 Belief taxonomy and trading volume

Our belief taxonomy also has sharp implications for trading dynamics. As is standard in models with belief heterogeneity, trading volume is closely related to the degree of disagreement among investors. We show that not only the level but also the *type* of disagreement—as classified in Definition 3—plays a central role in shaping the dynamics of trading volume over the business cycle.

5.3.1 Analytical characterization of turnover

We adopt *share turnover* as our measure of trading volume, following Lo and Wang (2000), defined as

$$\tau_t \equiv \frac{1}{2}\sum_{i=1}^{I} \mu_i \left| S_{i,t} - S_{i,t-1} \right| = \frac{1}{2}\sum_{i=1}^{I} \left| \omega_{i,t}\eta_{i,t} - \omega_{i,t-1}\eta_{i,t-1} \right|,$$

where we divide by two to avoid double counting trades, and the second equality uses $\mu_i S_{i,t} = \omega_{i,t}\eta_{i,t}$.

In the absence of belief heterogeneity, $\omega_{i,t} = 1$ and $\eta_{i,t} = \eta_{i,t-1}$, so $\tau_t = 0$. With heterogeneous beliefs, turnover depends on the extent of belief dispersion. To characterize this dependence analytically, we consider a small deviation from a homogeneous-beliefs benchmark. Specifically, assume

$$p^i_{ss'} = p^*_{ss'} + \delta^i_{ss'}\epsilon, \qquad \delta^i_{sH} + \delta^i_{sL} = 0,$$

where $p^*_{ss'}$ denotes the common belief in the benchmark and ϵ controls the degree of dispersion. For simplicity, we take the benchmark belief to be symmetric and i.i.d., $p^*_{ss'} = \frac{1}{2}$. Appendix A.10 discusses the general case.

In this neighborhood, the portfolio share and next-period wealth share of investor i admit the expansions

$$\omega_i(X,s) = 1 + \kappa_\omega\left[p^i_{sH} - \overline{p}^m_{sH}(X,s)\right] + \mathcal{O}(\epsilon^2), \qquad \eta'_i(X,s,s') = \eta_i + \eta_i\frac{p^i_{ss'} - \overline{p}^m_{ss'}(X,s)}{p^*_{ss'}} + \mathcal{O}(\epsilon^2),$$

where $\kappa_\omega > 0$ and $\overline{p}^m_{ss'}(X,s) \equiv \sum_{j=1}^I \eta_j p^j_{ss'}$ denotes market beliefs. These expressions highlight how differences in beliefs drive portfolios and wealth dynamics.

To understand the determinants of trading volume, it is useful to decompose individual trading into two distinct components. The following lemma expresses an investor's net purchases as the sum of a rebalancing motive and a relative belief-updating motive.

Lemma 1. *Investor i's net purchases of shares are given by*

$$\mu_i\Delta S_i(X,s,s';\epsilon) = \underbrace{\Delta\eta_i(X,s,s')}_{\text{rebalancing effect}} + \underbrace{\Delta\omega_i(X,s,s')\,\eta_i}_{\text{change-in-beliefs effect}} + \mathcal{O}(\epsilon^2), \tag{36}$$

where

$$\Delta\eta_i(X,s,s') \equiv \eta_i\frac{p^i_{ss'} - \overline{p}^m_{ss'}(X,s)}{p^*_{ss'}}, \qquad \Delta\omega_i(X,s,s') \equiv \kappa_\omega\left[\left(p^i_{s'H} - \overline{p}^m_{s'H}(X,s)\right) - \left(p^i_{sH} - \overline{p}^m_{sH}(X,s)\right)\right].$$

Lemma 1 shows that individual trading behavior is governed by two distinct forces. The *rebalancing effect* captures trades required to maintain a constant portfolio share following changes in relative wealth. This effect occurs because when an optimistic investor gains wealth after a favorable shock, she must purchase additional shares to keep her portfolio composition unchanged. The *change-in-beliefs effect* reflects active portfolio reallocation driven by revisions in relative beliefs across states: if an investor becomes more optimistic in state s' than in state s, she increases her exposure to the risky asset upon the transition.

Whether these two effects reinforce or offset each other depends on how beliefs evolve across states. This interaction is the key mechanism behind the state-dependent relationship between belief dispersion and trading volume.

Turnover and belief taxonomy. As in Section 3, we can express beliefs in terms of a persistence parameter, ρ_i, and a bias term, Δ_i:

$$p^i_{HH} = \frac{1+\rho_i}{2} + \Delta_i, \qquad p^i_{LH} = \frac{1-\rho_i}{2} + \Delta_i,$$

which implies that the conditional expectation of productivity growth is given by:

$$\mathbb{E}_{i,t}[x_{t+1}] = \overline{x}_i + \rho_i(x_t - \overline{x}_i), \tag{37}$$

where $\overline{x}_i$ is the subjective unconditional mean of x_t.

Consider first the case of rank-preserving beliefs, $\rho_i = \rho$, where investors differ on the level of optimism, Δ_i. In this case, the change-in-beliefs effect vanishes and turnover is driven solely by the rebalancing effect:

$$\tau(X, s, s'; \epsilon) = \sum_{i=1}^{I} \eta_i \left| p^i_{ss'} - \overline{p}^m_{ss'}(X, s) \right| + \mathcal{O}(\epsilon^2).$$

In this case, turnover is proportional to the average absolute deviation of beliefs, a natural measure of belief dispersion, so trading volume increases with disagreement.

Consider next the case of rank-alternating beliefs, $\Delta_i = \Delta$, where investors differ on the persistence parameter, ρ_i. In this case, the change-in-beliefs effect emerges when the economy switches states, $s \neq s'$, and it may either amplify or dampen the rebalancing effect. For instance, suppose the economy switches from H to L. Investors who were relatively optimistic in the boom lose wealth on impact and, at the same time, become relatively pessimistic in the downturn. Both forces push toward selling, so turnover rises sharply. By contrast, when the economy switches from L to H, the same investors become relatively optimistic and wish to raise their risky share, while rebalancing incentives work in the opposite direction because their wealth share decreases. Turnover is therefore more sensitive to disagreement in recessions than in expansions.

The next proposition formalizes this intuition and delivers the central result of this section: trading volume is increasing in belief dispersion, and this relationship is asymmetric over the business cycle under rank-alternating beliefs.

Proposition 6. *Suppose investors' beliefs are rank-alternating, that is, they are given by (37) with $\Delta_i = \Delta$. Turnover as the economy switches from s to s' is given by*

$$\tau(X, s, s'; \epsilon) = \frac{\zeta(s, s')}{2} \sum_{i=1}^{I} \eta_i \big|\rho_i - \rho(X)\big| + \mathcal{O}(\epsilon^2),$$

where $\rho(X) \equiv \sum_{i=1}^{I} \eta_i \rho_i$, and

$$\zeta(s, s') \equiv \begin{cases} \kappa_\omega + 1, & \text{if } s = H \text{ and } s' = L, \\ \big|\kappa_\omega - 1\big|, & \text{if } s = L \text{ and } s' = H, \\ 1, & \text{if } s = s'. \end{cases}$$

The key message from Proposition 6 is that turnover increases with belief dispersion and, furthermore, that this sensitivity is stronger in downturns. The assumption of rank-alternating beliefs is essential for generating this asymmetry. With rank-preserving beliefs—investors are relatively optimistic or pessimistic by the same amount in both states—the change-in-beliefs effect is absent, and turnover is driven primarily by rebalancing. Hence, disagreement does not generate a disproportionately strong response of trading volume in bad times. Thus, rank-alternating beliefs are key to inducing state-dependent dynamics of stock market turnover.

5.3.2 Empirical evidence

We now test the model's prediction that trading volume increases with belief dispersion and that this sensitivity is stronger during downturns.

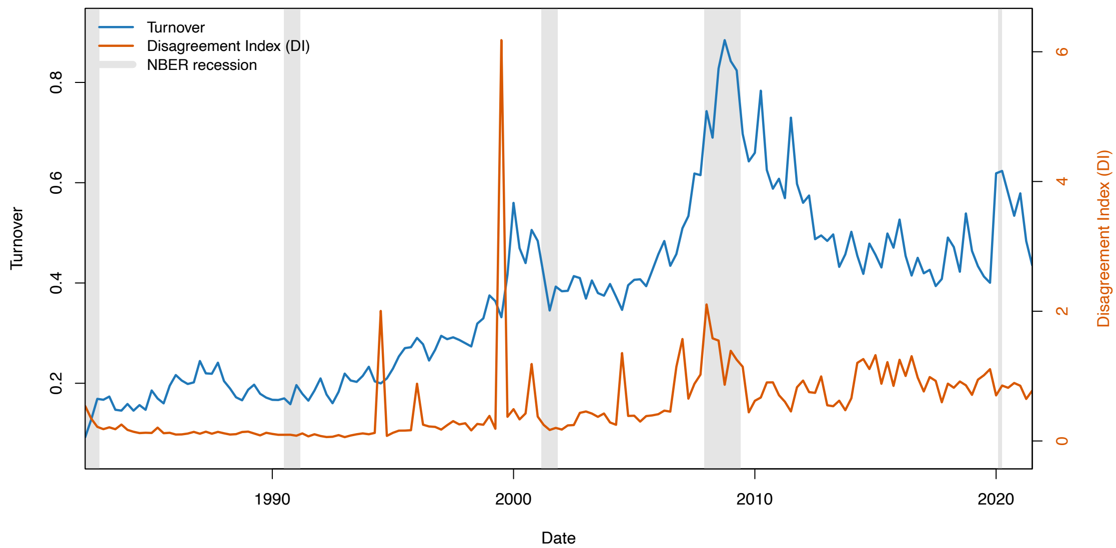
Measuring disagreement and turnover. The model links trading dynamics to disagreement about aggregate shocks. To construct an empirical analogue, we combine analyst forecasts from I/B/E/S with stock market turnover from CRSP.

Let $\mathbb{E}_{j,t}[\Delta e_{i,t+1}]$ denote analyst j's forecast of firm i's earnings growth at time t, and let Δe_t denote realized aggregate earnings growth. To isolate the analyst-specific component reflecting beliefs about aggregate dynamics, we estimate

$$\mathbb{E}_{j,t}[\Delta e_{i,t+1}] = \alpha_{i,t} + \beta_{0,j} + \beta_{1,j} \Delta e_t + u_{j,i,t}, \tag{38}$$

where $\alpha_{i,t}$ is a firm–time fixed effect, $\beta_{0,j}$ is an analyst-specific intercept, $\beta_{1,j}$ captures the

Figure 7: Disagreement Index and Stock Market Turnover



analyst's sensitivity to aggregate earnings growth, and $u_{j,i,t}$ is an error term.

The firm–time fixed effect absorbs firm-level information common to all analysts. The remaining analyst-specific terms therefore capture systematic differences in beliefs about aggregate earnings dynamics. We interpret the implied aggregate expectation of analyst j as

$$\mathbb{E}_{j,t}[\Delta e_{t+1}] = \beta_{0,j} + \beta_{1,j}\Delta e_t,$$

which mirrors the model's specification of expected productivity growth in Equation (37).

We then construct a time-varying *disagreement index* as the cross-sectional dispersion of these implied aggregate expectations:

$$DI_t = \overline{\sigma}_t\left[\beta_{0,j} + \beta_{1,j}\Delta e_t\right], \tag{39}$$

where $\overline{\sigma}_t[\cdot]$ denotes the cross-sectional standard deviation at time t.

While analyst-specific coefficients are constant over time, time changes in disagreement occur through Δe_t. This reflects the model's mechanism that dispersion in persistence beliefs interacts with the current state of the economy.

To measure trading activity, we follow Lo and Wang (2000) and compute value-weighted stock market turnover from CRSP.[17]

Figure 7 plots the time series of the disagreement index and turnover. Shaded areas correspond to NBER recessions. Disagreement rises sharply in downturns, periods in

[17]Details on construction are provided in Appendix O1.4.

Table 3: Turnover Regressions with Standardized Disagreement

	Baseline	Rec × DI	Rec × DI	Early Rec.	Early Rec.
	(1)	(2)	(3)	(4)	(5)
Disagreement (z-score)	0.075	0.054	0.054	0.069	0.069
	(0.047)	(0.039)	(0.039)	(0.047)	(0.047)
Recession		0.051			
		(0.048)			
Disagreement × Recession		0.176***	0.188***		
		(0.058)	(0.061)		
Early Rec. (first 2 qtrs)				0.014	
				(0.035)	
Disagreement × Early Rec.				0.095*	0.098*
				(0.051)	(0.051)
Marginal effect of DI in recession		0.230	0.242	0.164	0.167
Constant	0.311***	0.301***	0.305***	0.309***	0.309***
	(0.023)	(0.022)	(0.020)	(0.023)	(0.022)
Observations	158	158	158	158	158
R^2	0.212	0.352	0.344	0.235	0.235

Sample period: 1982Q2–2021Q3. Disagreement is standardized within sample. Newey–West (lag 4) standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

which turnover is also elevated.

Regression evidence. We formally assess the relationship using time-series regressions of turnover on the standardized disagreement index DI_t. Table 3 reports Newey–West standard errors with four lags: Column (1) shows a positive but statistically insignificant association between turnover and disagreement. Column (2) adds an NBER recession indicator and its interaction with disagreement. The interaction term is positive and highly significant, indicating that turnover becomes substantially more sensitive to disagreement in recessions. The implied marginal effect of disagreement during recessions is economically meaningful: a one-standard-deviation increase in DI_t is associated with an increase in turnover of approximately 0.23 (the sum of the main and interaction coefficients), a sizeable fraction of its sample mean (0.31). This is exactly what Proposition 6 predicts: the turnover–disagreement relationship strengthens in recessions.

Column (3) shows that the interaction effect remains substantial when the recession dummy is excluded, indicating that the result is driven by differences in slopes rather than by shifts in the intercept.

The model emphasizes the role of state transitions in generating this asymmetry. To proxy for transition periods, Columns (4) and (5) focus on the first two quarters of NBER recessions. Although the number of such observations is smaller, the interaction between disagreement and the transition indicator remains positive and statistically significant. This evidence is consistent with the model's prediction that turnover responds most strongly to disagreement at the onset of downturns.

The results are robust to controlling for aggregate market volatility (VIX) and market returns; these specifications are reported in Appendix O1.4. Overall, the evidence supports the model's central implication: while trading volume increases with belief dispersion, this sensitivity is substantially stronger in recessions, particularly around state transitions.

6 Quantitative analysis

We now consider the quantitative implications. We extend the baseline formulation in Section 3: First, productivity growth x_t now follows a more general process than the two-state process used to obtain analytical results. This extension is necessary to capture the empirical patterns of dividend growth. Second, we introduce a force that induces a non-degenerate stationary wealth distribution. A force like this is needed so that belief heterogeneity survives in the model's ergodic distribution. For convenience, we do so through Uzawa endogenous discounting.[18]

Belief process with a continuum of states. We start by specifying a continuous Markov process for x_t under both objective and subjective beliefs. Let $\hat{x}_t \equiv \log x_t$ denote log aggregate productivity growth. Under the objective measure, $\hat{x}_t$ follows:

$$\hat{x}_{t+1} = \mu_x + \sigma_x \epsilon_{x,t+1}. \tag{40}$$

where $\epsilon_{x,t+1} \overset{\text{iid}}{\sim} N(0,1)$. Thus, productivity growth is iid under the objective measure.

In turn, subjective beliefs about $\hat{x}_t$ for household $i \in \mathcal{I}$ are given by:

$$\hat{x}_{t+1} = \mu_{x,i} + \rho_{x,i}(\hat{x}_t - \mu_{x,i}) + v_{i,t} + \sigma_{x,i}\epsilon_{x,i,t+1}, \tag{41}$$

[18] Alternatively, we can obtain non-degeneracy through birth-death processes, for example.

where $\epsilon_{x,i,t+1} \overset{\text{iid}}{\sim} N(0,1)$. Beliefs deviate from the process (40) in two critical ways: First, the subjective conditional expectation responds to current productivity growth $\hat{x}_t$. This dimension is important to allow households to over- or under-react to past information, consistent with Fact 2 in Section 2. Overreaction is controlled by $\rho_{x,i}$. For instance, a positive $\rho_{x,i}$ implies the household overreacts to productivity news, given our assumption that productivity growth is iid under the objective measure.

Second, beliefs are also exposed to a persistent *sentiment shock* v_t, which evolve according to

$$v_{i,t+1} = \rho_{v,i} v_{i,t} + \sigma_{v,i} \epsilon_{v,i,t+1} \tag{42}$$

where $\epsilon_{v,i,t+1} \overset{\text{iid}}{\sim} N(0,1)$, and $\epsilon_{v,i,t}$ and $\epsilon_{x,i,t}$ are uncorrelated.

The sentiment is independent of TFP growth. Thus, this shock leads to fluctuations in the degree of optimism unrelated to fundamentals. Introducing this shock is necessary to quantitatively match the volatility of subjective TFP growth expectations relative to the objective one—Fact 1 in Section 2. While extrapolation, through higher values of $\rho_{x,i}$, increases the volatility of subjective expectations, it is insufficient, so the sentiment is needed.

We note that the belief system in this section is a hybrid of the one in our taxonomy. It encompasses both extrapolation and time-dependent optimism through sentiment shock. We let the survey data discipline the strength of each force.

Endogenous discounting. To ensure the wealth distribution is stationary, we assume that households' subjective discount rate responds to their consumption share:

$$\beta_{i,t} = \beta e^{-\kappa s_{i,t}}. \tag{43}$$

where $s_{i,t} \equiv \log \frac{\tilde{C}_{i,t}}{Y_t A_t}$ denotes the relative consumption of type i, and $Y_t \equiv h_t^{\alpha} - \xi e^{-\hat{x}_t} \frac{h_t^{1+\nu}}{1+\nu}$ denotes detrended net output. This assumption, a form of Uzawa (2017) preferences, implies that households' marginal propensity to consume increases with their share of consumption, ensuring that no investor type concentrates all the wealth asymptotically. Following Schmitt-Grohé and Uribe (2003), we assume that $\beta_{i,t}$ depends on the average consumption share of type-i households, so households take the process for $\beta_{i,t}$ as given.[19]

Model solution. Appendix O2 describes the characterization of the model with a continuum of states and Uzawa preferences. The qualitative mechanisms discussed in the

[19]This mechanism is often used in small open-economy models to ensure a stationary distribution of external debt.

binary-state case carry over to this richer environment. Away from log preferences, no closed-form solution is available. We therefore solve the model using a third-order perturbation around the non-stochastic steady state, following, for example, Van Binsbergen, Fernández-Villaverde, Koijen and Rubio-Ramírez (2012). A third-order approximation is necessary to generate time variation in risk premia.[20] Simulations are based on the pruned state-space representation of the higher-order solution (see Andreasen, Fernández-Villaverde and Rubio-Ramírez 2018).

Calibration. We use the following calibration, where parameters are expressed in quarterly terms. Preferences and technology parameters are standard. *Preferences*: We set $\beta = 0.978$ to match an unconditional annualized risk-free rate of 1%. The risk aversion is set to $\gamma = 10.0$ and the EIS to $\psi = 2.0$, typical values in the macro-finance literature. We choose the labor disutility parameter ξ to normalize the average hours to 1 and set the Frish elasticity to one, $\nu = 1$, a standard value in the literature. *Technology:* We set $\alpha = 0.66$ and choose μ_x and σ_x to match the average and standard deviation of annual consumption growth of 2.1% and 2.7%, respectively.

The calibration of the belief processes is novel. *Beliefs:* We focus on the case $I = 2$, and assume that households of type $i = 2$ have rational beliefs, which allows for some rational agents in the environment. We set $\mu_{x,1} = \mu_x$ and $\sigma_{x,1} = \sigma_x$, so households agree on the mean and conditional volatility of productivity growth. We set $\mu_2 = 0.1$, so the fraction of rational agents in the population is 10%, consistent with the fact that average beliefs in surveys of households and analysts show substantial deviations from rational expectations. We set $\rho_{v,1}$ and $\sigma_{v,1}$ to match the persistence of subjective dividend growth and the share of variance explained by movements in expectations, based on estimations of De La O and Myers (2021). In turn, we set $\rho_{x,1}$ to match the correlation between subjective expectations and current productivity growth observed in the data. This calibration is agnostic about the relevant belief system: the subjective belief moments pin down these parameters, determining whether non-rational beliefs are extrapolation or rank-preserving. Finally, we set $\kappa = 0.5\%$, consistent with the mean-reversion in consumption shares in Gârleanu and Panageas (2015).

6.1 Biases in productivity growth expectations

In the model, investors hold distorted beliefs about productivity growth. These distortions propagate to other variables—such as dividends and returns—through the

[20] For evidence on the accuracy of higher-order perturbation methods, see Caldara, Fernandez-Villaverde, Rubio-Ramirez and Yao 2012.

likelihood-ratio representation:

$$\mathbb{E}_{i,t}[Z_{t+1}] = \mathbb{E}_t[\ell_{i,t+1} Z_{t+1}], \tag{44}$$

where $\ell_{i,t+1}$ is the likelihood-ratio between subjective and objective beliefs. Hence, a single distortion in productivity growth beliefs induces distortions in expectations of all equilibrium objects.[21]

It is therefore important to verify that the model is consistent with the empirical properties of subjective productivity growth expectations.

Coibion–Gorodnichenko regressions. We assess this using Coibion–Gorodnichenko (CG) regressions (Coibion and Gorodnichenko, 2015), which test for systematic forecast biases by regressing forecast errors on forecast revisions:

$$\Delta prod_{t+h} - \mathbb{E}_{i,t}[\Delta prod_{t+h}] = c + \beta_{CG} \left(\mathbb{E}_{i,t}[\Delta prod_{t+h}] - \mathbb{E}_{i,t-1}[\Delta prod_{t+h}]\right) + u_{t+h}. \tag{45}$$

Under full-information rational expectations, $\beta_{CG} = 0$. A negative estimate indicates over-extrapolation: agents revise forecasts too strongly in the direction of recent news.

Empirical implementation. As direct survey data on TFP growth expectations are limited, we consider two alternative measures of productivity growth using quarterly data from the Survey of Professional Forecasters (SPF):

$$\Delta prod^{LP}_{t+1} = \Delta \hat{y}_{t+1} - \Delta \hat{n}_{t+1}, \tag{46}$$

$$\Delta prod^{TFP}_{t+1} = \Delta \hat{y}_{t+1} - \alpha\, \Delta \hat{n}_{t+1}, \tag{47}$$

where $\Delta \hat{y}$ is GDP growth, $\Delta \hat{n}$ is employment growth, and we set $\alpha = 2/3$.

The first measure corresponds to labor productivity growth, and the second is a TFP proxy consistent with how TFP growth is defined in the model. Table 4 reports the results. For labor productivity, we estimate $\hat{\beta}_{CG} = -0.63$, and for the TFP proxy, $\hat{\beta}_{CG} = -0.50$, both statistically significant.

[21] Cui, De La O and Myers (2024) estimate an empirical counterpart to the likelihood ratio and show that it explains a large fraction of cross-sectional variation in survey expectations.

Table 4: Coibion–Gorodnichenko Regressions for Productivity Measures

	Labor Productivity	TFP Proxy
Forecast revision coefficient β_{CG}	-0.63***	-0.50**
	(0.030)	(0.198)
Constant	-0.15	-0.25
	(0.289)	(0.259)
Observations	86	86
R^2	0.3697	0.0579
Adj. R^2	0.3622	0.0467

Notes: Each column reports OLS estimates of $\text{FE}_t = c + \beta\,\text{Revision}_t + u_t$, where $h = 1$ quarter. Reported standard errors (in parentheses) are Newey–West HAC with lag 4. Labor productivity uses $\Delta y - \Delta n$. TFP proxy uses $\Delta y - \alpha\Delta n$ with $\alpha = 2/3$. Sample: 2004Q1–2025Q2. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$ (based on HAC p-values).

Model comparison. We perform the same CG regression using simulated data from the calibrated model. The model implies

$$\hat{\beta}_{CG}^{\text{model}} = -0.695.$$

This value is very close to the empirical estimate for labor productivity and lies within one standard error of the estimate for the TFP proxy.

These results support the model's mechanism. The same distortions that drive endogenous movements in labor demand and risk premia also generate empirically realistic forecast biases in productivity growth.

Overall, even though the model is not directly calibrated to productivity-expectations moments, it reproduces the key bias pattern observed in the data.

6.2 Unconditional moments

Next, we study the model's ability to match unconditional moments. To isolate the role of each ingredient, we start from a stripped-down version of the model and progressively add features until we reach the complete model. This progression allows us to discuss the role of each ingredient in the model. Table 5 presents the results under each formulation. The table also reports ± 2 standard-error intervals for the data moments.

Column 1 presents the stripped-down version of the model with a single representative rational investor, for which hiring is done after TFP is known and labor supply

Table 5: Unconditional moments

	Rational Beliefs				Subjective Beliefs							
	Endowment		Production		Homogeneous		Heterogeneous		High elast.		Data (±2 s.e.)	
Variables	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**
Interest rate	10.2	0.0	10.0	0.7	10.1	1.3	1.7	6.1	1.6	6.1	[0.6, 3.0]	[3.7, 6.0]
Excess returns (equity)	0.7	2.7	1.2	7.2	1.2	8.3	5.2	15.8	5.3	15.6	[3.5, 9.3]	[15.6, 21.4]
Consumption growth	2.1	2.7	2.1	2.7	2.1	2.8	2.1	2.8	2.1	3.0	[1.2, 3.1]	[2.0, 3.4]
Dividend growth	2.1	2.7	2.1	9.3	2.1	10.0	2.1	10.0	2.1	9.8	[-0.3, 3.4]	[6.9, 11.2]
Log hours	0.0	0.0	0.2	0.0	0.2	1.3	0.2	1.2	0.6	1.9	–	[1.7, 2.7]

Note: The table reports annualized unconditional moments in percentage points. Column 1 is the endowment benchmark with rational beliefs; Column 2 adds production with labor chosen in advance under rational beliefs; Column 3 is the homogeneous-beliefs version of the production model; Column 4 is the heterogeneous-beliefs benchmark calibration; Column 5 is the high-labor-elasticity robustness. The data columns report point estimate ±2 bootstrap standard errors. Detailed data construction, sample windows, and bootstrap methodology are provided in the Data Appendix (Appendix O1.6).

is inelastic. This economy behaves as an Epstein-Zin version of the endowment economy studied by Mehra and Prescott (1985). The model fails to generate a sizeable equity premium and excess stock volatility. Moreover, without the hiring friction, the volatility of consumption and dividends coincide. Given that rational beliefs are iid, the price-dividend ratio is constant, so return volatility equals dividend volatility.

Column 2 introduces the timing of hires assumption and the elastic labor supply. The operating leverage channel endogenously generates differences in volatility between consumption and dividends in line with the data, even though this is not a targeted moment. With iid beliefs, there is no time variation in the price of risk, so the model fails to generate fluctuations in hours. Moreover, the hiring friction implies that dividend growth follows a mean-reverting process.[22] Without time-varying risk premia, mean-reversion in dividends causes the price-dividend ratio to negatively correlate with realized returns. This counterfactual behavior dampens return volatility relative to dividend volatility.

Column 3 considers the role of non-rational beliefs calibrated to match the survey data, while abstracting from differences in beliefs. In this case, the model does generate movements in hours, given that subjective beliefs are volatile, consistent with the discussion in Section 4. However, deviations from rational expectations are insufficient to generate an equity premium and stock return volatility in line with the data. Movements in subjective beliefs only attenuate the negative correlation between returns and the price-dividend ra-

[22]For models with mean-reverting dividend process, see e.g. Menzly, Santos and Veronesi (2004) and Santos and Veronesi (2006).

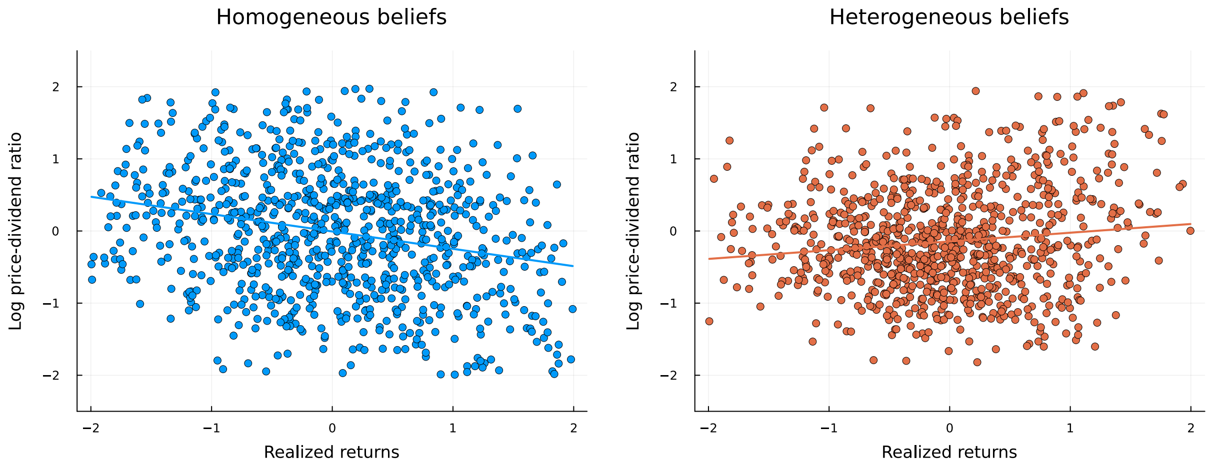


Figure 8: Price amplification: homogeneous vs. heterogeneous beliefs

Note: Figure shows a scatterplot of the log price-dividends and realized log returns in a 1,000-period simulation of the model. The left panel shows the simulation for the model with homogeneous beliefs, and the right panel shows the simulation for the model with heterogeneous beliefs. We standardized both variables and removed outliers to improve the visualization.

tio, so return volatility is still dampened relative to dividend volatility.

Column 4 introduces heterogeneity in beliefs. It brings along rational investors and investors with distorted beliefs. The model now generates the level and volatility of asset prices consistent with the evidence, with significant volatility in hours. To obtain a high equity premium, it is crucial to generate excess volatility of returns. This requires the price-dividend ratio to have the right comovement with returns. Figure 8 shows that this is the case for the model with heterogeneous beliefs, but not for the model with homogeneous beliefs. This success is explained by the time-varying movements in the relative wealth of investors.

Column 5 considers an alternative calibration with a higher labor supply elasticity, increasing it from 1.0 to 2.0, the upper limit of the estimates of the macro labor supply elasticity in Keane and Rogerson (2012). Increasing the labor supply elasticity is motivated by the presence of unmodeled frictions: it is akin to increasing the real wage rigidities. While asset-price figures do not change substantially, the volatility of hours almost doubles, aligning the volatility of hours with that of the data.

Overall, aside from a modest overprediction of risk-free-rate volatility, the model with heterogeneous beliefs matches key asset-pricing and business-cycle moments.

Extension with stochastic volatility. Appendix O2.3 extends the model to allow for stochastic volatility, following Bansal and Yaron (2004). Introducing volatility shocks modestly increases the equity premium and return volatility, while leaving macroeconomic moments largely unchanged. The qualitative conclusions of Table 5 remain intact.

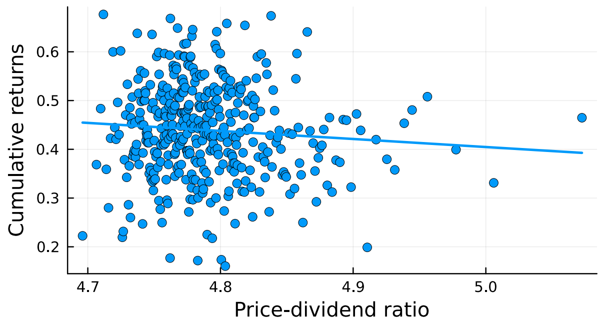


Figure 9: Return predictability

Dependent Variable:	cum_ex_returns			
Model:	(1)	(2)	(3)	(4)
Variables				
labor_income	-2.07		0.33	
	(0.94)		(1.31)	
p/d		-0.29		-0.04
		(0.12)		(0.17)
Observations	4,000	4,000	4,000	4,000

Table 6: Labor and p/d ratio regressions

Note: The left panel shows a binscatter plot based on model simulations of the price-dividend ratio and a measure of future cumulative returns, $\sum_{k=1}^{T} \rho^{k-1} r_{t+k}$, for $T = 60$ quarters. We sort the first 4,000 observations by the price-dividend ratio into 400 equal-count bins and plot bin averages. The right panel shows the regression coefficients of future cumulative excess returns for $T = 60$ on the price-dividend ratio and labor income. The data for the first two columns was simulated under the objective measure, while the data for the last two columns was simulated under subjective beliefs. Entries report Monte Carlo mean coefficients, and parentheses report the standard deviation across simulations.

6.3 Excess volatility and return predictability

The model's version with heterogeneous beliefs successfully generates empirically relevant levels of excess volatility, i.e., returns that are more volatile than cash flows. Following Cochrane (1992), we can use a Campbell-Shiller approximation to decompose the variance of the price-dividend ratio into a cash-flow and a discount-rate component:

$$Var[pd_t] = Cov\left[\sum_{k=1}^{\infty} \rho^{k-1} \Delta d_{t+k}, pd_t\right] - Cov\left[\sum_{k=1}^{\infty} \rho^{k-1} r_{t+k}, pd_t\right], \qquad (48)$$

where pd_t is the log price-dividend ratio, Δd_{t+1} denotes log dividend growth, and r_{t+1} denotes realized log returns. Expression (48) connects volatility and predictability. It shows that movements in the price-dividend ratio either predict changes in dividend growth or future returns. To quantify the relative importance of movements in discount rates, consider the share of variance explained by movements in expected returns, defined as $\beta_r \equiv -\frac{Cov\left[\sum_{k=1}^{\infty} \rho^k r_{t+k}, pd_t\right]}{Var[pd_t]}$. As shown, e.g., in Cochrane (2011), empirical evidence suggests that $\beta_r \approx 1$, so discount rates explain most of the price-dividend ratio variance.

Given that β_r corresponds to the slope of a regression of cumulative returns on the price-dividend ratio, the importance of discount rates to explain return volatility is tightly connected to the ability of the price-dividend ratio in predicting future long-run returns. Figure 9 shows that the model with heterogeneous beliefs can capture the predictability patterns observed in the data. The figure shows a strong negative association between the price-dividend ratio and cumulative returns in model-simulated data. Moreover, the coefficient β_r in models (4) and (5), as defined in Table 5, is given by $\beta_r = 0.86$ and $\beta_r = 0.89$, respectively. In contrast, we have $\beta_r = -0.15$ for model (2), so the price-dividend ratio predicts returns with the wrong sign, and $\beta_r = 0.26$ for model (3), which has the correct sign but movements in cash flows drive the majority of the fluctuations in the price-dividend ratio. Hence, only versions of the model with belief heterogeneity generate a degree of return predictability that aligns with the data.

Labor income and return predictability. Time variation in risk premia generates fluctuations in hours worked in our setting. As a result, in addition to the price-dividend ratio, movements in labor income should also predict excess returns. Table 6 reports the results of a regression of cumulative excess returns on the price-dividend ratio and labor income in our simulated data. Column 1 shows that periods of high labor income are associated with lower future returns. This finding is consistent with the evidence from Santos and Veronesi (2006), who show that a high labor income share predicts lower aggregate stock returns. Similarly, Belo et al. (2023) document that the aggregate hiring rate of publicly traded firms negatively predicts stock market excess returns.[23] Our model links this evidence to fluctuations in subjective beliefs. As discussed in Section 4, waves of optimism lead to both an increase in labor income and a compression of risk premia, jointly generating predictability from labor income and the price-dividend ratio.

Subjective vs. objective predictability. The predictability results above are from the perspective of an econometrician. Recent work by Nagel and Xu (2023) shows that return predictability looks very different under subjective beliefs: the standard predictors of returns are only weakly associated with survey-based measures of subjective risk premia. While expected excess returns are countercyclical under the objective measure, they appear acyclical under subjective beliefs.

Table 6 shows that the model replicates these empirical patterns: Columns 1 and 2 report predictability regressions using labor income and the price-dividend ratio as predictors under the objective measure. In contrast, Columns 3 and 4 present the same regressions when the model is simulated under subjective beliefs. Consistent with Nagel and Xu (2023), standard predictors are only weakly associated with future returns under subjective beliefs. The coefficient on labor income has the wrong sign and is statistically insignificant, while the coefficient on the price-dividend ratio is near zero. Thus, our

[23] Belo et al. (2014) and Favilukis and Lin (2016) show similar patterns for the cross-section of stock returns and U.S. states. For recent evidence using administrative data, see Meeuwis, Papanikolaou, Rothbaum and Schmidt (2024).

model generates subjective risk premia that are essentially acyclical, which aligns with survey evidence. As discussed in Section 4, this acyclicity arises because subjective expectations of future cash flows closely track asset price movements, dampening fluctuations in perceived risk premia.

7 Conclusion

When asked about the nature of business cycles, Thomas Sargent emphasized that progress requires quantitative models that discipline deviations from rational expectations and confront them with the data:[24]

> "[...] economists have been working hard to refine rational expectations theory. [...] An influential example of such work is the 1978 QJE paper by Harrison and Kreps. [...], for policymakers to know whether and how they can moderate bubbles, we need to have well-confirmed quantitative versions of such models up and running."

This paper responds to that call. We develop a tractable macro-finance framework in which heterogeneous and extrapolative beliefs interact with production and labor demand to generate endogenous movements in risk premia, asset prices, and real activity.

Quantitatively, the model matches a wide range of empirical patterns in survey expectations, trading volume, return predictability, and business-cycle dynamics. These results arise from the interaction between belief dispersion, wealth dynamics, and operating leverage.

Although our benchmark specification is deliberately parsimonious, with a simple productivity process and two investor types, the framework is substantially more general. The Markov structure accommodates richer technology dynamics, stochastic volatility, alternative belief processes, and additional investor types or frictions. The simplicity of the baseline clarifies the mechanism; it does not limit the model's scope.

Our framework provides a disciplined way to embed waves of optimism and pessimism into a standard macroeconomic environment while remaining flexible enough to incorporate additional amplification mechanisms. We view this as a step toward quantitatively credible models in which confidence fluctuations are measurable forces with testable implications.

[24] Interview with Thomas Sargent, *The Region*, August 26, 2010.

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Online Appendix

A Proofs

A.1 Derivation of the investor's modified problem

We start by providing a derivation of the investor's modified problem. We provide a characterization for the general case of $\nu \geq 0$

Proof. First, we adopt a change of variables and write the investor's problem as follows

$$V_{i,t} = \max_{\{\tilde{C}_{i,t}, h_{i,t}, B_{i,t}, S_{i,t}\}} (1-\beta) U\left(\tilde{C}_{i,t}\right) + \beta U\left(\Psi^{-1}\left(\mathbb{E}_{i,t}\left[\Psi\left(U^{-1}(V_{i,t+1})\right)\right]\right)\right), \tag{A.1}$$

subject to

$$\tilde{C}_{i,t} + Q_t S_{i,t} + B_{i,t} = R_{e,t} Q_{t-1} S_{i,t-1} + R_{f,t-1} B_{i,t-1} + W_t h_{i,t} - \xi_t \frac{h_{i,t}^{1+\nu}}{1+\nu}, \tag{A.2}$$

and the natural borrowing

$$(Q_t + \pi_t) S_{i,t-1} + R_{f,t-1} B_{i,t-1} + W_t h_{i,t} - \xi_t \frac{h_{i,t}^{1+\nu}}{1+\nu} \geq -\mathcal{H}_{i,t} \tag{A.3}$$

It is immediate that the optimal value of $h_{i,t}$ satisfies

$$W_t = \xi_t h_{i,t}^{\nu}. \tag{A.4}$$

We show next that, given $h_{i,t}$ satisfying (A.4), if the sequence $(\tilde{C}_{i,t}, B_{i,t}, S_{i,t})$ satisfies (A.2) and (A.3), then there exists $(N_{i,t}, \omega_{i,t})$ such that $(\tilde{C}_{i,t}, N_{i,t}, \omega_{i,t})$ satisfies (10) and $N_{i,t} \geq 0$. Conversely, if $(\tilde{C}_{i,t}, N_{i,t}, \omega_{i,t})$ satisfies (10) and $N_{i,t} \geq 0$, there exists $(B_{i,t}, S_{i,t})$ such that $(\tilde{C}_{i,t}, B_{i,t}, S_{i,t})$ satisfies (A.2) and (A.3).

From the definition of the return on human wealth, we have that $W_t h_{i,t} - \xi_t \frac{h_{i,t}^{1+\nu}}{1+\nu} = R_{h,t-1}\mathcal{H}_{i,t-1} - \mathcal{H}_{i,t}$, which allow us to write (A.2) and (A.3) as follows:

$$\tilde{C}_{i,t} + Q_t S_{i,t} + B_{i,t} + \mathcal{H}_{i,t} = N_{i,t}, \qquad N_{i,t} \geq 0. \tag{A.5}$$

We consider next the law of motion of total wealth:

$$N_{i,t+1} = \left[R_{e,t+1}\frac{Q_t S_{i,t}}{Q_t S_{i,t} + B_{i,t} + \mathcal{H}_{i,t}} + R_{f,t}\frac{B_{i,t}}{Q_t S_{i,t} + B_{i,t} + \mathcal{H}_{i,t}} + R_{h_i,t+1}\frac{\mathcal{H}_{i,t}}{Q_t S_{i,t} + B_{i,t} + \mathcal{H}_{i,t}}\right](N_{i,t} - \tilde{C}_{i,t}) . \tag{A.6}$$

As markets are dynamically complete, there exists replicating portfolios $(\omega_{h_i,t}, \omega_{e,t})$ such that

$$R_{k,t+1} = \omega_{k,t} R_{r,t+1} + (1 - \omega_{k,t}) R_{f,t}, \tag{A.7}$$

for $k \in \{h_i, e\}$.

Combining the previous two conditions, we obtain

$$N_{i,t+1} = \left[R_{r,t+1}\frac{\omega_{e,t} Q_t S_{i,t} + \omega_{h_i,t}\mathcal{H}_{i,t}}{N_{i,t} - \tilde{C}_{i,t}} + R_{f,t}\frac{B_{i,t} + (1-\omega_{e,t}) Q_t S_{i,t} + (1-\omega_{h_i,t})\mathcal{H}_{i,t}}{N_{i,t} - \tilde{C}_{i,t}}\right](N_{i,t} - \tilde{C}_{i,t}) . \tag{A.8}$$

Using the first condition in (A.5) to solve for $B_{i,t}$, we obtain

$$N_{i,t+1} = \left[\left(R_{r,t+1} - R_{f,t}\right)\omega_{i,t} + R_{f,t}\right](N_{i,t} - \tilde{C}_{i,t}) , \tag{A.9}$$

where $\omega_{i,t} \equiv \frac{\omega_{e,t} Q_t S_{i,t} + \omega_{h_i,t}\mathcal{H}_{i,t}}{N_{i,t} - \tilde{C}_{i,t}}$.

□

A.2 Proof of Lemma 2

The next lemma characterizes the value function, the consumption function, and the Euler equations of each investor.

Lemma 2 (Consumption and Euler equations)**.** *The household's value function takes the form: $V_i(N, X, s) = U(v_i(X, s)N)$, where $v_i(X, s)$ denotes the wealth multiplier. The consumption-wealth ratio $c_i(X, s) = \frac{\tilde{C}_i(N,X,s)}{N}$ and Euler equations for investor $i \in \mathcal{I}$ are given by*

(i) Consumption-wealth ratio:

$$c_i(X, s) = \frac{(\beta^{-1} - 1)^{\psi}\mathcal{R}_i(X, s)^{1-\psi}}{1 + (\beta^{-1} - 1)^{\psi}\mathcal{R}_i(X, s)^{1-\psi}}, \tag{A.10}$$

where $\mathcal{R}_i(X, s) \equiv \Psi^{-1}\left(\mathbb{E}_i\left[\Psi\left(v(X', s')R_i(X, s, s')\right)|X\right]\right)$.

(ii) Euler equation for an asset $j \in \{r, f\}$:

$$1 = \mathbb{E}_i\left[\Lambda_i(X, s, s')R_j(X, s, s')\right], \tag{A.11}$$

where, for $\theta \equiv \frac{1-\gamma}{1-\psi^{-1}}$, the investor's SDF is given by

$$\Lambda_i(X,s,s') = \beta^\theta \left(\frac{c_i(\chi(X,s,s'),s')N'}{c_i(X,s)N} \right)^{-\frac{\theta}{\psi}} R_i(X,s,s')^{-(1-\theta)}. \tag{A.12}$$

(iii) The wealth multipliers satisfy:

$$v_i(X,s) = U^{-1}\left[U\left(c_i(X,s)\right) + \beta U\left(\mathcal{R}_i(X,s)\left(1-c_i(X,s)\right)\right)\right]. \tag{A.13}$$

Proof. First, we verify that the value function takes the form (12). Given the conjecture about the value function, the Bellman equation for investor i can be written as

$$\frac{(v_i(X,s)N)^{1-\psi^{-1}}-1}{1-\psi^{-1}} = \max_{\tilde{c}_i,\omega_i}(1-\beta)\frac{(\tilde{c}_iN)^{1-\psi^{-1}}-1}{1-\psi^{-1}} + \beta\frac{\mathbb{E}_i\left[(v_i(X',s')N')^{1-\gamma}\right]^{\frac{1-\psi^{-1}}{1-\gamma}}-1}{1-\psi^{-1}}, \tag{A.14}$$

subject to $N' = R_i(X,s,s')(1-\tilde{c}_i)N$ and $N' \geq 0$.

The first-order conditions for the consumption-wealth ratio and the portfolio share are given by

$$(1-\beta)\tilde{c}_i^{-\psi^{-1}} = \beta\mathcal{R}_i(X,s)^{1-\psi^{-1}}(1-\tilde{c}_i)^{-\psi^{-1}} \tag{A.15}$$

$$0 = \mathbb{E}_i\left[(v_i(X',s')R_{i,n}(X,s,s'))^{-\gamma}v_i(X')(R_r(X,s,s')-R_f(X,s))\right] \tag{A.16}$$

where $\mathcal{R}_i(X,s) = \mathbb{E}_i\left[(v_i(X',s')R_i(X,s,s'))^{1-\gamma}|X,s\right]^{\frac{1}{1-\gamma}}$.

Given $\mathcal{R}_i(X,s)$, we can solve for the consumption-wealth ratio:

$$\tilde{c}_i(X,s) = \frac{(\beta^{-1}-1)^\psi\mathcal{R}_i(X,s)^{1-\psi}}{1+(\beta^{-1}-1)^\psi\mathcal{R}_i(X,s)^{1-\psi}}. \tag{A.17}$$

The envelope condition with respect to N is given by

$$v_i(X)^{1-1/\psi} = \beta\mathcal{R}_i(X)^{1-1/\psi}(1-\tilde{c}_i(X))^{-1/\psi} \Rightarrow \tilde{c}_i(X) = (1-\beta)^\psi v_i(X)^{1-\psi}. \tag{A.18}$$

From the optimality condition for the risky asset, we obtain

$$\mathbb{E}_i\left[(v_i(X',s')R_i(X,s,s'))^{1-\gamma}\right] = \mathbb{E}_i\left[v_i(X',s')^{1-\gamma}R_i(X,s,s')^{-\gamma}R_j(X,s,s')\right], \tag{A.19}$$

for $j \in \{r,f\}$.

Raising the envelope condition (A.18) to the power $\theta \equiv \frac{1-\gamma}{1-\psi^{-1}}$, using the definition of

$\mathcal{R}_i(X)$ and condition (A.19), we obtain

$$1 = \mathbb{E}_i \left[\beta^\theta \left(\frac{v_i(X', s')}{v_i(X, s')} \right)^{1-\gamma} R_i(X, s, s')^{-\gamma} R_j(X, s, s')(1 - \tilde{c}_i(X, s))^{-\theta/\psi} \right]. \tag{A.20}$$

Using the condition $v_i(X) = (1-\beta)^{\frac{1}{1-\psi^{-1}}} \tilde{c}_i(X)^{-\frac{\psi^{-1}}{1-\psi^{-1}}}$, we obtain the Euler equations

$$1 = \mathbb{E}_i \left[\beta^\theta \left(\frac{\tilde{c}_i(X', s')N'}{\tilde{c}_i(X, s)N} \right)^{-\frac{\theta}{\psi}} R_i(X, s, s')^{-(1-\theta)} R_j(X, s, s') \right]. \tag{A.21}$$

This concludes the derivation of the consumption-wealth ratio and the Euler equations for the two assets. It remains to check that the value function takes the form (12), which amounts to show that $v_i(X)$ indeed does not depend on N. Notice that $\tilde{c}_i(X, s)$ and $\omega_i(X, s)$ do not depend on N. We can then write the Bellman equation as follows:

$$v_i(X, s)^{1-\psi^{-1}} = (1-\beta)\tilde{c}_i(X, s)^{1-\psi^{-1}} + \beta \mathbb{E}_i \left[(v_i(X', s')R_i(X, s, s')(1 - \tilde{c}_i(X)))^{1-\gamma} \right]^{\frac{1-\psi^{-1}}{1-\gamma}}, \tag{A.22}$$

for $\psi \neq 1$ and

$$\log v_i(X, s) = (1-\beta) \log \tilde{c}_i(X, s) + \beta \log \mathbb{E}_i \left[(v_i(X', s')R_i(X, s, s')(1 - \tilde{c}_i(X, s)))^{1-\gamma} \right]^{\frac{1}{1-\gamma}}. \tag{A.23}$$

which is independent of N, which confirms our conjecture for the value function (12).

□

A.3 Proof of Lemma 3

The following lemma characterizes households' portfolio weight in terms of the economy-wide SDF, the market prices, and their beliefs.

Lemma 3 (Portfolio share)**.** *The shares of total wealth invested in the risky asset are*

$$\omega_i(X, s) = \frac{1}{\Delta R_r(X, s)} \left[\frac{\tilde{p}_i(X, s, H)}{p_{s,H} \Lambda(X, s, H)} - \frac{\tilde{p}_i(X, s, L)}{p_{s,L} \Lambda(X, s, L)} \right], \tag{A.24}$$

where $\tilde{p}_i(X, s, s')$ *is*

$$\tilde{p}_i(X, s, s') = \frac{(p^i_{ss'})^{\frac{1}{\gamma}} \left[v_i(\chi(X, s, s'), s') | R^e_r(X, s, s') | \right]^{\frac{1}{\gamma}-1}}{\sum_{\tilde{s}' \in \{L,H\}} (p^i_{s\tilde{s}'})^{\frac{1}{\gamma}} \left[v_i(\chi(X, s, \tilde{s}'), \tilde{s}') | R^e_r(X, s, \tilde{s}') | \right]^{\frac{1}{\gamma}-1}}.$$

Lemma 3, describes how portfolio shares depend on distorted probabilities, $\tilde{p}_i(X,s,s')$, and $p_{ss'} \times \Lambda(X,s,s')$. The portfolio share $\omega_i(X,s)$ in (A.24) is increasing in p^i_{sH}. This means that relatively optimistic investors hold more of the risky asset.

Proof. Let $X' = \chi(X,s,s')$. The optimal portfolio share satisfies the condition

$$\frac{p^i_{sL}}{p^i_{sH}} \frac{v_i(\chi(X,s,L),L)^{1-\gamma}}{v_i(\chi(X,s,H),H)^{1-\gamma}} \left(\frac{\omega_i(X,s)R^e_r(X,s,L)+1}{\omega_i(X,s)R^e_r(X,s,H)+1}\right)^{-\gamma} \frac{|R^e_r(X,s,L)|}{R^e_r(X,s,H)} = 1 \tag{A.25}$$

Raising both sides to $-\frac{1}{\gamma}$, we obtain

$$\left(\frac{p^i_{sL}}{p^i_{sH}}\right)^{-\frac{1}{\gamma}} \frac{v_i(\chi(X,s,L),L)^{1-\frac{1}{\gamma}}}{v_i(\chi(X,s,H),H)^{1-\frac{1}{\gamma}}} \frac{\omega_i(X,s)R^e_r(X,s,L)+1}{\omega_i(X,s)R^e_r(X,s,H)+1} \frac{|R^e_r(X,s,L)|^{-\frac{1}{\gamma}}}{R^e_r(X,s,H)^{-\frac{1}{\gamma}}} = 1 \tag{A.26}$$

Rearranging the expression above, we obtain

$$\omega_i(X,s) = \frac{\tilde{p}_i(X,s,H)}{|R^e_r(X,s,L)|} - \frac{\tilde{p}_i(X,s,L)}{R^e_r(X,s,H)}, \tag{A.27}$$

where

$$\tilde{p}_i(X,s,s') = \frac{(p^i_{ss'})^{\frac{1}{\gamma}} \left[v_i(\chi(X,s,s'),s')|R^e_r(X,s,s')|\right]^{\frac{1}{\gamma}-1}}{\sum_{s'\in\{L,H\}}(p^i_{ss'})^{\frac{1}{\gamma}} \left[v_i(\chi(X,s,s'),s')|R^e_r(X,s,s')|\right]^{\frac{1}{\gamma}-1}}. \tag{A.28}$$

The SDF in this economy is given by

$$\Lambda(X,s,s') = \frac{1}{p_{ss'}} \frac{1}{R_f(X,s)} \frac{|R_r(X,s,-s') - R_f(X,s)|}{\Delta R_r(X,s)}, \tag{A.29}$$

where $\Delta R_r(X,s) = R_r(X,s,H) - R_r(X,s,L)$.

We can then write $\omega_i(X,s)$ as follows

$$\omega_i(X,s) = \frac{1}{\Delta R_r(X,s)} \left[\frac{\tilde{p}_i(X,s,H)}{p_{s,H}\Lambda(X,s,H)} - \frac{\tilde{p}_i(X,s,L)}{p_{s,L}\Lambda(X,s,L)}\right]. \tag{A.30}$$

Log utility. The formula above simplifies under the case of log utility. First, we obtain $\tilde{p}_i(X,s,s') = p^i_{ss'}$. This allows us to write the portfolio share as follows:

$$\omega_i(X,s) = \frac{p^i_{s,H}}{|R^e_r(X,s,L)|} - \frac{p^i_{s,L}}{R^e_r(X,s,H)}, \tag{A.31}$$

using the expression for $\Lambda(X, s, s')$ and the fact that $R_r^e(X, s, L) < 0 < R_r^e(X, s, H)$ under log utility.

We can rewrite the expression above as follows:

$$\omega_i(X, s) = \frac{p_{s,H}^i R_r^e(X, s, H) + p_{s,L}^i R_r^e(X, s, L)}{|R_r^e(X, s, L)| R_r^e(X, s, H)} = \frac{\mathbb{E}_{i,s}[R_r^e(X, s, s')]}{|R_r^e(X, s, L)| R_r^e(X, s, H)}. \tag{A.32}$$

The denominator in the expression above can be written as follows:

$$|R_r^e(X, s, L)| R_r^e(X, s, H) = \frac{\mathcal{L}'(X, s) - x_L}{(1-\alpha)\mathcal{L}'(X, s)} \frac{x_H - \mathcal{L}'(X, s)}{(1-\alpha)\mathcal{L}'(X, s)}. \tag{A.33}$$

The risk-neutral expectation of productivity growth can be written as follows:

$$\mathcal{L}'(X, s) = p_{sH}^{RN}(X, s) x_H + p_{sL}^{RN}(X, s) x_L. \tag{A.34}$$

where $p_{ss'}^{RN}(X, s)$ is the risk-neutral probability of state s' given state s.

Combining the previous two expressions we obtain:

$$|R_r^e(X, s, L)| R_r^e(X, s, H) = p_{sH}^{RN}(X, s) p_{sL}^{RN}(X, s) \Delta R_r^e(X, s)^2, \tag{A.35}$$

where $\Delta R_r^e(X, s) = R_r^e(X, s, H) - R_r^e(X, s, L) = \frac{\Delta x}{(1-\alpha)\mathcal{L}'(X,s)}$. The expression above is the variance of the excess return on the risky asset under risk-neutral probabilities.

The risk-neutral probability of state s' given state s is given by

$$p_{ss'}^{RN}(X, s) = \frac{p_{ss'} \Lambda(X, s, s')}{\mathbb{E}_s[\Lambda(X, s, s')]} = \frac{R_r^e(X, s, -s')}{\Delta R_r^e(X, s)}. \tag{A.36}$$

We can then write the risk-neutral probabilities as follows:

$$p_{sL}^{RN}(X, s) = \frac{R_r^e(X, s, H)}{\Delta R_r^e(X, s)} = \frac{x_H - \mathcal{L}'(X, s)}{\Delta x}, \qquad p_{sH}^{RN}(X, s) = \frac{R_r^e(X, s, L)}{\Delta R_r^e(X, s)} = \frac{\mathcal{L}'(X, s) - x_L}{\Delta x}. \tag{A.37}$$

Diffusion-like approximation. To better interpret the expression for the portfolio share, it is useful to consider an approximation analogous to the continuous-time limit for diffusion processes. Given $R_r(X, s, s')$, probabilities $p_{ss'}^i$ for household i, and a small parameter

$\epsilon > 0$, we can find $\mu_{i,r}(X,s)$ and $\sigma_{i,r}(X,s)$ that satisfies the conditions

$$R_r^e(X,s,H) = \mu_{i,r}(X,s)\epsilon + \sqrt{\frac{p_{sL}}{p_{sH}}}\sigma_{i,r}(X,s)\sqrt{\epsilon}, \qquad R_r^e(X,s,L) = \mu_{i,r}(X,s)\epsilon - \sqrt{\frac{p_{sH}}{p_{sL}}}\sigma_{i,r}(X,s)\sqrt{\epsilon}, \tag{A.38}$$

which gives us the expected value and variance for household i:

$$\mathbb{E}_i[R_r^e(X,s,s')|X,s] = \mu_{i,r}(X,s)\epsilon, \qquad Var_i[R_r^e(X,s,s')|X,s] = \sigma_{i,r}^2(X,s)\epsilon. \tag{A.39}$$

Similarly, we can write $R_f(X,s) = 1 + r_f(X,s)\epsilon$.

From Equation (A.95), and assuming $\gamma = 1$, we obtain

$$\begin{aligned}\omega_i(X,s) &= R_f(X,s)\frac{p^i_{s,H}R_r^e(X,s,H) + p^i_{s,L}R_r^e(X,s,L)}{|R_r^e(X,s,L)|R_r^e(X,s,H)} \\ &= (1 + r_f(X,s)\epsilon)\frac{\mu_{i,r}(X,s)\epsilon}{\left(\sqrt{\frac{p_{sH}}{p_{sL}}}\sigma_{i,r}(X,s)\sqrt{\epsilon} - \mu_{i,r}(X,s)\epsilon\right)\left(\mu_{i,r}(X,s)\epsilon + \sqrt{\frac{p_{sL}}{p_{sH}}}\sigma_{i,r}(X,s)\sqrt{\epsilon}\right)},\end{aligned} \tag{A.40}$$

where we used the fact that $R_r^e(X,s,L) < 0$ by no-arbitrage.

In general, $(\mu_{i,r}(X,s), \sigma_{i,r}(X,s))$ and $p^i_{ss'}$ are functions of ϵ. Assuming that $\mu_{i,r}(X,s) = \mathcal{O}(1)$, $\sigma_{i,r}(X,s)) = \mathcal{O}(1)$, and $p^i_{ss'} = \mathcal{O}(1)$, we can write the expression $\omega_i(X,s)$ as follows:[25]

$$\omega_i(X,s) = \frac{\mu_{i,r}(X,s)}{\sigma_{i,r}^2(X,s)} + \mathcal{O}(\epsilon). \tag{A.41}$$

□

A.4 Proof of Proposition 1

Proof. First, we compute the Sharpe ratio on the risky asset. We will compute expectations using the objective measure, but a similar calculation gives the Sharpe ratio using the investors' subjective beliefs. The expected excess return is given by

$$\mathbb{E}\left[R_r^e(X,s,s')\right] = p_{sL}R_r^e(X,s,L) + p_{sH}R_r^e(X,s,H). \tag{A.42}$$

[25] These assumptions are analogous to the ones used by e.g. Merton (1992) to derive the continuous-time limit with diffusion processes. Allowing for rare events, $p^i_{ss'} = \mathcal{O}(\epsilon)$ for some s', would lead to a jump-diffusion process.

The variance of excess returns is given by

$$Var[R^e_r(X,s,s')] = p_{sL}p_{sH}\Delta R^e_r(X,s)^2. \tag{A.43}$$

The Sharpe ratio in the risky asset is then given by

$$\frac{\mathbb{E}[R^e_r(X,s,s')]}{\sqrt{Var[R^e_r(X,s,s')]}} = \sqrt{\frac{p_{sL}}{p_{sH}}}\frac{R^e_r(X,s,L)}{\Delta R^e_r(X,s)} + \sqrt{\frac{p_{sH}}{p_{sL}}}\frac{R^e_r(X,s,H)}{\Delta R^e_r(X,s)}. \tag{A.44}$$

We can write the expression above in terms of the economy's SDF. The SDF under the objective measure can be written as

$$\Lambda(X,s,L) = \frac{\mathbb{E}[\Lambda(X,s,s')]}{p_{sL}}\frac{R^e_r(X,s,H)}{\Delta R^e_r(X,s)}, \qquad \Lambda(X,s,H) = -\frac{\mathbb{E}[\Lambda(X,s,s')]}{p_{sH}}\frac{R^e_r(X,s,L)}{\Delta R^e_r(X,s)}. \tag{A.45}$$

Combining the expressions above, we obtain

$$\frac{\mathbb{E}[R^e_r(X,s,s')]}{\sqrt{Var[R^e_r(X,s,s')]}} = \sqrt{p_{sL}p_{sH}}\frac{\Lambda(X,s,L)-\Lambda(X,s,H)}{\mathbb{E}[\Lambda(X,s,s')]}. \tag{A.46}$$

We consider next how the Sharpe ratio affects the risk-neutral expectation of future productivity growth. The risk-neutral expectation of productivity is given by

$$\mathbb{E}^{Q}\left[x_{t+1}\right] = p_{sL}\frac{\Lambda(X,s,L)}{\mathbb{E}[\Lambda(X,s,s')]}x_L + p_{sH}\frac{\Lambda(X,s,H)}{\mathbb{E}[\Lambda(X,s,s')]}x_H. \tag{A.47}$$

The difference between the expected value of productivity under the physical measure and the risk-neutral measure is given by

$$\mathbb{E}[x_{t+1}] - \mathbb{E}^{Q}[x_{t+1}] = p_{sL}\frac{\mathbb{E}[\Lambda(X,s,s')]-\Lambda(X,s,L)}{\mathbb{E}[\Lambda(X,s,s')]}x_L + p_{sH}\frac{\mathbb{E}[\Lambda(X,s,s')]-\Lambda(X,s,H)}{\mathbb{E}[\Lambda(X,s,s')]}x_H. \tag{A.48}$$

Rearranging the expression above, we obtain

$$\mathbb{E}[x_{t+1}] - \mathbb{E}^{Q}[x_{t+1}] = p_{sL}p_{sH}\frac{\Lambda(X,s,L)-\Lambda(X,s,H)}{\mathbb{E}[\Lambda(X,s,s')]}\Delta x, \tag{A.49}$$

where $\Delta x = x_H - x_L$.

Using the expression for the Sharpe ratio, we obtain

$$\mathbb{E}^{Q}[x_{t+1}] = \mathbb{E}[x_{t+1}] - \sqrt{p_{sL}p_{sH}}\frac{\mathbb{E}[R_r^e(X,s,s')]}{\sqrt{Var[R_r^e(X,s,s')]}}\Delta x. \tag{A.50}$$

Using the fact that $\mathcal{L}'(X_t, s_t) = \mathbb{E}_t^{Q}[x_{t+1}]$ and $\sigma_s[x_{t+1}] = \sqrt{p_{sL}p_{sH}}\Delta x$, we obtain the result of Proposition 1. □

A.5 Proof of Proposition 2

Proof. We start by deriving the process for returns. From the market clearing condition for goods, we obtain

$$\frac{x_s h(\mathcal{L})^{\alpha} - \xi\frac{h(\mathcal{L})^{1+\nu}}{1+\nu}}{P(X,s)} = 1-\beta \tag{A.51}$$

The return on the surplus claim is given by

$$R_p(X,s,s') = \frac{x_s P(\chi(X,s,s'),s')}{P(X,s) - \left(x_s h(\mathcal{L})^{\alpha} - \xi\frac{h(\mathcal{L})^{1+\nu}}{1+\nu}\right)} = \frac{x_s}{\beta}\frac{x_{s'}h(\mathcal{L}'(X,s))^{\alpha} - \xi\frac{h(\mathcal{L}'(X,s))^{1+\nu}}{1+\nu}}{x_s h(\mathcal{L})^{\alpha} - \xi\frac{h(\mathcal{L})^{1+\nu}}{1+\nu}}. \tag{A.52}$$

Using the conditions in (20), we can rewrite the expression as follows

$$R_p(X,s,s') = \frac{x_s}{\beta}\frac{x_{s'}\mathcal{L}'(X,s)^{\frac{\alpha}{1+\nu-\alpha}} - \frac{\alpha}{1+\nu}\mathcal{L}'(X,s)^{\frac{1+\nu}{1+\nu-\alpha}}}{x_s\mathcal{L}^{\frac{\alpha}{1+\nu-\alpha}} - \frac{\alpha}{1+\nu}\mathcal{L}^{\frac{1+\nu}{1+\nu-\alpha}}}. \tag{A.53}$$

Note that the denominator in the expression above is positive if and only if $\mathcal{L} < \frac{1+\nu}{\alpha}x_s$. A sufficient condition is given by $\alpha x_H < x_L$, as shown below

$$\mathcal{L} \leq x_H < \frac{x_L}{\alpha} < \frac{1+\nu}{\alpha}x_s, \tag{A.54}$$

and, similarly, this condition guarantees that the numerator is also positive.

Interest rate. The interest rate satisfies the condition $R_f(X,s) = \mathbb{E}\left[\frac{\Lambda(X,s,s')}{\mathbb{E}[\Lambda(X,s,s')]}R_p(X,s,s')\right]$, so $R_f(X,s)$ is given by

$$R_f(X,s) = \left(1 - \frac{\alpha}{1+\nu}\right)\frac{x_s}{\beta}\frac{\mathcal{L}'(X,s)^{\frac{1+\nu}{1+\nu-\alpha}}}{x_s\mathcal{L}^{\frac{\alpha}{1+\nu-\alpha}} - \frac{\alpha}{1+\nu}\mathcal{L}^{\frac{1+\nu}{1+\nu-\alpha}}}, \tag{A.55}$$

using the fact that $\mathbb{E}\left[\frac{\Lambda(X,s,s')}{\mathbb{E}[\Lambda(X,s,s')]}x_{s'}\right] = \mathcal{L}'(X,s)$.

The expression above is increasing in $\mathcal{L}'(X,s)$, decreasing in x_s, and it is increasing in $\mathcal{L}$ for $s = L$.

Risk premium. The risk asset's excess return is given by

$$\frac{R_p(X,s,s')}{R_f(X,s)} = \frac{1}{1-\frac{\alpha}{1+\nu}} \frac{x_{s'} - \frac{\alpha}{1+\nu}\mathcal{L}'(X,s)}{\mathcal{L}'(X,s)}. \tag{A.56}$$

The conditional risk premium is then given by

$$\mathbb{E}_s[R_p^e(X,s,s')] = \frac{1}{1-\frac{\alpha}{1+\nu}} \frac{\mathbb{E}_s[x_{s'}] - \mathcal{L}'(X,s)}{\mathcal{L}'(X,s)}, \tag{A.57}$$

given the definition $R_p^e(X,s,s') \equiv \frac{R_p(X,s,s') - R_f(X,s)}{R_f(X,s)}$.

□

A.6 Proof of Proposition 3 and Corollary 1

Proof. We start by deriving the expression for the SDF. From the no-arbitrage conditions, we obtain

$$\Lambda(X,s,s') = \frac{1}{p_{ss'}} \frac{|R_r^e(X,s,-s')|}{\Delta R_r(X,s)}. \tag{A.58}$$

The excess return on the risky asset and the return difference are given by

$$R_r^e(X,s,s') = \frac{x_{s'} - \mathcal{L}'(X,s)}{(1-\alpha)\mathcal{L}'(X,s)}, \qquad \Delta R_r(X,s) = \frac{R_f(X,s)\Delta x}{(1-\alpha)\mathcal{L}'(X,s)}. \tag{A.59}$$

Combining the previous two expressions, we obtain

$$\Lambda(X,s,s') = \frac{1}{p_{ss'}} \frac{|x_{-s'} - \mathcal{L}'(X,s)|}{R_f(X,s)\Delta x}. \tag{A.60}$$

Portfolio share. Under log utility, the SDF for investor i is given by

$$\Lambda_i(X,s,s') = \beta\left(R_i(X,s,s')(1-(1-\beta))\right)^{-1} = R_i(X,s,s')^{-1}, \tag{A.61}$$

using the fact that consumption growth equals the growth rate of the household's net worth.

Using the fact that $R_i(X,s,s') = R_f(X,s)\,[1+\omega_i(X,s)R_r^e(X,s,s')]$, and the expression linking the individual and economy-wide SDFs, we obtain

$$\frac{1}{R_f(X,s)}\frac{1}{1+\omega_i(X,s)R_r^e(X,s,s')} = \frac{1}{p_{ss'}^i}\frac{|x_{-s'}-\mathcal{L}'(X,s)|}{R_f(X,s)\Delta x}. \tag{A.62}$$

Taking the ratio for $s'=H$ and $s'=L$, we obtain

$$\frac{1+\omega_i(X,s)R_r^e(X,s,L)}{1+\omega_i(X,s)R_r^e(X,s,H)} = \frac{p_{sL}^i}{p_{sH}^i}\frac{\mathcal{L}'(X,s)-x_L}{x_H-\mathcal{L}'(X,s)}. \tag{A.63}$$

Rearranging the expression above, we obtain

$$p_{sH}^i(x_H-\mathcal{L}'(X,s))\,[1+\omega_i(X,s)R_r^e(X,s,L)] = [1+\omega_i(X,s)R_r^e(X,s,H)]\,p_{sL}^i(\mathcal{L}'(X,s)-x_L). \tag{A.64}$$

Solving for $\omega_i(X,s)$, we obtain

$$\omega_i(X,s) = \frac{p_{sH}^i(x_H-\mathcal{L}'(X,s))+p_{sL}^i(x_L-\mathcal{L}'(X,s))}{p_{sH}^i(\mathcal{L}'(X,s)-x_H)R_r^e(X,s,L)+p_{sL}^i(\mathcal{L}'(X,s)-x_L)R_r^e(X,s,H)} \tag{A.65}$$

Using the expression for $R_r^e(X,s,s')$, we obtain

$$\omega_i(X,s) = \frac{p_{sH}^i\frac{x_H-\mathcal{L}'(X,s)}{(1-\alpha)\mathcal{L}'(X,s)}+p_{sL}^i\frac{x_L-\mathcal{L}'(X,s)}{(1-\alpha)\mathcal{L}'(X,s)}}{\frac{x_H-\mathcal{L}'(X,s)}{(1-\alpha)\mathcal{L}'(X,s)}\frac{\mathcal{L}'(X,s)-x_L}{(1-\alpha)\mathcal{L}'(X,s)}} = \frac{\mathbb{E}_{i,s}[R_r^e(X,s,s')]}{Var_{RN,s}[R_r^e(X,s,s')]}. \tag{A.66}$$

We can write the portfolio share as follows:

$$\omega_i(X,s) = (1-\alpha)\mathcal{L}'(X,s)\frac{p_{sH}^i(x_H-\mathcal{L}'(X,s))+p_{sL}^i(x_L-\mathcal{L}'(X,s))}{(x_H-\mathcal{L}'(X,s))(\mathcal{L}'(X,s)-x_L)} \tag{A.67}$$

Demand for risk. The demand for risk in this economy is given by

$$\sum_{i=1}^{I}\eta_i\sigma_s[R_{i,n}(X,s,s')] = \sqrt{p_{sL}p_{sH}}\left[\frac{p_{sH}(X,s)}{p_{sH}\Lambda(X,s,H)}-\frac{p_{sL}(X,s)}{p_{sL}\Lambda(X,s,L)}\right], \tag{A.68}$$

where $p_{ss'}(X,s) = \sum_{i=1}^{I}\eta_{i,t}p_{ss'}^i$, using the fact that $\sigma_s[R_r(X,s,s')] = \sqrt{p_{sH}p_{sL}}\Delta R_r(X,s)$ and the results in Lemma 3.

Using the expression for the SDF, the demand for risk can be written as

$$\sum_{i=1}^{I} \eta_i \sigma_s[R_{i,n}(X,s,s')] = \sigma_s[x_{s'}] \frac{\frac{1+\nu-\alpha}{1+\nu}\frac{x_s}{\beta}}{x_s \mathcal{L}^{\frac{\alpha}{1+\nu-\alpha}} - \frac{\alpha}{1+\nu}\mathcal{L}^{\frac{1+\nu}{1+\nu-\alpha}}} \left[\frac{p_{sH}(X,s)\mathcal{L}'(X,s)^{\frac{1+\nu}{1+\nu-\alpha}}}{\mathcal{L}'(X,s) - x_L} - \frac{p_{sL}(X,s)\mathcal{L}'(X,s)^{\frac{1+\nu}{1+\nu-\alpha}}}{x_H - \mathcal{L}'(X,s)} \right], \tag{A.69}$$

given $\sigma_s[x_{s'}] = \sqrt{p_{sL}p_{sH}}\Delta x$.

The first term inside brackets in the expression above is decreasing in $\mathcal{L}'(X,s)$ if and only if the following condition holds

$$\frac{1+\nu}{1+\nu-\alpha}\mathcal{L}'(X,s)^{\frac{1+\nu}{1+\nu-\alpha}-1}(\mathcal{L}'(X,s) - x_L) - \mathcal{L}'(X,s)^{\frac{1+\nu}{1+\nu-\alpha}} < 0 \iff \mathcal{L}'(X,s) < \frac{1+\nu}{\alpha}x_L, \tag{A.70}$$

which holds, given that $\mathcal{L}'(X,s) \leq x_H < \frac{1+\nu}{\alpha}x_L$.

Therefore, the demand for risk is decreasing in $\mathcal{L}'(X,s)$. As $\mathcal{L}'(X,s)$ is decreasing in the Sharpe ratio of the risky asset, then the demand for risk is increasing in the Sharpe ratio.

Supply of risk. The volatility of returns is given by

$$\sigma_s[R_p(X,s,s')] = \frac{x_s}{\beta}\frac{\sigma_s[x_{s'}]\mathcal{L}'(X,s)^{\frac{\alpha}{1+\nu-\alpha}}}{x_s\mathcal{L}^{\frac{\alpha}{1+\nu-\alpha}} - \frac{\alpha}{1+\nu}\mathcal{L}^{\frac{1+\nu}{1+\nu-\alpha}}}, \tag{A.71}$$

which is increasing in $\mathcal{L}'(X,s)$ and decreasing in x_s.

Equilibrium. Combining supply and demand for risk, we obtain

$$\frac{x_s}{\beta}\frac{\sigma_s[x_{s'}]\mathcal{L}'(X,s)^{\frac{\alpha}{1+\nu-\alpha}}}{x_s\mathcal{L}^{\frac{\alpha}{1+\nu-\alpha}} - \frac{\alpha}{1+\nu}\mathcal{L}^{\frac{1+\nu}{1+\nu-\alpha}}} = \frac{1+\nu-\alpha}{1+\nu}\frac{x_s}{\beta}\frac{\sigma_s[x_{s'}]\mathcal{L}'(X,s)^{\frac{1+\nu}{1+\nu-\alpha}}}{x_s\mathcal{L}^{\frac{\alpha}{1+\nu-\alpha}} - \frac{\alpha}{1+\nu}\mathcal{L}^{\frac{1+\nu}{1+\nu-\alpha}}}\left[\frac{p_{sH}(X,s)}{\mathcal{L}'(X,s) - x_L} - \frac{p_{sL}(X,s)}{x_H - \mathcal{L}'(X,s)}\right]. \tag{A.72}$$

The left-hand side is strictly increasing in $\mathcal{L}'(X,s)$, while the right-hand side is strictly decreasing in $\mathcal{L}'(X,s)$ in the interval $x_L < \mathcal{L}'(X,s) < x_H$. The right-hand side converges to $+\infty$ as $\mathcal{L}'(X',s)$ approaches x_L from above, and it converges to $-\infty$ as $\mathcal{L}'(X,s)$ approaches x_H from below. Therefore, there exists a unique value of $\mathcal{L}'(X,s)$ solving the equation above in this interval. Note that the two curves intersect again for $\mathcal{L}'(X,s) > x_H$, which can be seen by noticing that the right-hand is decreasing in $\mathcal{L}'(X,s)$ for $\mathcal{L}'(X,s) > x_H$ and converges to $+\infty$ as $\mathcal{L}'(X,s)$ approaches x_H from above. Therefore, the economically relevant solution corresponds to the smallest of the two points of intersection.

Rearranging the expression above, we obtain

$$1 = \frac{1+\nu-\alpha}{1+\nu}\mathcal{L}'(X,s)\frac{p_{sH}(X,s)(x_H - \mathcal{L}'(X,s)) - p_{sL}(X,s)(\mathcal{L}'(X,s) - x_L)}{(\mathcal{L}'(X,s) - x_L)(x_H - \mathcal{L}'(X,s))}. \tag{A.73}$$

We then obtain a quadratic equation for $\mathcal{L}'(X,s)$:

$$\frac{\alpha}{1+\nu}\mathcal{L}'(X,s)^2 - \left[\left(1 - \frac{1+\nu-\alpha}{1+v}p_{sH}(X,s)\right)x_H + \left(1 - \frac{1+\nu-\alpha}{1+v}p_{sL}(X,s)\right)x_L\right]\mathcal{L}'(X,s) + x_L x_H = 0 \tag{A.74}$$

The equilibrium value is given by the smallest root of the equation above.

□

A.7 Proof of Proposition 4

Proof. Consider the case where labor can be chosen conditional on the current productivity level. In this case, hours and detrended profits are given by

$$h(X,s) = \left(\frac{\alpha x_s}{\xi}\right)^{\frac{1}{1-\alpha}}, \qquad \pi(X,s) = (1-\alpha)\left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} x_s^{\frac{1}{1-\alpha}}. \tag{A.75}$$

Notice that hours and profits are independent of beliefs.

Under log utility, returns for the risky asset and the risk-free asset are given by

$$R_r(X,s,s') = \frac{x_s}{\beta}\frac{\tilde{x}_{s'}}{\tilde{x}_s}, \qquad R_f(X,s) = \frac{x_s}{\beta}\frac{\tilde{\mathcal{L}}'(X,s)}{\tilde{x}_s}, \tag{A.76}$$

where $\tilde{\mathcal{L}}'(X,s) \equiv \mathbb{E}^Q[\tilde{x}_{s'}]$, where $\mathbb{E}^Q[\cdot]$ denotes the expectation under the risk-neutral measure, and $\tilde{x}_s = x_{s'}^{\frac{1}{1-\alpha}}$.

The excess return on the risky asset is then given by

$$R_r^e(X,s,s') = \frac{\tilde{x}_{s'} - \tilde{\mathcal{L}}(X,s)}{\tilde{\mathcal{L}}(X,s)}. \tag{A.77}$$

The portfolio share is given by

$$\omega_i(X,s) = p_{sH}^i \frac{\tilde{\mathcal{L}}'(X,s)}{\tilde{\mathcal{L}}'(X,s) - \tilde{x}_L} - p_{sL}^i \frac{\tilde{\mathcal{L}}'(X,s)}{x_H - \tilde{\mathcal{L}}'(X,s)}. \tag{A.78}$$

The market clearing for the risky asset implies

$$\overline{p}^m_{sH} \frac{\tilde{\mathcal{L}}'(X,s)}{\tilde{\mathcal{L}}'(X,s) - \tilde{x}_L} = 1 + \overline{p}^m_{sL} \frac{\tilde{\mathcal{L}}'(X,s)}{x_H - \tilde{\mathcal{L}}'(X,s)}. \tag{A.79}$$

The left-hand side is strictly decreasing in $\tilde{\mathcal{L}}'(X,s)$, it approaches $+\infty$ as $\tilde{\mathcal{L}}'(X,s)$ approaches x_L from above, and it approaches $\frac{\overline{p}^m_{sH} x_H}{x_H - x_L}$ as $\tilde{\mathcal{L}}'(X,s)$ approaches x_H from below. The right-hand side is strictly increasing in $\tilde{\mathcal{L}}'(X,s)$, it approaches $+\infty$ as $\tilde{\mathcal{L}}'(X,s)$ approaches x_H from below, and it approaches $\frac{\overline{p}^m_{sL} x_L}{x_H - x_L}$ as $\tilde{\mathcal{L}}'(X,s)$ approaches x_L from above. Hence, there exists a unique value of $\tilde{\mathcal{L}}'(X,s)$ solving the equation above.

Therefore, as market beliefs become more optimistic, the interest rate increases and the risk premium decreases. However, there is no effect on labor demand or return volatility.

□

A.8 Proof of Proposition 5

Proof. The portfolio share of investor i is given by

$$\omega_i(X,s) = \frac{p^i_{s,H}}{|R^e_r(X,s,L)|} - \frac{p^i_{s,L}}{R^e_r(X,s,H)}, \tag{A.80}$$

The return on the investor's portfolio is given by

$$R_i(X,s,s') = R_f(X,s)\left[1 + \omega_i(X,s) R^e_r(X,s,s')\right] \tag{A.81}$$

$$= R_f(X,s)\left[1 + (x_{s'} - \mathcal{L}'(X,s))\left(\frac{p^i_{sH}}{\mathcal{L}'(X,s) - x_L} - \frac{p^i_{sL}}{x_H - \mathcal{L}'(X,s)}\right)\right]. \tag{A.82}$$

We can write the expression above as follows

$$R_i(X,s,L) = \frac{\Delta x R_f(X,s)}{x_H - \mathcal{L}'(X,s)} p^i_{sL}, \qquad R_i(X,s,h) = \frac{\Delta x R_f(X,s)}{\mathcal{L}'(X,s) - x_L} p^i_{sH}. \tag{A.83}$$

Wealth share dynamics. The share of wealth of investor i is given by

$$\eta'_i(X,s,s') = \frac{\eta_i R_{i,n}(X,s,s')}{\sum_{j=1}^I \eta_j R_{j,n}(X,s,s')} = \frac{\eta_i p^i_{ss'}}{\sum_{j=1}^I \eta_j p^j_{ss'}} = \eta_i \frac{p^i_{ss'}}{p_{ss'}(X)}. \tag{A.84}$$

The case of arbitrary risk aversion. Consider the case of unit-EIS, so the consumption-wealth ratio is still given by $c_i(X,s) = 1 - \beta$, but arbitrary risk aversion $\gamma > 0$. In this case, Propostion 3 still holds. Moreover, $\tilde{\eta}_i(X,s) = \eta_i$.

As shown in Appendix A.3, the portfolio share of investor i is given by

$$\omega_i(X,s) = \frac{\tilde{p}_i(X,s,H)}{|R^e_r(X,s,L)|} - \frac{\tilde{p}_i(X,s,L)}{R^e_r(X,s,H)}, \tag{A.85}$$

where

$$\tilde{p}_i(X,s,s') = \frac{(p^i_{ss'})^{\frac{1}{\gamma}}\left[v_i(\chi(X,s,s'),s')|R^e_r(X,s,s')|\right]^{\frac{1}{\gamma}-1}}{\sum_{s'\in\{L,H\}}(p^i_{ss'})^{\frac{1}{\gamma}}\left[v_i(\chi(X,s,s'),s')|R^e_r(X,s,s')|\right]^{\frac{1}{\gamma}-1}}. \tag{A.86}$$

A derivation similar to the one above shows that the wealth share of investor i evolves according to

$$\eta'_i(X,s,s') = \eta_i \frac{\tilde{p}^i_{ss'}}{\tilde{p}^m_{ss'}(X,s)}, \tag{A.87}$$

where $\tilde{p}^m_{ss'}(X,s) \equiv \sum_{j=1}^I \eta_j \tilde{p}^j_{ss'}(X,s)$. Hence, $\frac{\partial \eta'_i(X,s,s')}{\partial \eta_i} > 0$.

□

A.9 Proof of Corollary 2

Proof. Consider an economy that starts at $s = H$ with wealth distribution $\{\eta_i\}_{i=1}^I$ which switches to the low state after either one period (early transition) or two periods (late transition). To simplify notation, we write $p_{ss'}(X)$ to denote $\overline{p}^m_{ss'}(X,s)$.

Market beliefs on the low state in the case of an early transition are given by

$$p_{LH}(X') = \sum_{i=1}^I \eta'_i p^i_{LH} = \sum_{i=1}^I \eta_i \frac{p^i_{HL}}{p_{HL}(X)} p^i_{LH}, \tag{A.88}$$

and market beliefs on the low state in the case of a late transition are given by

$$p_{LH}(X'') = \sum_{i=1}^I \eta''_i p^i_{LH} = \sum_{i=1}^I \eta'_i \frac{p^i_{HL}}{p_{HL}(X')} p^i_{LH}, \tag{A.89}$$

where $\eta'_i = \eta_i \frac{p^i_{HH}}{p_{HH}(X)}$.

Note that if investor i is optimistic, $p^i_{HH} > p_{HH}(X)$, then $\eta'_i > \eta_i$ and $p^{-i}_{HL}(X') \leq p^{-i}_{HL}(X)$, where $p^{-i}_{HL}(X) \equiv \frac{1}{1-\eta_i}\sum_{j\neq i}\eta_j p^j_{HL}$. Hence, the following inequality holds:

$$\eta'_i \frac{p^i_{HL}}{p_{HL}(X')} = \frac{\eta'_i p^i_{HL}}{\eta'_i p^i_{HL} + (1-\eta'_i)p^{-i}_{HL}(X')} > \frac{\eta_i p^i_{HL}}{\eta_i p^i_{HL} + (1-\eta_i)p^{-i}_{HL}(X)} = \eta_i \frac{p^i_{HL}}{p_{HL}(X)}. \tag{A.90}$$

Therefore, there is more weight on the beliefs of investors who were optimistic in the original state in the case of a late transition. In the case of rank-preserving beliefs, these agents are also optimistic in the low state, so the market is more optimistic under a late transition:

$$p_{LH}(X'') > p_{LH}(X'). \tag{A.91}$$

Alternatively, the market is now more pessimistic after a late transition in the case of rank-alternating beliefs:

$$p_{LH}(X'') < p_{LH}(X'). \tag{A.92}$$

A similar argument shows that, under rank-preserving beliefs, the market is more pessimistic after a late transition when the economy starts at state $s = L$:

$$p_{HH}(X'') = \sum_{i=1}^{I} \eta_i' \frac{p_{LH}^i}{p_{LH}(X')} p_{HH}^i < \sum_{i=1}^{I} \eta_i \frac{p_{LH}^i}{p_{LH}(X)} p_{HH}^i = p_{HH}(X), \tag{A.93}$$

where $\eta_i' = \eta_i \frac{p_{LL}^i}{p_{LL}(X)}$. Alternatively, the market is more optimistic under a late transition in the case of rank-alternating beliefs.

□

A.10 Proof of Lemma 1

Proof. Turnover is given by

$$\tau(X, s, s') = \frac{1}{2} \sum_{i=1}^{I} |\omega_i(X', s')\eta_i'(X, s, s') - \omega_i(X, s)\eta_i|, \tag{A.94}$$

where $X' = \chi(X, s, s')$ and $\eta_i'(X, s, s') = \eta_i \frac{p_{ss'}^i}{\overline{p}_{ss'}^m(X,s)}$.

Solving for the portfolio share. Using the expression for the economy-wide SDF and Equation (A.24), we can write the portfolio share as follows

$$\omega_i(X, s) = \frac{p_{sH}^i}{|R_r^e(X, s, L)|} - \frac{p_{sL}^i}{R_r^e(X, s, H)}. \tag{A.95}$$

The excess return on the risky asset can be written as follows:

$$R_r^e(X, s, s') = \frac{x_{s'} - \mathcal{L}'(X, s)}{(1 - \alpha)\mathcal{L}'(X, s)}. \tag{A.96}$$

Combining the previous expressions, we obtain

$$\omega_i(X,s) = (1-\alpha)\left[p^i_{sH}\frac{\mathcal{L}'(X,s)}{\mathcal{L}'(X,s)-x_L} - p^i_{sL}\frac{\mathcal{L}'(X,s)}{x_H-\mathcal{L}'(X,s)}\right], \tag{A.97}$$

which is strictly decreasing in $\mathcal{L}'(X,s)$ and $\omega_i(X,s) > 1$ if and only if $p^i_{sH} > \overline{p}^m_{sH}(X,s)$.

Turnover is then given by

$$\tau(X,s,s') = (1-\alpha)\sum_{i=1}^{I}\eta_i\left|\left(\frac{p^i_{s'H}\mathcal{L}'(X',s')}{\mathcal{L}'(X',s')-x_L} - \frac{p^i_{s'L}\mathcal{L}'(X',s')}{x_H-\mathcal{L}'(X',s')}\right)\frac{p^i_{ss'}}{\overline{p}^m_{ss'}(X,s)} - \left(\frac{p^i_{sH}\mathcal{L}'(X,s)}{\mathcal{L}'(X,s)-x_L} - \frac{p^i_{sL}\mathcal{L}'(X,s)}{x_H-\mathcal{L}'(X,s)}\right)\right| \tag{A.98}$$

Perturbation. It is useful to parameterize the dispersion in beliefs as follows:

$$p^i_{ss'} = p^*_{ss'} + \epsilon\delta^i_{ss'}, \tag{A.99}$$

where $\delta^i_{sH} + \delta^i_{sL} = 0$.

Notice that all equilibrium variables now depend on ϵ. For instance, the average probability of the high state can be written as

$$\overline{p}^m_{sH}(X;\epsilon) = p^*_{sH} + \delta_{sH}(X,s)\epsilon + \mathcal{O}(\epsilon^2), \tag{A.100}$$

where $\delta_{sH}(X,s) \equiv \sum_{i=1}^{I}\eta_i\delta^i_{sH}$.

The risk-neutral expectation of productivity growth is a function of market beliefs, $\mathcal{L}'(X,s;\epsilon) = f(\overline{p}^m_{sH}(X))$, where $f(p)$ satisfies the condition

$$1 = (1-\alpha)\left[p\frac{f(p)}{f(p)-x_L} - (1-p)\frac{f(p)}{x_H-f(p)}\right] \Rightarrow f'(p) = \frac{\frac{f(p)}{f(p)-x_L} + \frac{f(p)}{x_H-f(p)}}{p\frac{x_L}{(f(p)-x_L)^2} + (1-p)\frac{x_H}{(x_H-f(p))^2}}. \tag{A.101}$$

Let $\mathcal{L}^*(X,s) \equiv \mathcal{L}'(X,s;0)$ denote the value of $\mathcal{L}'(X,s)$ when $\epsilon = 0$. In this case, we can drop the dependence on X and simply write $\mathcal{L}^*(s)$, as $\mathcal{L}'(X,s)$ would only depend on the state s. We can then expand $\mathcal{L}'(X,s;\epsilon)$ in ϵ to obtain:

$$\mathcal{L}'(X,s;\epsilon) = \mathcal{L}^*(s) + \tilde{\mathcal{L}}(X,s)\epsilon + \mathcal{O}(\epsilon^2), \tag{A.102}$$

where $\tilde{\mathcal{L}}(X,s) = f'(p^*_{sH})\sum_{i=1}^{I}\eta_i\delta^i_{sH}$, where $f'(\cdot) > 0$.

We can then write the portfolio share of investor i as follows

$$\omega_i(X,s;\epsilon) = 1 + \left[\theta_{\omega,1}(s)\delta^i_{sH} - \theta_{\omega,2}(s)\delta_{sH}(X)\right]\epsilon + \mathcal{O}(\epsilon^2), \tag{A.103}$$

where $\theta_{\omega,1}(s) > 0$ and $\theta_{\omega,2}(s) > 0$

$$\theta_{\omega,1}(s) \equiv (1-\alpha)\left(\frac{\mathcal{L}^*(s)}{\mathcal{L}^*(s)-x_L} + \frac{\mathcal{L}^*(s)}{x_H-\mathcal{L}^*(s)}\right) \tag{A.104}$$

$$\theta_{\omega,2}(s) \equiv (1-\alpha)\left[\frac{p^*_{sH}x_L}{(\mathcal{L}^*(s)-x_L)^2} + \frac{p^*_{sL}x_H}{(x_H-\mathcal{L}^*(s))^2}\right] f'(p^*_{sH}). \tag{A.105}$$

Using the expression for $f'(\cdot)$, we obtain that $\theta_{\omega,1} = \theta_{\omega,2}$. We can then write $\omega_i(X,s;\epsilon)$ as follows:

$$\omega_i(X,s;\epsilon) = 1 + \theta_{\omega,1}(s)\left[\delta^i_{sH} - \delta_{sH}(X)\right]\epsilon + \mathcal{O}(\epsilon^2), \tag{A.106}$$

The evolution of wealth is given by

$$\eta'_i(X,s,s';\epsilon) = \eta_i + \eta_i\frac{\delta^i_{ss'} - \delta_{ss'}(X)}{p^*_{ss'}}\epsilon + \mathcal{O}(\epsilon^2) \tag{A.107}$$

Let $p_H(X,s,s';\epsilon) = \sum_{i=1}^I \eta'_i(X,s,s';\epsilon)p^i_{s'H}$ denote the market-implied probability of the high state after a transition from state s to state s', then

$$p_H(X,s,s';\epsilon) = p^*_{s'H} + \delta_{s'H}(X)\epsilon + \mathcal{O}(\epsilon^2), \tag{A.108}$$

using the fact that $\sum_{i=1}^I \eta_i\frac{\delta^i_{ss'}-\delta_{ss'}(X)}{p^*_{ss'}}p^*_{s'H} + \sum_{i=1}^I \eta_i\delta^i_{s'H} = \sum_{i=1}^I \eta_i\delta^i_{s'H} \equiv \delta_{s'H}(X)$.

The portfolio share next period is given by

$$\omega'_i(X,s,s';\epsilon) = 1 + \theta_{\omega,1}(s')\left[\delta^i_{s'H} - \delta_{s'H}(X)\right]\epsilon + \mathcal{O}(\epsilon^2). \tag{A.109}$$

Investor i's net purchases of shares is given by

$$\begin{aligned}\Delta S_i(X,s,s';\epsilon) = \eta_i &\left[\frac{\delta^i_{ss'} - \delta_{ss'}(X)}{p^*_{ss'}} + \theta_{\omega,1}(s')\left(\delta^i_{s'H} - \delta_{s'H}(X)\right)\right]\epsilon \\ &- \theta_{\omega,1}(s)\eta_i\left[\delta^i_{sH} - \delta_{sH}(X)\right]\epsilon + \mathcal{O}(\epsilon^2)\end{aligned} \tag{A.110}$$

For simplicity, suppose that investors believe productivity growth to be iid in the reference economy, that is, $p^*_{Ls'} = p^*_{Hs'}$. We can then write the

$$\mu_i\Delta S_i(X,s,s';\epsilon) = \left[\underbrace{\Delta\omega_i(X,s,s')\eta_i}_{\text{change-in-beliefs effect}} + \underbrace{\Delta\eta_i(X,s,s')}_{\text{rebalancing effect}}\right]\epsilon + \mathcal{O}(\epsilon^2), \tag{A.111}$$

where, given $\theta_{\omega,1}(L) = \theta_{\omega,1}(H) \equiv \kappa_\omega$,

$$\Delta\omega_i(X,s,s') \equiv \kappa_\omega \left[\left(\delta^i_{s'H} - \delta_{s'H}(X,s) \right) - \left(\delta^i_{sH} - \delta_{sH}(X) \right) \right] \tag{A.112}$$

$$\Delta\eta_i(X,s,s') \equiv \eta_i \frac{\delta^i_{ss'} - \delta_{ss'}(X)}{p^*_{ss'}}. \tag{A.113}$$

□

A.11 Proof of Proposition 6

Proof. Turnover is given by

$$\tau(X,s,s';\epsilon) = \frac{1}{2}\sum_{i=1}^{I} \eta_i \left| \frac{\tilde{\delta}^i_{ss'}}{p^*_{ss'}} + \kappa_\omega \left(\tilde{\delta}^i_{s'H} - \tilde{\delta}^i_{sH} \right) \right| \epsilon + \mathcal{O}(\epsilon^2), \tag{A.114}$$

where $\tilde{\delta}^i_{ss'} = \delta^i_{ss'} - \delta_{ss'}(X)$.

Suppose $s = s'$, then

$$\begin{aligned}
\tau(X,s,s';\epsilon) &= \frac{1}{2}\sum_{i=1}^{I} \eta_i \frac{\left|\tilde{\delta}^i_{ss'}(X)\right|}{p^*_{ss'}} \epsilon + \mathcal{O}(\epsilon^2) && \text{(A.115)}\\
&= \frac{1}{2}\left[\sum_{i=1}^{I} \eta_i \frac{\tilde{\delta}^i_{ss'}(X)}{p^*_{ss'}} \mathbf{1}_{\tilde{\delta}^i_{ss'}(X)\geq 0} - \sum_{i=1}^{I} \eta_i \frac{\tilde{\delta}^i_{ss'}(X)}{p^*_{ss'}} \mathbf{1}_{\tilde{\delta}^i_{ss'}<0} \right] \epsilon + \mathcal{O}(\epsilon^2) && \text{(A.116)}\\
&= \frac{1}{2}\left[\eta_B \frac{\tilde{\delta}^B_{ss'}(X)}{p^*_{ss'}} + \eta_S \frac{|\tilde{\delta}^S_{ss'}(X)|}{p^*_{ss'}} \right] \epsilon + \mathcal{O}(\epsilon^2). && \text{(A.117)}
\end{aligned}$$

where

$$\eta_B \equiv \sum_{i=1}^{I} \eta_i \mathbf{1}_{\tilde{\delta}^i_{ss'}(X)\geq 0}, \qquad \tilde{\delta}^B_{ss'}(X) \equiv \frac{1}{\eta_B}\sum_{i=1}^{I} \eta_i \tilde{\delta}^i_{ss'}(X) \mathbf{1}_{\tilde{\delta}^i_{ss'}(X)\geq 0}, \tag{A.118}$$

$$\eta_S \equiv \sum_{i=1}^{I} \eta_i \mathbf{1}_{\tilde{\delta}^i_{ss'}(X)<0}, \qquad \tilde{\delta}^S_{ss'}(X) \equiv \frac{1}{\eta_S}\sum_{i=1}^{I} \eta_i \tilde{\delta}^i_{ss'}(X) \mathbf{1}_{\tilde{\delta}^i_{ss'}(X)<0}. \tag{A.119}$$

We can write turnover in this case as follows

$$\tau(X,s,s';\epsilon) = \eta_B \eta_S \frac{\delta^B_{ss'}(X) - \delta^S_{ss'}(X)}{p^*_{ss'}} \epsilon + \mathcal{O}(\epsilon^2), \tag{A.120}$$

using the fact that $\eta_B \tilde{\delta}^B_{ss'}(X) + \eta_S \tilde{\delta}^S_{ss'}(X) = 0$ and $\tilde{\delta}^B_{ss'}(X) = \eta_S(\delta^B_{ss'}(X) - \delta^S_{ss'}(X))$.

Heterogeneous persistence. We consider next the special case where investors agree about the unconditional mean of x, but they disagree about the persistence of the aggregate productivity growth.

The stationary distribution of beliefs for investor i is given by

$$\overline{p}^i_L = \frac{p^i_{HL}}{p^i_{LH} + p^i_{HL}}. \tag{A.121}$$

We assume that $\overline{p}^i_L$ is equalized across investors, so all investors agree about the unconditional mean of x_t. Note this implies that the likelihood ratio p^i_{LH}/p^i_{HL} is equalized across investors. The unconditional mean is given by

$$\overline{x} = \frac{p^i_{HL}}{p^i_{LH} + p^i_{HL}} x_L + \frac{p^i_{LH}}{p^i_{LH} + p^i_{HL}} x_H. \tag{A.122}$$

The expected value of x_{t+1} relative to the mean $\overline{x}$ conditional on $x_t = x_L$ is given by

$$\begin{aligned} \mathbb{E}_i[x_{t+1} - \overline{x} | x_t = x_L] &= p^i_{LL}(x_L - \overline{x}) + p^i_{LH}(x_H - \overline{x}) && \text{(A.123)} \\ &= \left[1 + p^i_{LH} \frac{x_H - x_L}{x_L - \overline{x}}\right] (x_L - \overline{x}) && \text{(A.124)} \\ &= \left[1 - (p^i_{LH} + p^i_{HL})\right] (x_L - \overline{x}), && \text{(A.125)} \end{aligned}$$

using the fact that $\overline{x} - x_L = \frac{p^i_{LH}}{p^i_{LH} + p^i_{HL}} (x_H - x_L)$

We obtain a similar expression conditioning on $x_t = x_H$ instead:

$$\begin{aligned} \mathbb{E}_i[x_{t+1} - \overline{x} | x_t = x_H] &= p^i_{HL}(x_L - \overline{x}) + p^i_{HH}(x_H - \overline{x}) && \text{(A.126)} \\ &= \left[1 - p^i_{HL} \frac{x_H - x_L}{x_H - \overline{x}}\right] (x_H - \overline{x}) && \text{(A.127)} \\ &= \left[1 - (p^i_{LH} + p^i_{HL})\right] (x_H - \overline{x}), && \text{(A.128)} \end{aligned}$$

using the fact that $x_H - \overline{x} = \frac{p^i_{HL}}{p^i_{LH} + p^i_{HL}} (x_H - x_L)$.

Let $\hat{x}_t = x_t - \overline{x}$, we can then write

$$\mathbb{E}_i[\hat{x}_{t+1} | \hat{x}_t] = \rho_i \hat{x}_t, \tag{A.129}$$

where $\rho_i \equiv 1 - (p^i_{LH} + p^i_{HL}) = p^i_{HH} - p^i_{LH}$.

Given that investors agree about the unconditional mean of x, we are able to pin down beliefs as a function of θ_i:

$$p^i_{LH} = \overline{p}_H(1 - \rho_i), \qquad p^i_{HH} = \overline{p}_H + \overline{p}_L\rho_i. \tag{A.130}$$

Corollary. Under the assumption investors agree about the unconditional mean of x_t, we have that

$$p^i_{LH} - p_{LH}(X) = -\overline{p}_H(\rho_i - \rho(X)), \qquad p^i_{HH} - p_{HH}(X) = \overline{p}_L(\rho_i - \rho(X)), \tag{A.131}$$

where $\rho(X) \equiv \sum_{i=1}^{I} \eta_i \rho_i$.

Notice that we have that $\tilde{\delta}^i_{ss'}(X)\epsilon = p^i_{ss'} - p_{ss'}(X)$, which gives us

$$\tilde{\delta}^i_{LH}(X)\epsilon = -p^*_H(\rho_i - \rho(X)), \qquad \tilde{\delta}^i_{HH}(X)\epsilon = (1 - p^*_H)(\rho_i - \rho(X)). \tag{A.132}$$

We can then write turnover in the case $s = L$ and $s' = H$ as follows:

$$\tau(X, L, H; \epsilon) = \frac{1}{2}\,|\kappa_\omega - 1| \sum_{i=1}^{I} \eta_i |\rho_i - \rho(X)| + \mathcal{O}(\epsilon^2). \tag{A.133}$$

Consider now the case $s = H$ and $s' = L$:

$$\tau(X, H, L; \epsilon) = \frac{1}{2}\,|\kappa_\omega + 1| \sum_{i=1}^{I} \eta_i |\rho_i - \rho(X)| + \mathcal{O}(\epsilon^2), \tag{A.134}$$

Suppose now that $s = s'$, then

$$\tau(X, H, H; \epsilon) = \frac{1}{2}\left|\frac{p^*_L}{p^*_H}\right| \sum_{i=1}^{I} \eta_i |\rho_i - \rho(X)| + \mathcal{O}(\epsilon^2) \tag{A.135}$$

$$\tau(X, L, L; \epsilon) = \frac{1}{2}\left|\frac{p^*_H}{p^*_L}\right| \sum_{i=1}^{I} \eta_i |\rho_i - \rho(X)| + \mathcal{O}(\epsilon^2). \tag{A.136}$$

□

B Hand-to-Mouth Workers

This section considers a version of the model in which labor is supplied by hand-to-mouth workers rather than by investors. We show that this modification leaves our main results unchanged. In particular, in the special case $\nu = 0$ considered in Section 4, the equilibrium allocation and asset prices are identical to those in the baseline model.

Environment. The economy is populated by two types of households: investors and workers.

There is a finite number of investor types, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$, with masses $\{\mu_i\}$ satisfying $\sum_i \mu_i = 1$. Investors behave exactly as in the baseline model, except that we assume their labor disutility is infinite, so that they supply no labor in equilibrium: $h_{i,t} = 0$.

In addition, there is a unit mass of workers. Workers are hand-to-mouth and consume all their labor income:

$$C_{w,t} = W_t h_{w,t}.$$

Workers have GHH preferences, and their labor supply condition is

$$W_t = \xi_t h_{w,t}^{\nu}, \qquad \xi_t = \xi A_{t-1},$$

as in the baseline model.

Characterization. Investor i solves Problem (1). Hence, the characterization of investors' portfolio and consumption choices is unchanged relative to the baseline model. The firm's problem is also unchanged.

Labor market clearing implies

$$h_t = h_{w,t}.$$

Since the worker labor supply condition coincides with the labor supply equation in the baseline model, the equilibrium determination of hours and wages is identical.

Because workers do not save, financial market clearing conditions are unchanged:

$$\sum_{i=1}^{I} \mu_i B_{i,t} = 0, \qquad \sum_{i=1}^{I} \mu_i S_{i,t} = 1.$$

The only difference arises in goods market clearing:

$$C_{w,t} + \sum_{i=1}^{I} \mu_i C_{i,t} = A_t h_t^{\alpha}.$$

Using $C_{w,t} = W_t h_t = \xi_t h_t^{1+\nu}$, we obtain

$$\sum_{i=1}^{I} \mu_i C_{i,t} = A_t h_t^{\alpha} - \xi_t h_t^{1+\nu}.$$

In the baseline model, where investors supply labor, the goods market clearing condition can be written in terms of net consumption $\tilde{C}_{i,t} \equiv C_{i,t} - \xi_t \frac{h_t^{1+\nu}}{1+\nu}$:

$$\sum_{i=1}^{I} \mu_i \tilde{C}_{i,t} = A_t h_t^{\alpha} - \xi_t \frac{h_t^{1+\nu}}{1+\nu}.$$

Equivalence when $\nu = 0$. When $\nu = 0$, labor income equals labor disutility in the baseline model. In this case, the two goods market clearing conditions coincide exactly. Hence, equilibrium allocations, asset prices, wealth dynamics, and the evolution of market beliefs are identical to those in the baseline economy.

Economic intuition. Separating workers and investors therefore does not alter the mechanism driving our results. What matters is the timing of hiring, not who supplies labor.

When $\nu > 0$, investor consumption becomes more volatile in the hand-to-mouth specification, because labor income accrues to workers, whose consumption is smoother than output. As a result, investors absorb a larger share of aggregate risk, consistent with the evidence on the consumption of stockholders (see, e.g., Mankiw and Zeldes 1991 and Parker 2001).[26] This *strengthens* the operating-leverage channel without overturning any of the qualitative results.

More generally, the mechanism linking beliefs, risk premia, and labor demand operates through asset pricing and the firm's first-order condition, and does not rely on investors supplying labor directly.

C Connection with Search Models

This appendix clarifies how our labor-demand block relates to a canonical search-and-matching model. In particular, we present a generalized formulation that nests both our baseline model and the search model in Shimer (2010) as special cases.

[26] This mechanism is reminiscent of, e.g., Danthine and Donaldson (2002) where limited market participation of workers, combined with firms insuring workers against labor risk, creates a similar operating-leverage channel.

The mapping between our "labor chosen one-period in advance" formulation and the search model provides three useful insights. First, it shows that predetermined labor arises naturally in search environments. Second, our baseline specification corresponds to the limit of a search model in which recruiting costs vanish and workers receive no surplus. This limit delivers a particularly tractable benchmark. Third, the mapping clarifies how the neutrality result emphasized by Shimer (2010) relates to movements in the risk-neutral expectation of productivity growth in our model, thereby connecting the labor volatility puzzle to asset-pricing mechanisms.

C.1 Environment

The firm. Consider a representative firm employing h_t workers at time t. Each period the firm chooses recruiting intensity v_t, defined as the fraction of workers allocated to recruiting. Recruiting intensity v_t implies that the firm attracts $h_t v_t \mu(\theta_t)$ new workers on average, where θ_t denotes labor-market tightness and $\mu(\theta_t)$ is the job-filling rate. Workers separate exogenously at rate s and are paid wage W_t.

Employment evolves according to

$$h_{t+1} = (1-s)h_t + h_t v_t \mu(\theta_t). \tag{C.1}$$

Output is produced using employed workers net of recruiting costs:

$$Y_t = A_t h_t^{\alpha}(1-\varepsilon v_t)^{\alpha},$$

where $\varepsilon \in [0,1]$ governs recruiting costs. The case $\varepsilon = 0$ corresponds to no recruiting costs, while $\varepsilon = 1$ implies that recruiters generate no output.

The firm solves

$$J_t(h_t) = \max_{v_t} \left\{ A_t h_t^{\alpha}(1-\varepsilon v_t)^{\alpha} - W_t h_t + \mathbb{E}_t[\Lambda_{t,t+1} J_{t+1}(h_{t+1})] \right\}, \tag{C.2}$$

subject to (C.1), where $\Lambda_{t,t+1}$ is the stochastic discount factor.

Because recruiting decisions affect next period's employment, labor is predetermined, as in Section 3. Relative to our baseline, the search formulation introduces explicit adjustment costs through ε.

C.2 Optimality conditions

Let $J_{h,t} \equiv \partial J_t / \partial h_t$. The first-order condition for recruiting intensity is

$$\alpha \varepsilon A_t h_t^{\alpha} (1 - \varepsilon v_t)^{\alpha - 1} = \mathbb{E}_t [\Lambda_{t,t+1} J_{h,t+1} h_t \mu(\theta_t)] \,. \tag{C.3}$$

The envelope condition is

$$J_{h,t} = \alpha A_t h_t^{\alpha-1} (1 - \varepsilon v_t)^{\alpha} - W_t + \mathbb{E}_t [\Lambda_{t,t+1} J_{h,t+1} (1 - s + v_t \mu(\theta_t))] \,. \tag{C.4}$$

Combining these expressions yields the recruiting Euler equation

$$\alpha A_t h_t^{\alpha-1} (1 - \varepsilon v_t)^{\alpha-1} \varepsilon = \mu(\theta_t) \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\alpha A_{t+1} h_{t+1}^{\alpha-1} (1 - \varepsilon v_{t+1})^{\alpha-1} \left(1 + \frac{1-s}{\mu(\theta_{t+1})} \varepsilon \right) - W_{t+1} \right) \right] . \tag{C.5}$$

Wage setting. We assume firms have all bargaining power and workers receive no surplus. Wages satisfy

$$W_t = C_t^{\chi} \xi_t h_t^{\nu} , \tag{C.6}$$

where χ controls income effects. The case $\chi = 0$ corresponds to GHH preferences, while $\chi = 1$ corresponds to separable log utility.

C.3 The Shimer benchmark

Impose the restrictions

$$\alpha = 1, \quad \varepsilon = 1, \quad \chi = 1, \quad \xi_t = \xi, \quad \nu = 0, \quad \Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}} .$$

Under these assumptions, the recruiting condition simplifies to

$$1 = \mu(\theta_t) \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\left(1 + \frac{1-s}{\mu(\theta_t)} \right) x_{t+1} - w_{t+1} \right) \right] , \tag{C.7}$$

where $x_{t+1} = A_{t+1}/A_t$ and $w_{t+1} = W_{t+1}/A_t$.

Define the risk-neutral expectations

$$\mathcal{L}_t \equiv \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\mathbb{E}_t[\Lambda_{t,t+1}]} x_{t+1} \right] , \quad \mathcal{W}_t \equiv \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\mathbb{E}_t[\Lambda_{t,t+1}]} w_{t+1} \right] .$$

The recruiting condition becomes

$$1 = (1 - s + \mu(\theta_t))\frac{\mathcal{L}_t}{R_{f,t}} - \mu(\theta_t)\frac{\mathcal{W}_t}{R_{f,t}}. \tag{C.8}$$

Proposition 7 (Labor volatility puzzle)**.** *There exists an equilibrium in which θ_t, h_t, v_t, W_t/A_t, and C_t/A_t are constant. Consequently, employment does not respond to productivity shocks.*

This reproduces the neutrality logic emphasized by Shimer (2010). With log utility and separable preferences, high expected (risk-neutral) productivity growth is offset by corresponding high discounting, keeping hiring incentives constant.

C.4 Labor chosen one-period in advance

Consider instead

$$\varepsilon = 0, \quad \chi = 0, \quad \xi_t = \xi A_{t-1}.$$

The recruiting Euler equation reduces to

$$0 = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\alpha A_{t+1}h_{t+1}^{\alpha-1} - W_{t+1}\right)\right], \tag{C.9}$$

which coincides with the labor-demand condition in Equation (**??**).

With GHH preferences, wages are predetermined, and the condition can be written as

$$\alpha\mathcal{L}_t h_{t+1}^{\alpha-1} = w_{t+1}, \tag{C.10}$$

where $\mathcal{L}_t$ is the risk-neutral expectation of productivity growth.

Comparison

The generalized search model nests both the Shimer benchmark and our baseline as special cases.

In the Shimer benchmark, (discounted) risk-neutral expectations are constant, and employment is insulated from productivity shocks. In our framework, belief heterogeneity and asset-pricing forces generate time variation in $\mathcal{L}_t$. Fluctuations in risk premia therefore break neutrality and allow productivity and belief shocks to propagate into employment.

Thus, our model embeds the search benchmark but identifies a financial channel — operating through risk-neutral valuation — that resolves the labor volatility puzzle while remaining consistent with observed asset-pricing dynamics.

Supplementary Appendix

O1 Data Appendix

O1.1 Fact 1: Volatility of Expectations (Dividend Growth)

Overview. This subsection describes the replication of Fact 1 in Section 2 of the paper: the variance share of (i) an "objective" dividend-growth forecast constructed from dividend dynamics and (ii) a survey-based "subjective" dividend-growth forecast.

Inputs. The replication uses two quarterly time series from the dividend-growth expectations dataset:

- *Realized next-year dividend growth* $\Delta d_{t,t+4}$ (realized next-year log dividend growth at quarterly dates);
- *Subjective expectations* $\mathbb{E}^{sub}[\Delta d_{t,t+4}]$ (I/B/E/S survey expectations of next-year log dividend growth).

To construct the objective forecast, we use the Shiller historical dataset and its real dividend series to build quarterly dividend growth.

Sample. The baseline replication uses a quarterly sample spanning 2003Q1–2021Q3 (75 observations), dictated by the overlap between the realized next-year dividend-growth series and the survey expectations series. The variance ratios are computed on the common non-missing sample for each expectations measure.

Objective expectations construction. Let g_t^d denote quarterly log dividend growth constructed from Shiller real dividends by taking the end-of-quarter (last monthly) observation in each quarter and forming log differences. We estimate an AR(1) on g_t^d:

$$g_t^d = a + \rho g_{t-1}^d + \varepsilon_t.$$

Given the estimated (a, ρ), it maps quarterly growth into an implied one-year-ahead expectation by iterating the AR(1) forward and summing the implied quarterly growth rates:

$$\mathbb{E}^{obj}[\Delta d_{t,t+4}] \equiv \sum_{h=1}^{4} \mathbb{E}_t[g_{t+h}^d], \qquad \mathbb{E}_t[g_{t+h}^d] = \mu + \rho^h \left(g_t^d - \mu\right),\ \mu = \frac{a}{1-\rho}.$$

Table O.1: Objective Dividend-Growth Expectations from AR(1): Sample Window Robustness

Sample window	Observations (N)	Variance ratio
Full (1871Q2–2022Q2)	605	0.0665
Post-1947 (1947Q1–2022Q2)	302	0.0599
Post-1976 (1976Q1–2022Q2)	186	0.0501
Post-2003 (2003Q1–2022Q2)	78	0.0495

Notes: The objective expectation is constructed from an AR(1) estimated on quarterly log dividend growth, $\Delta d_{t+1} = a + \rho \Delta d_t + \varepsilon_{t+1}$. The one-year-ahead expectation is the implied four-quarter forecast, $E_t[\Delta d_{t,t+4}]$. The reported variance ratio is $\text{Var}(E_t[\Delta d_{t,t+4}]) / \text{Var}(\Delta d_{t,t+4})$.

The resulting objective series is then aligned to the quarterly dates in the dividend-growth expectations dataset by matching year/quarter identifiers.

Variance ratios. For each expectations measure $m \in \{obj, sub\}$, the replication computes:

$$\frac{\text{Var}\big[\mathbb{E}^m[\Delta d_{t,t+4}]\big]}{\text{Var}[\Delta d_{t,t+4}]},$$

using the common sample for which both the realized series $\Delta d_{t,t+4}$ and the corresponding expectation series are non-missing (objective and subjective samples coincide in the default run).

Outputs and replicated numbers. We report the variance ratios implied by the baseline inputs used for the paper. The dividend-growth variance ratios are:

$$\frac{\text{Var}\big[\mathbb{E}^{obj}[\Delta d_{t,t+4}]\big]}{\text{Var}[\Delta d_{t,t+4}]} = 0.0486\ (\approx 0.05), \qquad \frac{\text{Var}\big[\mathbb{E}^{sub}[\Delta d_{t,t+4}]\big]}{\text{Var}[\Delta d_{t,t+4}]} = 0.8268\ (\approx 0.83).$$

O1.2 Fact 1 Robustness: Long-Sample Objective Expectations

Overview. This subsection reports long-sample robustness checks for the objective dividend-growth expectation in Fact 1. We keep the target object fixed,

$$\frac{\text{Var}\big[\mathbb{E}^{obj}[\Delta d_{t,t+4}]\big]}{\text{Var}[\Delta d_{t,t+4}]},$$

Table O.2: Objective Dividend-Growth Expectations: *p/d*-Only vs VAR

Sample window	N (*p/d* only)	Var. ratio (*p/d* only)	N (VAR)	Var. ratio (VAR)
Full (1871Q1–2022Q2)	606	0.0695	605	0.1356
Post-1947 (1947Q1–2022Q2)	302	0.0020	302	0.0936

Notes: The *p/d*-only specification is the direct horizon-4 regression, $\Delta d_{t,t+4} = a + b\,(p/d)_t + \varepsilon_{t+4}$. The VAR specification is a quarterly VAR(1) in $(\Delta d_t, p/d_t)$, and $E_t[\Delta d_{t,t+4}]$ is obtained from the model-implied four-step forecasts. In both columns, the reported statistic is $\text{Var}(E_t[\Delta d_{t,t+4}])\,/\text{Var}(\Delta d_{t,t+4})$.

and vary only the forecasting specification and sample window.

Data and quarterly construction. All robustness exercises use the Shiller historical market series. We convert monthly data to quarterly frequency by taking end-of-quarter (last monthly) observations of real prices and real dividends, then define:

$$\Delta d_t = \log D_t - \log D_{t-1}, \qquad \Delta d_{t,t+4} = \log D_{t+4} - \log D_t, \qquad pd_t = \log P_t - \log D_t.$$

AR(1)-implied objective expectation. The first robustness follows the same approach as the baseline Fact 1 construction. We estimate

$$\Delta d_{t+1} = a + \rho \Delta d_t + \varepsilon_{t+1},$$

form the implied four-quarter expectation $E_t[\Delta d_{t,t+4}]$, and compute the variance ratio across alternative sample windows. Results are reported in Table O.1.

***p/d*-only versus VAR benchmark.** The second robustness compares two objective forecasting approaches in long samples. (i) A direct horizon-4 regression:

$$\Delta d_{t,t+4} = a + b\, pd_t + \varepsilon_{t+4}.$$

(ii) A quarterly VAR(1) in $(\Delta d_t, pd_t)$, from which we construct $E_t[\Delta d_{t,t+4}]$ using model-implied four-step forecasts. Table O.2 reports both variance ratios for the full and post-1947 samples.

Interpretation. The AR(1)-implied objective expectation remains low-variance in long samples (around 0.05–0.07), close to the baseline Fact 1 estimate. The direct *p/d*-only approach contributes little in the post-1947 sample, while the VAR allows richer short-run dynamics and yields slightly larger objective-expectation variance ratios. In all specifications, movements in expectations, as inferred by an econometrician, explain only a small fraction of the total variation in realized dividend growth.

O1.3 Fact 3: I/B/E/S Expectations and Labor (Firm-Level)

Overview. This subsection describes the firm-level empirical exercise behind Fact 3 in Section 2: regressions of realized labor outcomes on lagged earnings-growth expectations from I/B/E/S with firm-clustered standard errors.

Data sources. The replication combines WRDS extracts from:

- Compustat Annual (`comp.funda`): `gvkey`, `datadate`, `fyear`, `emp`, `xlr`;
- CRSP monthly stock file (`crsp.msf`): `permno`, `date`, `ret`;
- CRSP–Compustat link table (`crsp.ccmxpf_linktable`);
- I/B/E/S Summary Statistics (`ibes.statsum_epsus`);
- WRDS I/B/E/S–CRSP historical link (`wrdsapps.ibcrsphist`);
- S&P 500 historical membership list (local `DSP500LIST_v4.csv`).

Sample selection. The baseline replication applies the following filters:

i. Compustat observations with non-missing `gvkey` and `datadate`;

ii. CRSP observations with valid monthly returns (`ret` > -1);

iii. I/B/E/S observations restricted to `measure=EPS`, horizons `FPI` $\in \{6,7,8,9\}$, and non-missing `statpers` and forecast value;

iv. I/B/E/S–CRSP links with `score`≤ 2;

v. CRSP–Compustat linking by valid link-date intervals;

vi. S&P 500 membership filter by `permno`-date using historical entry/exit dates from `DSP500LIST_v4.csv`.

Variable construction. For firm i and year t:

- Payroll outcome:

$$Y_{it}^{pay} = \frac{\Delta xlr_{it}}{\sigma_i(\Delta xlr)}, \qquad \Delta xlr_{it} = xlr_{it} - xlr_{i,t-1}.$$

- Workers outcome:

$$Y_{it}^{emp} = \frac{\Delta emp_{it}}{\sigma_i(\Delta emp)}, \qquad \Delta emp_{it} = emp_{it} - emp_{i,t-1}.$$

- Expectations regressor: from I/B/E/S, quarter-level expected one-year earnings is constructed as the sum of available EPS forecasts for $\{6, 7, 8, 9\}$, then transformed to firm-level annual changes and standardized:

$$X_{it} = \frac{\Delta \widehat{earn}_{it}}{\sigma_i(\Delta \widehat{earn})}, \qquad \Delta \widehat{earn}_{it} = \widehat{earn}_{it} - \widehat{earn}_{i,t-1}.$$

  The regression uses $X_{i,t-1}$ (lagged one year).

- Return controls from CRSP:

$$ret12m_{it} = \prod_{m=1}^{12}(1 + r_{i,t-m+1}) - 1, \qquad ret12m\_lag_{it} = ret12m_{i,t-1}.$$

To avoid mechanically extreme standardized values, we require at least three non-missing first differences per firm and drop observations with absolute standardized values above 50.

Regression specifications. The six reported specifications are:

$$\begin{aligned}
&(1) \quad Y_{it}^{pay} = a + \beta X_{i,t-1} + \varepsilon_{it},\\
&(2) \quad Y_{it}^{pay} = a + \beta X_{i,t-1} + \gamma\, ret12m_lag_{it} + \varepsilon_{it},\\
&(3) \quad Y_{it}^{pay} = a + \beta X_{i,t-1} + \delta\, ret12m_{it} + \varepsilon_{it},\\
&(4) \quad Y_{it}^{emp} = a + \beta X_{i,t-1} + \varepsilon_{it},\\
&(5) \quad Y_{it}^{emp} = a + \beta X_{i,t-1} + \gamma\, ret12m_lag_{it} + \varepsilon_{it},\\
&(6) \quad Y_{it}^{emp} = a + \beta X_{i,t-1} + \delta\, ret12m_{it} + \varepsilon_{it}.
\end{aligned}$$

Standard errors are clustered at the firm (`gvkey`) level.

Common-sample implementation. The replication code optionally enforces a common sample across all six columns (same observations in payroll/workers regressions and both return controls) and optionally restricts fiscal-year ranges to align with legacy implementations.

O1.4 Turnover–Disagreement Analysis (IBES + CRSP)

Overview. The turnover analysis combines (i) a quarterly stock-market turnover measure built from CRSP, (ii) an analyst-level disagreement index built from IBES micro forecasts, and (iii) NBER business-cycle dates used to define recession indicators. The final regression sample is quarterly and spans 1982Q2–2021Q3.

Quarterly turnover from CRSP. Turnover is constructed from monthly CRSP stock data as follows:

i. Keep NYSE common stocks: `EXCHCD`=1 and `SHRCD` $\in \{10, 11\}$.

ii. For each stock-month, compute turnover as

$$\text{turnover}_{i,t} = \frac{\text{VOL}_{i,t}}{\text{SHROUT}_{i,t}},$$

with `SHROUT` converted to shares (`SHROUT`$\times$1000).

iii. Winsorize stock-level turnover cross-sectionally at each month at the 99th percentile.

iv. Compute monthly equal-weighted and value-weighted turnover across stocks, where value weights use lagged market equity ($|\text{PRC}_{i,t}| \times \text{SHROUT}_{i,t}$, lagged one month).

v. Aggregate monthly turnover within each quarter by summing the three monthly values and multiplying by 100:

$$qvw_t = 100 \sum_{m \in t} \text{VWTurnover}_m.$$

The regressions use qvw_t as the dependent variable.

IBES disagreement index (DI). Let e_{ijt} denote analyst j's one-year-ahead earnings-growth forecast for firm i at quarter t. We estimate:

$$e_{ijt} = \alpha_{it} + \gamma_j + \beta_j g_t + \varepsilon_{ijt},$$

where g_t is aggregate one-year-ahead realized log earnings growth (from DeLao's earnings-growth series), α_{it} is a firm×quarter fixed effect, and (γ_j, β_j) are analyst-specific coefficients.

The index used in turnover regressions is

$$DI_t = \text{sd}_j(\omega_{jt}), \qquad \omega_{jt} = \gamma_j + \beta_j g_t,$$

computed across active analysts in each quarter.

IBES sample restrictions. The IBES micro sample applies:

i. `MEASURE=EPS`, `FPI=1`;

ii. non-missing analyst ID, firm ID, and forecast value;

iii. quarter-level winsorization of forecasts (1st/99th percentiles);

iv. analyst-quarter coverage filter: at least 3 firms per analyst-quarter;

v. quarter coverage filter: at least 10 analysts in the quarter.

These filters, together with availability of g_t, imply a DI sample from 1982Q2 to 2021Q3.

Recession indicators and regression specifications. NBER peaks and troughs are used to define:

- R_t: recession dummy;
- $F2_t$: dummy for the first two quarters of each recession spell.

The five turnover specifications are:

$$\begin{aligned}
&(1) \quad qvw_t = c + \beta DI_t + u_t,\\
&(2) \quad qvw_t = c + \beta DI_t + \delta R_t + \theta(DI_t \times R_t) + u_t,\\
&(3) \quad qvw_t = c + \beta DI_t + \theta(DI_t \times R_t) + u_t,\\
&(4) \quad qvw_t = c + \beta DI_t + \delta F2_t + \theta(DI_t \times F2_t) + u_t,\\
&(5) \quad qvw_t = c + \beta DI_t + \theta(DI_t \times F2_t) + u_t.
\end{aligned}$$

Table O.3: Turnover Regressions with Standardized Disagreement (Robustness)

	Baseline	Rec × DI	Rec × DI	Early Rec.	Early Rec.
	(1)	(2)	(3)	(4)	(5)
Disagreement (z-score)	0.058	0.039	0.039	0.049	0.049
	(0.045)	(0.037)	(0.037)	(0.043)	(0.043)
Recession		0.012			
		(0.054)			
Disagreement × Recession		0.174***	0.176***		
		(0.051)	(0.053)		
Early Rec. (first 2 qtrs)				-0.060	
				(0.044)	
Disagreement × Early Rec.				0.127***	0.121**
				(0.046)	(0.056)
VIX	0.009***	0.007**	0.007***	0.009***	0.009***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
Market return	-0.032	0.008	0.010	-0.042	-0.035
	(0.141)	(0.117)	(0.114)	(0.131)	(0.131)
Marginal effect of DI in recession		0.213	0.215	0.176	0.170
Constant	0.180***	0.201***	0.198***	0.174***	0.179***
	(0.057)	(0.060)	(0.053)	(0.060)	(0.060)
Observations	127	127	127	127	127
R^2	0.292	0.404	0.403	0.335	0.328
Adjusted R^2	0.274	0.379	0.384	0.307	0.306

Sample period: 1990Q1–2021Q3. Disagreement is standardized within sample. Newey–West (lag 4) standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Reported standard errors are Newey–West HAC with lag 4.

Robustness. As a robustness check, we control for aggregate market volatility (VIX) and market returns in the turnover regressions. The results are reported in Table O.3. We observe that the main results are robust to these controls: the interaction term with the recession indicator is still positive and statistically significant.

O1.5 Productivity-Growth Coibion–Gorodnichenko Regressions (SPF)

Overview. We construct one-quarter-ahead forecast errors and forecast revisions for two productivity measures from SPF level forecasts and realized macro data, then estimate CG regressions at horizon $h = 1$.

Forecast inputs from SPF. Using SPF mean-level forecasts for RGDP and EMP:

$$F_t \Delta y_{t+1} = 400 \log\left(\frac{RGDP2_t}{RGDP1_t}\right), \qquad F_t \Delta n_{t+1} = 400 \log\left(\frac{EMP2_t}{EMP1_t}\right),$$

and similarly for two-quarters-ahead forecasts:

$$F_t \Delta y_{t+2} = 400 \log\left(\frac{RGDP3_t}{RGDP2_t}\right), \qquad F_t \Delta n_{t+2} = 400 \log\left(\frac{EMP3_t}{EMP2_t}\right).$$

Productivity forecast measures. Define:

$$F_t \Delta prod_{t+1}^{LP} = F_t \Delta y_{t+1} - F_t \Delta n_{t+1},$$

$$F_t \Delta prod_{t+1}^{TFP} = F_t \Delta y_{t+1} - \alpha F_t \Delta n_{t+1}, \qquad \alpha = \tfrac{2}{3}.$$

Forecast revisions are:

$$Revision_t^{LP} = F_t \Delta prod_{t+1}^{LP} - F_{t-1} \Delta prod_{t+1}^{LP},$$

$$Revision_t^{TFP} = F_t \Delta prod_{t+1}^{TFP} - F_{t-1} \Delta prod_{t+1}^{TFP},$$

with $F_{t-1} \Delta prod_{t+1}$ obtained by lagging the t-dated two-quarters-ahead forecast.

Realized series. Realized GDP growth is constructed from real-time GDP vintages; realized employment growth is built from quarterly averages of monthly payroll employment (PAYEMS):

$$\Delta y_{t+1} = 400 \log\left(\frac{RGDP_{t+1}}{RGDP_t}\right), \qquad \Delta n_{t+1} = 400 \log\left(\frac{PAYEMS_{t+1}}{PAYEMS_t}\right).$$

Hence:

$$\Delta prod_{t+1}^{LP} = \Delta y_{t+1} - \Delta n_{t+1}, \qquad \Delta prod_{t+1}^{TFP} = \Delta y_{t+1} - \alpha \Delta n_{t+1}.$$

CG regressions. For each measure $m \in \{LP, TFP\}$, estimate:

$$FE_t^m = c + \beta_{CG}^m \, Revision_t^m + u_t,$$

where $FE_t^m = \Delta prod_{t+1}^m - F_t \Delta prod_{t+1}^m$. Reported standard errors are Newey–West HAC with lag 4.

Sample. The productivity CG table uses 2004Q1–2025Q2 (86 quarterly observations in each column).

O1.6 Construction of Table 5 Data Moments and Bootstrap Intervals

Overview. The data targets in Table 5 are constructed by ingesting the raw series described below and computing the moments and bootstrap uncertainty intervals reported in the table. All moments are reported in annualized percentage points.

Data sources (key series). For the moments requested in Table 5, we use:

- *Excess returns and dividend growth*: Shiller historical market dataset (CSV mirror), using real equity returns and nominal long rates.
- *Consumption growth*: BEA/FRED annual real nondurables and services series, `DNDGRG3A086NBEA` and `DSERRG3A086NBEA`, divided by annual population `B230RC0A052NBEA`.

Series and definitions. We construct five data objects:

i. *Real interest rate*: annual ex-post real long rate from Shiller,

$$r_t^{real} = r_t^{nominal} - \pi_t,$$

where $r_t^{nominal}$ is the Shiller long rate and π_t is realized annual inflation computed as a log change in the Shiller CPI (December-to-December, in our baseline construction).

ii. *Excess equity return*: annual real equity return minus the real interest rate.

iii. *Consumption growth*: annual log growth of real per-capita nondurables plus services consumption,

$$\Delta c_t = 100\Delta \log\left(\frac{DNDGRG3A086NBEA_t + DSERRG3A086NBEA_t}{B230RC0A052NBEA_t}\right).$$

iv. *Dividend growth*: annual log growth of annual-average real dividends from Shiller.

v. *Log-hours volatility*: standard deviation of HP-filtered ($\lambda = 100$) annual log total hours, where annual hours are built from CPS/BLS series as

$$hours_t = \frac{LNU02005053_t \times LNU02005054_t}{LNU00000000_t - LNU00000097_t}.$$

We access these monthly CPS series via the BLS API, construct $hours_t$ at the monthly frequency, and then take annual averages before applying the HP filter to the annual log series. This construction mirrors Rogerson and Shimer (2011).

Baseline sample windows used for Table 5. The replication uses:

- financial and dividend moments: 1890–2025;
- consumption growth: 1945–2025 (paper-table definition above);
- hours volatility: 1976–2025.

Annualization and reported moments. For each variable x_t, we report

$$\widehat{\mu}_x = \text{mean}(x_t), \qquad \widehat{\sigma}_x = \text{sd}(x_t),$$

already scaled in annual percentage points by construction.

Bootstrap standard errors and intervals. To report uncertainty in the data column, we compute moving-block bootstrap standard errors for both $\widehat{\mu}_x$ and $\widehat{\sigma}_x$:

- number of bootstrap replications: $B = 1000$;
- seed: 12345;
- block length: $l = \lfloor n^{1/3} \rfloor$ (unless manually overridden), where n is the sample size of the variable.

For each replication b, we resample contiguous blocks of annual observations (years) with replacement, form a pseudo-sample $x_t^{(b)}$, and compute $\widehat{\mu}_x^{(b)}$ and $\widehat{\sigma}_x^{(b)}$. Standard errors are:

$$\widehat{se}(\widehat{\mu}_x) = sd_b\left(\widehat{\mu}_x^{(b)}\right), \qquad \widehat{se}(\widehat{\sigma}_x) = sd_b\left(\widehat{\sigma}_x^{(b)}\right).$$

Table 5 reports ± 2 s.e. intervals:

$$\left[\widehat{\mu}_x - 2\widehat{se}(\widehat{\mu}_x),\, \widehat{\mu}_x + 2\widehat{se}(\widehat{\mu}_x)\right], \qquad \left[\widehat{\sigma}_x - 2\widehat{se}(\widehat{\sigma}_x),\, \widehat{\sigma}_x + 2\widehat{se}(\widehat{\sigma}_x)\right].$$

O2 Numerical solution of the quantitative model

This appendix summarizes the quantitative model in Section 6 in a form that is directly usable for computation. The objective is to solve for equilibrium policy and pricing functions in a heterogeneous-beliefs economy with Epstein–Zin preferences, production with hiring chosen one period in advance, and a stationary wealth distribution generated by Uzawa discounting.

Objects to compute. Given the state X_t, the key endogenous objects are the risk-neutral expectation of productivity growth $\mathcal{L}_t$, the surplus-claim price-dividend ratio n_t, the consumption-claim price-dividend ratios $\{n_{i,t}\}_{i=1}^{I}$, and consumption shares $\{s_{i,t}\}_{i=1}^{I}$ (equivalently, Pareto weights $\{\lambda_{i,t}\}$). All other quantities (SDFs, returns, hours, wages, and output) follow from these objects.

In contrast to the simpler baseline model in the main text, which is conveniently described in terms of wealth shares, the quantitative solution tracks heterogeneity using Pareto weights (or, equivalently, consumption shares). This representation is convenient for the perturbation-based solution.

Computation. An exact closed-form solution is unavailable away from log preferences. We therefore compute the solution using a *third-order perturbation* around the nonstochastic (deterministic) steady state, as described in Section 6. The third-order approximation is indispensable to generate time variation in risk premia.

Concretely, we take a Taylor expansion of the equilibrium conditions below around the steady state and solve for the coefficients of the resulting policy rules for the endogenous variables as functions of the state. We then simulate the implied nonlinear state-space system using the approximated decision rules to compute moments and impulse responses. For simulation, we use a *pruned* state-space representation of the higher-order solution to avoid spurious explosive dynamics induced by truncation of the perturbation expansion (see Andreasen et al. 2018).

O2.1 Equilibrium conditions

Aggregate productivity growth. Aggregate log productivity growth, $\hat{x}_t \equiv \log x_t$, follows the following process under the objective measure:

$$\hat{x}_t = \underbrace{\mu_x + \rho_x(\hat{x}_{t-1} - \mu_x)}_{\overline{x}_{t-1}} + \sigma_{x,t-1}\epsilon_{x,t}. \tag{O2.1}$$

Aggregate productivity follows the following process under the beliefs of investor i:

$$\hat{x}_t = \underbrace{\mu_{x,i} + \rho_{x,i}(\hat{x}_{t-1} - \mu_{x,i}) + v_{i,t-1}}_{\overline{x}_{i,t-1}} + \sigma_{x,i,t-1}\epsilon_{x,i,t}, \tag{O2.2}$$

where the belief shock $v_{i,t}$ follows the process:

$$v_{i,t} = \rho_{v,i} v_{i,t-1} + \sigma_{v,i}\epsilon_{v,i,t}. \tag{O2.3}$$

Following Bansal and Yaron (2004), we allow for stochastic volatility:

$$\sigma^2_{x,t} = q_t, \qquad \sigma^2_{x,i,t} = q_t \frac{\overline{\sigma}^2_{x,i}}{\overline{\sigma}^2_x}, \qquad i = 1, \ldots, I, \tag{O2.4}$$

where q_t follows an AR(1) process in levels:

$$q_t = \bar{q} + \rho_q(q_{t-1} - \bar{q}) + \sigma_q \epsilon_{q,t}, \qquad \epsilon_{q,t} \sim \mathcal{N}(0,1). \tag{O2.5}$$

In the baseline specification, we consider the special case with constant volatility: $q_t \equiv \bar{q}$, in which $\sigma_{x,t} = \overline{\sigma}_x$ and $\sigma_{x,i,t} = \overline{\sigma}_{x,i}$. In a robustness exercise, we allow for stochastic volatility, where the volatility shocks $\epsilon_{q,t}$ may be correlated with productivity and belief shocks, with $\text{Corr}(\epsilon_{x,t}, \epsilon_{q,t}) = \rho_{xq}$ and $\text{Corr}(\epsilon_{v,t}, \epsilon_{q,t}) = \rho_{qv}$.

Likelihood ratios. We assume that subjective beliefs are absolutely continuous with respect to the true distribution, so for any random variable g_{t+1} we can write:

$$\mathbb{E}_{i,t}[g_{t+1}] = \mathbb{E}_t[\ell_{i,t+1} g_{t+1}], \tag{O2.6}$$

where $\ell_{i,t+1}$ denotes the Radon–Nikodym derivative of the subjective probability density with respect to the true probability. We also assume that all shocks are Gaussian, so we can write this derivative as the likelihood ratio of two Gaussians:

$$\ell_{i,t+1} = \frac{\sigma_{x,t}}{\sigma_{x,i,t}} e^{-\frac{(\hat{x}_{t+1}-\bar{x}_{i,t})^2}{2\sigma_{x,i,t}^2} + \frac{(\hat{x}_{t+1}-\bar{x}_t)^2}{2\sigma_{x,t}^2}}, \tag{O2.7}$$

where $\sigma_{x,t}$ and $\sigma_{x,i,t}$ denote the objective and subjective conditional standard deviations of $\hat{x}_{t+1}$ given information at time t.

Preferences: EZ+ Uzawa. To ensure that a non-degenerate stationary wealth distribution exists in this economy, we assume that investors have external Uzawa preferences, such that the discount factor for household i is given by:

$$\beta_{i,t} = \beta e^{-\kappa s_{i,t}}, \tag{O2.8}$$

where $s_{i,t} \equiv \log \frac{\tilde{C}_{i,t}}{Y_t A_t}$ denotes the relative consumption of type i, and $Y_t \equiv h_t^{\alpha} - \xi e^{-\hat{x}_t} \frac{h_t^{1+\nu}}{1+\nu}$ denotes detrended net output. When $\kappa > 0$, the discount factor is decreasing in the consumption share of agent i, so households become more impatient as their consumption increases.

The stochastic discount factor for household i is given by

$$\Lambda_{i,t+1} = \beta_{i,t}^{\theta} \left(\frac{\tilde{C}_{i,t+1}}{\tilde{C}_{i,t}} \right)^{-\frac{\theta}{\psi}} \left(\frac{n_{i,t+1}+1}{n_{i,t}} \frac{\tilde{C}_{i,t+1}}{\tilde{C}_{i,t}} \right)^{\theta-1}, \tag{O2.9}$$

using the fact that $R_{i,n,t+1} = \frac{N_{i,t+1}}{N_{i,t}-\tilde{C}_{i,t}} = \frac{n_{i,t+1}+1}{n_{i,t}} \frac{\tilde{C}_{i,t+1}}{\tilde{C}_{i,t}}$, where $n_{i,t} \equiv \frac{N_{i,t}}{\tilde{C}_{i,t}} - 1$ represents the price-dividend ratio on a claim on household i's consumption.

We can then write the SDF as follows:

$$\Lambda_{i,t+1} = \beta_{i,t}^{\theta} \left(e^{\Delta s_{i,t+1} + \Delta y_{t+1} + \hat{x}_{t+1}} \right)^{-\gamma} \left(\frac{n_{i,t+1}+1}{n_{i,t}} \right)^{\theta-1}, \tag{O2.10}$$

where $y_t \equiv \log Y_t$.

Consumption claim's price-dividend ratio. The pricing condition for household's portfolio return is given by

$$1 = \mathbb{E}_{i,t}\left[\Lambda_{i,t+1} R_{i,n,t+1}\right]. \tag{O2.11}$$

We can write the expression above as follows:

$$1 = \mathbb{E}_t\left[\ell_{i,t+1}\beta^{\theta} e^{-\theta\kappa s_{i,t}}\left(e^{\Delta s_{i,t+1}+\Delta y_{t+1}+\hat{x}_{t+1}}\right)^{1-\gamma}\left(\frac{n_{i,t+1}+1}{n_{i,t}}\right)^{\theta}\right]. \tag{O2.12}$$

Optimal risk sharing. The optimal risk sharing condition can be written as follows:

$$\ell_{i,t+1}\Lambda_{i,t+1} = \ell_{1,t+1}\Lambda_{1,t+1}. \tag{O2.13}$$

We can write this condition as follows:

$$\ell_{i,t+1}\beta_{i,t}^{\theta}\left(\frac{\tilde{C}_{i,t+1}}{\tilde{C}_{i,t}}\right)^{-\gamma}\left(\frac{n_{i,t+1}+1}{n_{i,t}}\right)^{\theta-1} = \ell_{1,t+1}\beta_{1,t}^{\theta}\left(\frac{\tilde{C}_{1,t+1}}{\tilde{C}_{1,t}}\right)^{-\gamma}\left(\frac{n_{1,t+1}+1}{n_{1,t}}\right)^{\theta-1}, \tag{O2.14}$$

It is useful to work with a recursive formulation of the problem. Let $\hat{\lambda}_{i,t}$ denote the Pareto weight of household i, and write the optimal risk sharing condition as follows:

$$\hat{\lambda}_{i,t-1}\ell_{i,t}\tilde{C}_{i,t}^{-\gamma}(n_{i,t}+1)^{\theta-1} = \hat{\lambda}_{1,t-1}\ell_{1,t}\tilde{C}_{1,t}^{-\gamma}(n_{1,t}+1)^{\theta-1}. \tag{O2.15}$$

where $\hat{\lambda}_{i,t}$ denote the *Pareto weight* on household i, which evolves according to the law of motion:

$$\hat{\lambda}_{i,t} = \zeta_t\hat{\lambda}_{i,t-1}\beta_{i,t}^{\theta}\ell_{i,t}\left(\frac{n_{i,t}+1}{n_{i,t}}\right)^{\theta-1}, \tag{O2.16}$$

where ζ_t is a scaling process.

In the special case of CRRA preferences, $\theta = 1$, constant discount factor, $\kappa = 0$, and rational expectations, $\ell_{i,t} = 1$, relative Pareto weights are constant and the optimal risk sharing condition boils down to the expression with time-separable preferences: $\hat{\lambda}_i\tilde{C}_{i,t}^{-\gamma} = \hat{\lambda}_1\hat{C}_{1,t}^{-\gamma}$.

Taking the ratio of Eq. (O2.15) at $t+1$ and t, we obtain

$$\frac{\hat{\lambda}_{i,t}}{\hat{\lambda}_{i,t-1}}\frac{\ell_{i,t+1}}{\ell_{i,t}}\left(\frac{\tilde{C}_{i,t+1}}{\tilde{C}_{i,t}}\right)^{-\gamma}\left(\frac{n_{i,t+1}+1}{n_{i,t}+1}\right)^{\theta-1} = \frac{\hat{\lambda}_{1,t}}{\hat{\lambda}_{1,t-1}}\frac{\ell_{1,t+1}}{\ell_{1,t}}\left(\frac{\tilde{C}_{1,t+1}}{\tilde{C}_{1,t}}\right)^{-\gamma}\left(\frac{n_{1,t+1}+1}{n_{1,t}+1}\right)^{\theta-1}, \tag{O2.17}$$

which coincides with $\ell_{i,t+1}\Lambda_{i,t+1} = \ell_{1,t+1}\Lambda_{1,t+1}$. We can choose $\hat{\lambda}_{i,-1}$ such that condition (O2.15) is satisfied at $t = 0$.

From Eq. (O2.15) and the market clearing for goods, $\sum_{j=1}^{I} \mu_j \tilde{C}_{j,t} = A_t Y_t$, we can solve for relative consumption:

$$\frac{\tilde{C}_{i,t}}{A_t Y_t} = \frac{\hat{\lambda}_{i,t-1}^{\frac{1}{\gamma}} \left[\ell_{i,t}(n_{i,t}+1)^{\theta-1}\right]^{\frac{1}{\gamma}}}{\sum_{j=1}^{I} \mu_j \hat{\lambda}_{j,t-1}^{\frac{1}{\gamma}} \left[\ell_{j,t}(n_{j,t}+1)^{\theta-1}\right]^{\frac{1}{\gamma}}}. \tag{O2.18}$$

Taking logs, we can write the expression above as follows:

$$s_{i,t} - s_{1,t} = \lambda_{i,t-1} - \lambda_{1,t-1} + \frac{1}{\gamma}\left[\log\frac{\ell_{i,t}}{\ell_{1,t}} + (\theta-1)\log\frac{n_{i,t}+1}{n_{1,t}+1}\right], \tag{O2.19}$$

where $\lambda_{i,t} \equiv \frac{1}{\gamma}\log\hat{\lambda}_{i,t}$, and $\sum_{i=1}^{I}\mu_i e^{s_{i,t}} = 1$. We can solve for $s_{1,t}$ in terms of $\tilde{s}_{i,t} \equiv s_{i,t} - s_{1,t}$, $i = 2, \ldots, I$, using the condition

$$s_{1,t} = -\log\left(\mu_1 + \sum_{i=2}^{I}\mu_i e^{\tilde{s}_{i,t}}\right). \tag{O2.20}$$

The law of motion of $\lambda_{i,t}$ is given by

$$\lambda_{i,t} = \lambda_{i,t-1} + \frac{1}{\gamma}\left[-\theta\kappa s_{i,t} + \log\ell_{i,t} + (\theta-1)\log(1+n_{i,t}^{-1}) + \log\beta^{\theta}\zeta_t\right]. \tag{O2.21}$$

We are free to choose the value for ζ_t. For instance, we can set $\zeta_t = \beta^{-\theta}$, so the last term inside brackets above is equal to zero. Alternatively, we can set ζ_t such that $\lambda_{1,t} = 0$.

Asset pricing. The price of a risk-free bond is given by

$$P_{f,t} = \mathbb{E}_t\left[\ell_{i,t+1}\Lambda_{i,t+1}\right], \tag{O2.22}$$

and the risk-free rate is given by $R_{b,t} = P_{f,t}^{-1}$.

Let $R_{r,t+1}$ denote the return on a claim on total surplus, or net output Y_t. Let n_t denote the price-dividend ratio on the surplus claim, we then write the return as follows:

$$R_{r,t+1} = \frac{n_{t+1}+1}{n_t}e^{\Delta y_{t+1}+\hat{x}_{t+1}}. \tag{O2.23}$$

The pricing condition for the surplus claim is given by

$$1 = \mathbb{E}_t\left[\ell_{i,t+1}\Lambda_{i,t+1}R_{r,t+1}\right]. \tag{O2.24}$$

Production. The key endogenous object $\mathcal{L}_t$ is the risk-neutral expectation of productivity growth. It evolves according to:

$$\mathcal{L}_t = R_{b,t}\mathbb{E}_t\left[\ell_{i,t+1}\Lambda_{i,t+1}e^{\hat{x}_{t+1}}\right] \qquad \Rightarrow \qquad \mathbb{E}_t\left[\ell_{i,t+1}\Lambda_{i,t+1}(e^{\hat{x}_{t+1}} - \mathcal{L}_t)\right] = 0. \tag{O2.25}$$

Hours and wages are given by:

$$h_t = \left(\frac{\alpha\mathcal{L}_{t-1}}{\xi}\right)^{\frac{1}{1+\nu-\alpha}}, \qquad w_t = \xi\left(\frac{\alpha\mathcal{L}_{t-1}}{\xi}\right)^{\frac{\nu}{1+\nu-\alpha}}. \tag{O2.26}$$

Output is given by

$$e^{y_t} = h_t^{\alpha} - \xi e^{-\hat{x}_t}\frac{h_t^{1+\nu}}{1+\nu}. \tag{O2.27}$$

O2.2 Summary of equilibrium conditions

Roadmap. The equilibrium system can be organized as:

- **State dynamics:** the exogenous laws of motion for $\hat{x}_t$, q_t, and $v_{i,t}$, plus endogenous evolution of $\mathcal{L}_t$ and Pareto weights $\lambda_{i,t}$.
- **Risk sharing and pricing:** the consumption-claim Euler equations for each type and the surplus-claim pricing equation.
- **Production block:** hours, wages, and detrended net output Y_t.

State dynamics:

$$\hat{x}_t = \mu_x + \rho_x(\hat{x}_{t-1} - \mu_x) + \sigma_{x,t-1}\epsilon_{x,t} \tag{O2.28}$$

$$q_t = \bar{q} + \rho_q(q_{t-1} - \bar{q}) + \sigma_q\epsilon_{q,t} \tag{O2.29}$$

$$v_{i,t} = \rho_{v,i}v_{i,t-1} + \sigma_{v,i}\epsilon_{v,i,t}, \qquad i = 1, \ldots, I \tag{O2.30}$$

$$\mathcal{L}_t = R_{b,t}\mathbb{E}_t\left[\ell_{1,t+1}\Lambda_{1,t+1}e^{\hat{x}_{t+1}}\right] \tag{O2.31}$$

$$\lambda_{i,t} = \lambda_{i,t-1} + \frac{1}{\gamma}\left[-\theta\kappa(s_{i,t} - s_{1,t}) + \log\frac{\ell_{i,t}}{\ell_{1,t}} + (\theta-1)\log\frac{1+n_{i,t}^{-1}}{1+n_{1,t}^{-1}}\right], \qquad i = 2, \ldots, I \tag{O2.32}$$

where $\lambda_{1,t} = 0$. State vector: $X_t = (\hat{x}_t, q_t, \mathcal{L}_{t-1}, \{v_{i,t}\}_{i=1}^{I}, \{\lambda_{i,t}\}_{i=2}^{I})$.

Equilibrium conditions:

$$1 = \mathbb{E}_t \left[\ell_{i,t+1} \beta^\theta e^{-\theta\kappa s_{i,t}} \left(e^{\Delta s_{i,t+1} + \Delta y_{t+1} + \hat{x}_{t+1}} \right)^{1-\gamma} \left(\frac{n_{i,t+1}+1}{n_{i,t}} \right)^\theta \right], \qquad i = 1, \ldots, I \tag{O2.33}$$

$$s_{i,t} - s_{1,t} = \lambda_{i,t-1} + \frac{1}{\gamma} \left[\log \frac{\ell_{i,t}}{\ell_{1,t}} + (\theta - 1) \log \frac{n_{i,t}+1}{n_{1,t}+1} \right], \qquad i = 2, \ldots I \tag{O2.34}$$

$$1 = \sum_{i=1}^{I} \mu_i e^{s_{i,t}} \tag{O2.35}$$

$$R_{b,t}^{-1} = \mathbb{E}_t \left[\ell_{1,t+1} \Lambda_{1,t+1} \right] \tag{O2.36}$$

$$1 = \mathbb{E}_t \left[\ell_{1,t+1} \Lambda_{1,t+1} \frac{n_{t+1}+1}{n_t} e^{\Delta y_{t+1} + \hat{x}_{t+1}} \right] \tag{O2.37}$$

$$\overline{R}_{r,t} = \mathbb{E}_t \left[\frac{n_{t+1}+1}{n_t} e^{\Delta y_{t+1} + \hat{x}_{t+1}} \right] \tag{O2.38}$$

$$h_t = \left(\frac{\alpha \mathcal{L}_{t-1}}{\xi} \right)^{\frac{1}{1+\nu-\alpha}} \tag{O2.39}$$

$$w_t = \xi \left(\frac{\alpha \mathcal{L}_{t-1}}{\xi} \right)^{\frac{\nu}{1+\nu-\alpha}} \tag{O2.40}$$

$$e^{y_t} = h_t^\alpha - \xi e^{-\hat{x}_t} \frac{h_t^{1+\nu}}{1+\nu}, \tag{O2.41}$$

where

$$\Lambda_{i,t+1} = \beta^\theta e^{-\theta\kappa s_{i,t} - \gamma(\Delta s_{i,t+1} + \Delta y_{t+1} + \hat{x}_{t+1})} \left(\frac{n_{i,t+1}+1}{n_{i,t}} \right)^{\theta-1}, \qquad \ell_{i,t+1} = \frac{\sigma_{x,t}}{\sigma_{x,i,t}} e^{-\frac{(\hat{x}_{t+1} - \overline{x}_{i,t})^2}{2\sigma_{x,i,t}^2} + \frac{(\hat{x}_{t+1} - \overline{x}_t)^2}{2\sigma_{x,t}^2}}, \tag{O2.42}$$

$\overline{x}_t = \mu_x + \rho_x(\hat{x}_t - \mu_x)$, and $\overline{x}_{i,t} = \mu_{x,i} + \rho_{x,i}(\hat{x}_t - \mu_{x,i}) + v_{i,t}$.

O2.3 Extension: Stochastic Volatility

In this subsection, we extend the quantitative model to allow for stochastic volatility, following Bansal and Yaron (2004). We consider two cases. In the first case, investors share common beliefs about the volatility process and volatility follows a standard variance-level stochastic volatility (SV) process. In the second case, investors have heterogeneous beliefs about volatility. In this specification, productivity growth remains iid under the objective measure, so there is no objective stochastic volatility, but investors perceive time-varying volatility.

We also consider two correlation structures: (i) volatility shocks that are orthogonal to productivity and belief shocks, and (ii) volatility shocks that are negatively correlated with belief shocks. The latter captures episodes in which a negative sentiment shock is

Table O.4: Unconditional moments: Extension with stochastic volatility

	Baseline		**SV: Common Beliefs**				**SV: Heterogeneous Beliefs**					
	High elast.		$\rho_{qv} = 0.0$		$\rho_{qv} = -0.9$		$\rho_{qv} = 0.0$		$\rho_{qv} = -0.9$		**Data (± 2 s.e.)**	
Variables	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**
Interest rate	1.6	6.1	1.5	6.3	1.6	6.0	1.7	6.3	1.8	6.1	[0.6, 3.0]	[3.7, 6.0]
Excess returns (equity)	5.3	15.6	5.7	17.4	5.7	18.1	5.5	16.9	5.1	15.4	[3.5, 9.3]	[15.6, 21.4]
Consumption growth	2.1	3.0	2.2	2.9	2.2	2.9	2.2	3.0	2.2	2.9	[1.2, 3.1]	[2.0, 3.4]
Dividend growth	2.1	9.8	2.2	9.7	2.2	9.7	2.2	9.8	2.2	9.8	[-0.3, 3.4]	[6.9, 11.2]
Log hours	0.6	1.9	0.5	1.9	0.6	1.9	0.5	1.9	0.6	1.9	–	[1.7, 2.7]

Note: Model moments are annualized and reported in percentage points. The baseline column corresponds to Model 5 (High elasticity) in Table 5. "SV: Common Beliefs" uses a Bansal–Yaron style variance-level stochastic volatility specification in which both investors share the same volatility process. "SV: Heterogeneous Volatility Beliefs" assumes no objective stochastic volatility, while one investor perceives volatility to follow a variance-level stochastic volatility process. ρ_{qv} denotes the correlation between volatility shocks and belief shocks. Data intervals report the same ± 2 bootstrap standard-error ranges used in Table 5.

associated with both lower expected productivity growth and higher (perceived or objective) volatility.

Table O.4 reports the results. The first column reproduces the high labor-elasticity baseline from Table 5. The next two columns introduce stochastic volatility under common beliefs. Stochastic volatility increases the equity premium and return volatility relative to the baseline, while leaving consumption growth, dividend growth, and hours close to their benchmark values. Introducing a negative correlation between volatility and belief shocks reduces interest-rate volatility, in line with the upper bound of the empirical interval.

The final two columns allow for heterogeneous beliefs about volatility. Relative to the baseline, equity premia and return volatility again increase modestly. Interest-rate volatility is somewhat higher under heterogeneous volatility beliefs, but this effect is attenuated when volatility shocks are negatively correlated with belief shocks.

Overall, stochastic volatility acts primarily as an amplification mechanism for the equity premium and return volatility. Importantly, the main qualitative features of the baseline model remain intact. This extension shows that the core results are not driven by the absence of stochastic volatility and that introducing volatility shocks does not overturn the model's implications for macroeconomic quantities.