

The Fragility of Government Funding Advantage*

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April 27, 2026

Abstract

US federal debt plays a special financial role giving the US government a funding advantage compared to the private sector. Is this an immutable feature of US treasuries or a fragile equilibrium outcome? To answer this, we build a model where government funding advantage emerges endogenously from the financial sector's ability to use treasuries to hedge risk. Financial regulation can amplify the hedging properties of US treasuries by creating captive demand in bad times but only if the government runs stable fiscal policy to protect long-run treasury prices. Ultimately, the government cannot simultaneously choose: (i) high funding advantage, (ii) financial sector stability, and (iii) monetary-fiscal policy that destabilizes treasury prices. Balancing these tradeoffs has far-reaching macroeconomic and welfare implications.

JEL CLASSIFICATION: E31, E43, G12, N21, N41

KEY WORDS: Convenience Yields, Government Debt, Financial Repression, Safe Assets

*Previously circulated as “Convenience Yields and Financial Repression”. We are very grateful to the comments and discussion from Fernando Alvarez, Mark Aguiar, Jaroslav Borovička, Anmol Bhandari (discussant), Francesco Bianchi (discussant), Markus Brunnermeier, Lars Hansen, Greg Kaplan, Arvind Krishnamurthy (discussant), Moritz Lenel, Carolin Pflueger, Facundo Piguillem (discussant), Thomas J. Sargent, Emil Verner (discussant), and Annette Vissing-Jorgensen. We are also thankful for the comments from participants in the NBER conference on The Economics of Debt and Deficit, the Stanford SITE Conference of Government Financing, the Blue Collar Working Group at the University of Chicago, the BSE Summer Conference, the SED Annual Meeting, the Richmond Fed Conference on Sovereign Debt, the Reserve Bank of Australia Annual, International Conference, and macro/finance seminars at the City University of New York (CUNY), University of Chicago, University of Maryland, and University of Virginia. The views expressed here are those of the authors and do not necessarily represent the views of the Federal Reserve Board or its staff.

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“The temptation to rely on inflation or financial repression to reduce the debt burden will surely grow.”

—Janet L. Yellen, *Remarks on the Future of the Fed, AEA Meetings, Jan 2026.*

1 Introduction

US federal debt plays a special role in the economy and so has given the US government a funding advantage, often summarized by the spread between the yield on high-grade US corporate bonds and comparable US Treasuries. Different views have emerged about where this funding advantage comes from. At one extreme, many macro-finance models treat US funding advantage as an immutable feature of the economic environment and encoded the non-pecuniary benefits of holding US debt into agent preferences or the market structure. This means the government can easily “exploit” the funding advantage to increase spending. At the other extreme, US policy makers have historically argued that US funding advantage emerged as part of a complicated collection of financial-monetary policies that have shaped financial sector demand for US treasuries. When viewed in this way, generating and exploiting a funding advantage is closely interconnected with government policy, fragile in execution, and imposes far reaching impacts on the macroeconomy. It links the stability of the financial sector to the stability of the government budget constraint. It distorts the portfolio of the financial sector, potentially increasing default and crowding out private liquidity creation and productive investment. In this paper, we study the mechanics, limitations, and trade-offs associated with how government policies influence demand for long-term government debt.

When studying US funding advantage, the macro-finance literature has been strongly influenced by historical asset pricing data appearing to show a relatively stable, downward sloping equilibrium relationship between funding advantage and government Debt-to-GDP for the period 1920-2007 (e.g. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#)). This has led to many papers that model government funding advantage using a time-invariant bond-in-the-utility function that is calibrated to match the historical data. However, in Section 2, we use new data from [Lehner, Payne, Shurtleff and Szőke \(2025\)](#) and show that this apparently stable relationship comes from mismeasurement. Instead, there are low-frequency changes to the level and curvature of the equilibrium relationships that correlate with historical policy regime changes. US funding advantage appeared in the late 1860s, well before Bretton-Woods, and fell to zero during the high inflation of the 1970s, despite the emergence of US dollar denominated debt as the international reserve asset.

We build a general equilibrium structural model that endogenizes the connections between financial sector regulation, fiscal policy, the return process on long-term government

debt, and government funding advantage. Our environment is a stochastic neoclassical growth model extended to include a morning sub-period where households need liquidity services provided by a risky banking sector (referred to as the “secondary” asset market) and an afternoon subperiod where there are no frictions (referred to as the “primary” asset market). The economy is populated by households who need bank deposits to be able to trade and produce in the morning sub-period. Banks issue deposits and equity to households and invest in government and corporate bonds. The main friction is that banks cannot issue uncollateralized IOUs with each other to settle withdrawals (and redeposits) in the morning market. Instead, they must rely on bond sales and costly equity raising from the households, both of which become more costly in the crisis states of the world when the banking sector faces systemic liquidity shocks. The frictions make our environment a “second-best” world with two interconnected asset pricing distortions: a “liquidity spread” on financial sector liabilities and a “hedging spread” on assets that can help banks to self-insure against high withdrawal shocks. Absent financial regulation, additional financial frictions, or government debt devaluation, this is an economy where government and corporate bonds are equally useful/useless for hedging risks in the secondary market. That is, government debt does not have an immutable, special role in the economy.

We use our environment to study how government policies can both create and destroy a special role for government debt in the financial sector. In Section 3 we focus on financial regulations that nest the historical US experience from the National Banking Era (1862-1913) to the more recent Basel III weighted leverage ratio. We show how, in an environment without inflation risk, these government portfolio restrictions in the secondary market determine which asset plays the role of a hedging asset for the financial system. If the regulations place more weight on holding government debt, which we refer to as regulatory privilege (or “financial repression”), then banks end up crowding into government debt in the bad state of the world when the regulatory constraint binds more. In this sense, the government regulation can create “captive” counter-cyclical demand for its debt. This leads to an appreciation of the price of government debt in the morning market in bad states and so makes government debt a good “hedge” against aggregate shocks. Consequently, banks voluntarily increase their government debt in the afternoon market to self-insure against morning market shocks, which also leads to banks taking on higher leverage and so, in equilibrium, having more need for government bonds to hedge their risk. The end result is that the price of government debt is inflated in the primary asset market. In this sense, the government funding advantage comes not just from forcing banks to hold the asset but also from the hedging role that Treasury’s end up playing.

In Section 4, we show how the combination of regulatory privilege and inflation risk erode

the government’s funding advantage. This is because regulatory privilege ties the solvency of the banking sector to the stability of the government debt prices while at the same time unstable fiscal policy destabilizes government debt prices. This means that the banks are left with a no-win trade-off: if they don’t purchase government debt, then they violate the regulatory restrictions on backing deposits but if they purchase government debt, then the government’s monetary-fiscal policy forces them to take losses and pay negative dividends in the states of the world when they can least afford it. In the morning market, banks respond by raising equity and exiting the deposit market, which means the morning hedging demand disappears. In the afternoon market, banks respond to the loss of a “safe-bond” by using inefficient short-duration storage assets to bank their deposit creation. The end result is that the banking sector issues fewer deposits (the extensive margin) and less debt to back the deposits that remain (the extensive margin). This erodes the government’s captive demand in the banking sector and so the government’s funding advantage disappears. It is important to understand that this decrease in funding advantage is not coming from a simple inflation risk premium emerging on government debt (since corporate debt has the same inflation risk) but instead occurs because government debt no longer plays a special role in interbank market and so no longer provides a non-pecuniary benefit.

We use our model of endogenous government funding advantage to study the macroeconomic tradeoffs for a government choosing restrictions on the financial sector to finance a fiscal rule. Our model leaves the government with complicated trade-offs, which we summarize as a “trilemma” that the government cannot choose all three of: (i) high funding advantage, (ii) a well-functioning financial sector (profitable and stable), and (iii) monetary-fiscal policy that leads to systematic real debt devaluation (e.g. “default”, “counter-cyclical” issuance, “inflation”). Intuitively, this Trilemma says that when a government uses the financial sector to generate funding advantage, then it intertwines the balance sheets of the banking sector and the government, which constrains the range of feasible government policies. If the government sets a high regulatory privilege to generate a high funding advantage (choosing part 1 of the trilemma), then it either needs to run monetary-fiscal policy that targets bond price stability (giving up part 3 of the trilemma) or it will put the financial sector into distress (giving up part 2 of the trilemma). In this sense, our model of endogenous funding advantage both gives the government more freedom to expand funding advantage but also highlights that this freedom rests on a stable monetary-fiscal environment.

We close our paper by studying counterfactual financing of large government spending shocks in a calibrated version of our model. The four major historical increases in the US Debt-to-GDP ratio are the Civil War, World War I, World War II, and the Global Financial Crisis. A striking feature of these episodes is that there only one example where

the large increase in US Debt-to-GDP actually coincides with a decrease in government funding advantage: World War I. By contrast the very large increases in funding advantage during the Civil War and the Global Financial Crisis actually coincide with large increase the funding advantage. Through the lens of our model, this occurs because, during both the Civil War and the Global Financial Crisis, the financial system was reorganized to help finance the government. In both cases, this ended up leading to government debt becoming a better asset for hedging aggregate risk. In this sense, the government both increased the supply of government debt but also intervened in financial markets to increase demand. We perform a counterfactual experiment in which the US government finances World War I using a similar approach to the Civil War and the Global Financial Crisis. We find that the government is able to decrease taxation but at the cost of both lower investment and lower banking sector liquidity creation.

Related Literature: Our paper is part of a large literature studying financial and fiscal policies in non-Ricardian macroeconomic models. A recent branch of this literature studies the “fiscal-sustainability” of government debt taking fiscal policy and private sector pricing kernels as given (e.g. [Jiang, Lustig, Stanford, Van Nieuwerburgh and Xiaolan \(2022\)](#); [Chen, Jiang, Lustig, Van Nieuwerburgh and Xiaolan \(2022\)](#)) or deriving private sector pricing kernels from a model with incomplete markets that generate a premium on government debt (e.g. [Reis \(2021b\)](#), [Reis \(2021a\)](#), [Brunnermeier, Merkel and Sannikov \(2022\)](#)). Our paper studies the feasibility and costs of using financial regulation as a means to “choose” private sector pricing kernels that increase government fiscal capacity. Another branch of this literature studies fiscal-monetary connections (e.g. [Sargent and Wallace \(1981\)](#)) and the “fiscal theory of the price level” papers such as [Leeper \(1991\)](#), [Sims \(1994\)](#), [Woodford \(1994\)](#), [Cochrane \(2023\)](#), [Bianchi, Faccini and Melosi \(2023\)](#)). Unlike in these papers, government debt in our model is partially backed by financial regulation that creates captive demand within the financial sector and so makes government debt a safe asset. Ultimately, this means that fiscal policy not only backs government debt through the surplus process but also through its effectiveness as a safe asset. In this sense, we bring the fiscal cost of generating a funding cost spread onto the equilibrium path.

We are also part of a long literature attempting to understand how the government and the financial sector can create safe assets (e.g. [Holmstrom and Tirole \(1997\)](#), [Gorton and Odonez \(2013\)](#), [Gorton \(2017\)](#), [He, Krishnamurthy and Milbradt \(2016\)](#), [He, Krishnamurthy and Milbradt \(2019\)](#), [Choi, Kirpalani and Perez \(2022\)](#), [Bonfanti and Marcucci \(2025\)](#)) and the macroeconomic implications of safe asset creation (e.g. [Caballero, Farhi and Gourinchas \(2008\)](#), [Caballero, Farhi and Gourinchas \(2017\)](#), [Caballero and Farhi \(2018\)](#)). Our contribu-

tion to this literature is to connect an endogenous safe asset model to financial, fiscal, and monetary policy in a general equilibrium macroeconomy.

Our model focuses on how financial institutions manage the settlement risks that arise from maturity transformation (e.g. [Diamond and Dybvig \(1983\)](#), [Freeman \(1996a,b\)](#), [Gertler and Kiyotaki \(2010\)](#), [Williamson \(2012\)](#), [Gale and Yorulmazer \(2020\)](#), [Bianchi and Bigio \(2022, 2026\)](#)). The underlying friction in our environment arises from the inability to move resource from the depositors who have excess resources to the depositors who need additional resources. This is similar to in [Holmström and Tirole \(1998\)](#) but with frictional financial banks who face costs when they intermediate the process. Like [Gertler and Kiyotaki \(2010\)](#) and [Bianchi and Bigio \(2022\)](#), we integrate settlement frictions in the banking sector into a general equilibrium business cycle model. We don't attempt to innovate on the existing banking literature but instead show how financial-fiscal-monetary policy can create or erode inelastic bank demand for long-term government bonds in a standard bank model. Although we focus on banks, the forces we discuss would translate into other financial intermediary models where a financial intermediary faces financial frictions that non-pecuniary demand for long-term bonds that help them to hedge risk.

Our historical comparisons extend existing studies on the convenience yield (e.g. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Choi et al. \(2022\)](#)) back to the mid nineteenth century. This makes us part of a literature attempting to connect historical time series for asset prices to government financing costs (e.g. [Payne, Szőke, Hall and Sargent \(2023b\)](#), [Payne, Szőke, Hall and Sargent \(2023a\)](#), [Jiang, Lustig, Van Nieuwerburgh and Xiaolan \(2021\)](#), [Jiang, Lustig, Van Nieuwerburgh and Xiaolan \(2020\)](#)). Our focus on modeling the hedging properties of government debt is complementary to the empirical work of [Acharya and Laarits \(2023\)](#).

Outline: Section 2 documents the empirical evidence. Section 3 describes and characterizes our baseline model of captive demand and funding advantage. Section 4 introduces inflation risk and establishes the trilemma. Section 5 discusses how other commonly used models cannot account for the 1970s collapse in funding advantage. Section 6 presents the counterfactual fiscal experiment.

2 The Nature of Government Funding Advantage

In this section, we define the government funding advantage and discuss different views of its origins and equilibrium behavior. We argue that the defense of the popular bond-in-the-utility model rests on a fragile foundation: an apparent stable, policy-invariant equilibrium

relationship that dissolves once important institutional features of the US Treasury market are taken into account. Our new historical time series from [Lehner et al. \(2025\)](#) is more consistent with a policy-dependent picture of US government funding advantage.

2.1 Defining Government Funding Advantage

Suppose both the government and the corporate sector issue long-term bonds that pay a fraction ω of the remaining outstanding balance each period, so their average maturity is $1/\omega$. The bonds trade at prices q_t^b and q_t^h respectively. We say that the government has a funding advantage if it can sell its debt at a higher price than the private sector, $q_t^b > q_t^h$, even though the bonds promise the same cash flow stream. In yield terms, funding advantage can be summarized by the spread:

$$\chi_t := \left(-\omega \log(q_t^h) \right) - \left(-\omega \log(q_t^b) \right) > 0,$$

so the government can borrow at a cheaper rate than the private sector.¹ The object χ_t plays an important role in macro-finance, because its size determines the extent to which the government can issue debt unbacked by future tax revenue. Formally, let $\xi_{t,t+s}$ denote a stochastic discount factor that discounts payoffs between period t and $t+s$. Each period t , the government raises taxes τ_t , spends G_t , and issues long-term debt B_t . Define the period- t goods raised through new debt issuance as $\Delta_t := q_t^b(B_t - (1-\omega)B_{t-1})$ and write the inter-temporal government budget constraint as (see Appendix A):

$$\begin{aligned} (\omega + (1-\omega)q_t^b)B_{t-1} &= \mathbb{E}_t \left[\underbrace{\sum_{s=0}^{\infty} \xi_{t,t+s} (\tau_{t+s} - G_{t+s})}_{(i)} \right] \\ &+ \underbrace{\left(1 - \exp\left(\frac{-\chi_t}{\omega}\right) \right) (1-\omega)q_t^b B_{t-1} + \mathbb{E}_t \left[\sum_{s=0}^{\infty} \xi_{t,t+s} \left(1 - \exp\left(\frac{-\chi_{t+s}}{\omega}\right) \right) \Delta_{t+s} \right]}_{(ii)}. \end{aligned}$$

This equation states that the market value of outstanding debt (left-hand side) equals the present discounted value of future primary surpluses (term (i)) plus the present discounted value of the *convenience revenue*—the implicit “tax” the government earns by issuing debt

¹We consider the spread χ_t as a measure of the government *funding advantage* relative to the private sector rather than a measure of a premium on debt relative to non-debt assets more broadly. As such, it is conceptually different from the Treasury “box spread” (e.g. [van Binsbergen, Diamond and Grotteria \(2022\)](#)) or a “convenience yield” that many papers take to refer to the spread between expectation of the household’s stochastic discount factor and the yield on Treasuries (e.g. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#)). Under some assumptions, government funding advantage and convenience yield might coincide, but this distinction does not matter for our purposes.

at a lower cost than the private sector (term (ii)). The second term captures the contribution of the government funding advantage to debt capacity: the larger and more persistent the spread χ_t , the greater the share of debt that is backed by convenience revenue rather than fiscal surpluses. Understanding how various policies shape the sequence $\{\chi_t\}_{t \geq 0}$ is therefore central to understanding the government’s borrowing capacity and choice set.

2.2 Two Views of Government Funding Advantage

Researchers and policy makers have long been preoccupied by attempts to understand and control US government funding advantage, which has spawned a wide range of explanations for its origins and persistence. As an equilibrium relative price, the funding advantage is determined by the interplay of Treasury supply and demand. The supply side is uncontroversial: issuance is a policy instrument, and *ceteris paribus*, expanding the amount of outstanding debt compresses the government funding advantage by eroding the scarcity value of Treasuries. The demand side is less straightforward and more contested. At one extreme, we have the views of historical policy makers who have argued that government funding advantage is part of their choice set through the way they set financial-monetary-fiscal policy. At the other extreme, we have many macro-finance models, such as bond-in-utility (BIU) or bond-in-advance (BIA), that impose a policy-invariant Treasury demand function.

(I). Policy maker view: US policy makers have long sought to intervene in the Treasury market to reduce federal borrowing costs below the “prevailing market rates”. During the National Banking Era (1862–1913), Treasury Secretary Salmon Chase and Senator John Sherman pursued this goal by creating a captive market for Treasuries, requiring banks to back money creation with long-term government debt.² A generation later, World War I Treasury Secretary William McAdoo deployed conversion options, tax incentives, and “moral suasion” to hold government borrowing costs down and stabilize nominal returns on Treasuries, on the premise that bond price stability were essential to sustaining US funding advantage during the war—an argument that Federal Reserve Board research director Emanuel Goldenweiser would echo when advocating for yield curve control during World War II.³ Con-

²Lowenstein (2022) vividly describes how the Treasury Secretary turned to financial regulation as an instrument to finance the Civil War: “Chase was doggedly trying to raise money. [...] The banks were the only natural customers, and they would purchase only at a discount. [...] [Chase] needed a comprehensive and enduring solution. He therefore revived his proposal to create a new system of national banks designed, by charter, to lend to the government” (p. 145).

³On McAdoo’s conversion options and their role in stabilizing Liberty Bond prices, see Garbade (2012); Kennedy (2004) notes that “McAdoo’s unwillingness to rely either on heavy taxation *or on market-rate borrowing* led directly to the effort to mobilize emotions instead [...]” (p. 106). Emphasis added. The same logic of price stabilization resurfaced in 1941: Emanuel Goldenweiser proposed to the FOMC that “a

versely, financial commentators during the 1970s argued that the loss of the nominal anchor had eroded US funding advantage.⁴ More recently, the literature has pointed to Basel III and Dodd-Frank as modern instances of macroprudential regulations generating structural demand for government debt (e.g., [Gorton \(2017\)](#), [Begenau and Landvoigt \(2022\)](#)). Across these episodes, a common thread emerges: Treasury demand is shaped by government policy, whether through captive demand engineered via financial sector regulation, or through predictable returns maintained by a (promise of a) stable fiscal-monetary framework.

(II). Stable demand view: A popular motivation in the macro-finance literature on long-term government debt is a striking empirical pattern: historical asset pricing data appears to reveal a relatively stable, downward-sloping relationship between the US funding advantage, χ , and debt-to-GDP over the period 1920–2007 (e.g., [Krishnamurthy and Vissing-Jorgensen \(2012\)](#)). We replicate this pattern in the top panel of Figure 1, which plots the most commonly used measure of the funding advantage against the market value of debt-to-GDP. This regularity led many researchers to set aside institutional features of the Treasury market and the financial sector, focusing instead on reduced-form models that generate the downward-sloping relationship through a canonical scarcity effect—potentially shifted by exogenous demand shocks orthogonal to government policy. The workhorse framework in the macro-finance and safe-asset literatures reflects this view: a bond-in-the-utility model with exogenous demand shocks. The implicit interpretation is that, throughout the 20th century, movements in the funding advantage largely reflected changes in Treasury issuance, i.e., movements along a stable demand curve that was independent of government policy and only occasionally shifted by external forces.

2.3 Revisiting A Key Equilibrium Relationship

In [Lehner et al. \(2025\)](#), we show that the apparently stable equilibrium relationship in the top panel of Figure 1 is an artifact of mismeasurement. The most commonly used measure of government funding advantage in the literature is an *index-based* spread, constructed as the difference between (i) the Moody’s Seasoned AAA-rated corporate bond index, drawn from a sample of bonds with more than 20 years to maturity, and (ii) the Federal Reserve Board’s long-term US government bond yield index, constructed as the average yield on

definite rate be established for long-term Treasury offerings, with the understanding that it is the policy of the Government not to advance this rate during the emergency. [...] When the public is assured that the rate will not rise [and so the price will not fall], prospective investors will realize that there is nothing to be gained by waiting, and a flow of funds into Government securities [...] may be confidently expected” ([Federal Open Market Committee, 1941](#), p. 8). See also [Garbade \(2021\)](#) on this episode.

⁴See the Carter Bonds Episode (1978-1979), as well as [Kaufman \(1986\)](#), and [Goodfriend \(1993\)](#).

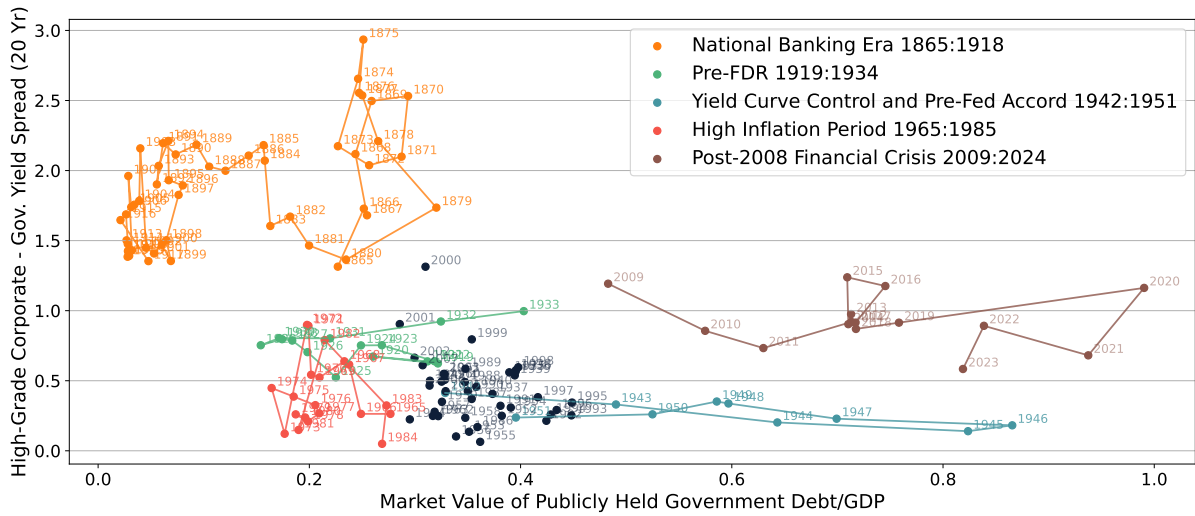
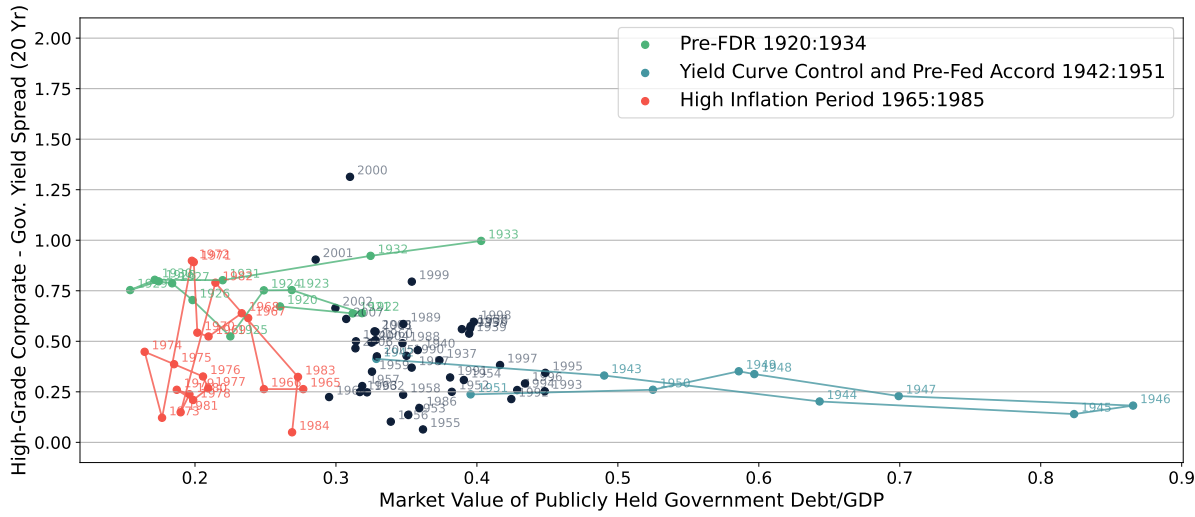
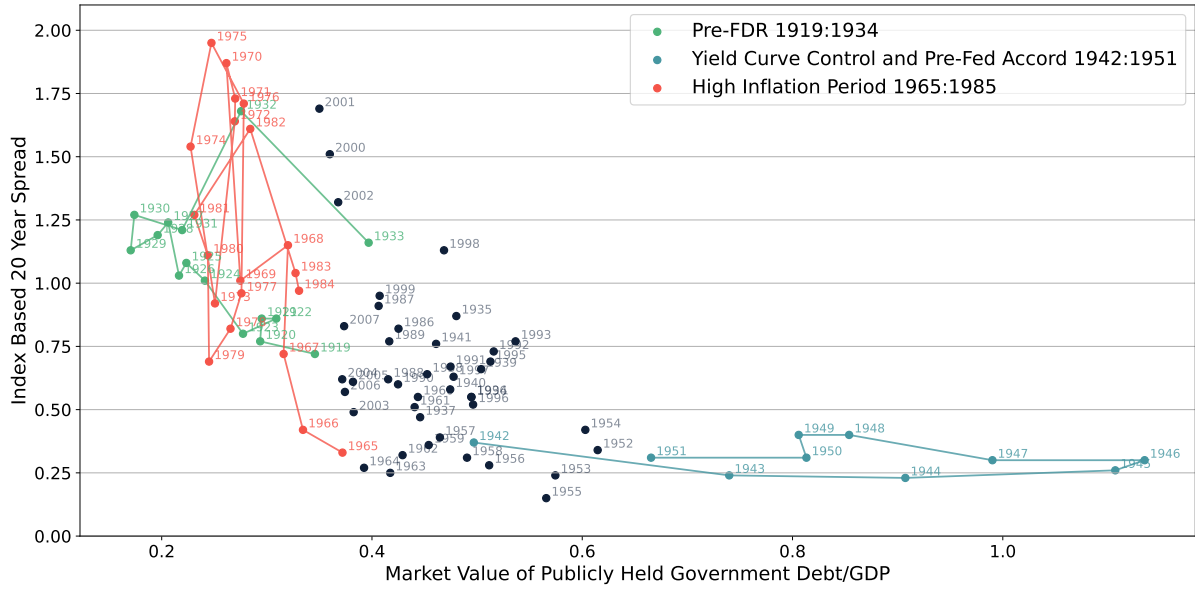


Figure 1: (a) The top panel replicates [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) using their data and time period from 1920-2007. (b) The middle panel uses our estimate of the spread to replicate the top panel. (c) The bottom panel uses our estimate for our full sample from 1865-2024.

all outstanding government bonds neither due nor callable in less than 10 years. This spread, however, does not compare like with like: in the 1960s-1970s it conflated the yield on a nominal corporate bond with the yield on a government “flower bond” that embedded both an implicit inflation hedge and a complex tax advantage. The apparent stability of the spread-quantity relationship in the top panel therefore reflects, in large part, the varying inflation-hedge premium embedded in Treasury flower bonds rather than any deep structural connection between government funding costs and the debt-to-GDP ratio. After correcting for these differences, a very different picture emerges. The middle panel of Figure 1 plots the corrected funding spread against debt-to-GDP for 1920-2007 and reveals almost no stable connection between the two series. The dependence on quantities is, at best, a weak predictor of equilibrium changes in the funding advantage.

The high-inflation episode of the 1970s and early 1980s (the red dots and lines) provides a particularly sharp illustration of what is at stake. In the top panel, the index-based spread appears to rise as inflation begins to erode the real value of long-term government debt after 1965, creating the impression that the US was moving along a stable, downward-sloping demand curve for Treasuries—consistent with the macro-finance models with a policy-invariant demand function. Our corrected series tells a fundamentally different story: as government debt was progressively devalued by inflation, the US funding advantage fell to approximately zero, its lowest value in the entire sample. Rather than moving along a stable demand curve, the 1970s episode looks like a breakdown—or a sharp leftward shift—of the equilibrium relationship itself. The US lost its funding advantage precisely when inflation became unanchored. This is consistent with policy-maker concerns that long-term price stability is a precondition for sustaining a government funding advantage.

We emphasize that none of these scatter plots identifies a demand function. They are therefore entirely consistent with the large applied microeconomic literature that uses a variety of instruments to establish that demand for Treasuries is downward-sloping in the market value of Treasury holdings.⁵ In fact, all of our structural modeling will generate demand functions that match this microeconomic evidence. What our results do challenge is the macro-finance practice of targeting a stable equilibrium relationship between quantities and spreads as an identification strategy. Interpreted through the lens of a bond-in-the-utility model, the difference between the top and middle panels implies that exogenous demand shifters—rather than movements along a stable pricing function—account for the preponderance of variation in the data. Any model that treats these shifters as a residual, or abstracts from them entirely, will draw misleading conclusions about the government’s

⁵See e.g., [Greenwood, Hanson and Stein \(2015\)](#), [Jiang, Richmond and Zhang \(2025\)](#), [Krishnamurthy and Li \(2023\)](#), [Jansen, Li and Schmid \(2025\)](#), [Cavaleri \(2024\)](#)

borrowing capacity.

The richness of the corrected series is further apparent when we extend the sample back to 1865, as shown in the bottom panel of Figure 1. Over this long horizon, there are pronounced low-frequency changes in both the level and curvature of the equilibrium relationship that correspond closely to changes in financial regulatory regimes emphasized by economic historians. The US funding advantage emerged in the late 1860s—well before the Bretton Woods system institutionalized the dollar’s reserve-currency role—and collapsed to zero during the high inflation of the 1970s, despite the dollar’s entrenched position as the dominant international reserve asset. These patterns suggest that the funding advantage is shaped by forces that operate at a lower frequency than the business cycle and that cut across conventional exchange-rate regimes.

Which kind of equilibrium relationship macro-finance is targeting has far-reaching implications, which raises the stakes on getting it correct in empirical work and macroeconomic modeling. If the US faces a stable, policy-invariant demand function for its debt, then a wide range of government actions—inflating away existing debt obligations, restructuring the financial sector, altering reserve requirements—leave the government’s monopoly position as a safe-asset supplier essentially intact. By contrast, if we think the US faces a policy-variant equilibrium pricing function, then the government faces a very different choice set. It can potentially expand the equilibrium pricing function for government debt but it can equally erode it through policies that raise return risk or crowd out private safe-asset demand. This opens up a collection of interesting questions about how government policies interact with government funding advantage. Can the government systematically intervene in government debt markets to change its funding advantage? What is the role of return risk? What is the actual choice set faced by a government? Which combinations of financial regulation, monetary policy, and fiscal policy are consistent with increasing household welfare?

These are inherently general equilibrium questions and require a structural model. In the next section, we build one. We endogenize the government funding advantage by focusing on the role that Treasuries play in helping financial intermediaries manage liquidity and risk. In the model, the shape of the equilibrium spread-quantity relationship is governed by the first-order conditions of profit-maximizing banks, which means the government can shift this relationship either through forced holdings and financial regulation or through fiscal and monetary policies that destabilize bond prices. This provides a micro-foundation for the demand shifters apparent in the bottom two panels of Figure 1, and allows us to nest the many narratives advanced by policy makers within a single unified framework connecting financial regulation, market structure, and monetary-fiscal policy. The result is a model

well-suited to speak to the policy-variant nature of the equilibrium pricing function that our new data series reveals.

3 A Model of Government Funding Advantage

In this section, we outline a tractable macro-finance model in which government funding advantage is endogenous and policy dependent. Conceptually, we do this by modeling an environment in which financial assets can take on a special role to help banks manage contract settlement frictions and their associated risks. If government bonds take on this role, then they trade at a higher price and lower yield than a private sector bond with an identical cash-flow stream, thereby giving the government a funding advantage. We use our model to show how financial regulation can be used to make government bonds (or other bonds) a “safe asset” and so make become the asset banks prefer for managing settlement risks.

Formally, we study a stochastic neoclassical growth model with a banking sector that engages in maturity transaction. Households need deposits in order to trade and produce input goods. Banks provide these deposits but cannot costlessly raise equity when settling deposit withdrawals. This is the main financial friction in our environment. It ultimately exposes banks to the risk of costly equity issuance and/or costly asset sales in the “crisis” states when households need more liquidity. The frictions make our environment a “second-best” world with two interconnected asset pricing distortions: a “liquidity spread” on financial sector liabilities and a “hedging spread” on assets that can help banks to self-insure against high withdrawal shocks. We assume that absent financial regulation, government and highest-grade corporate debt are equally useful for hedging settlement risks in the interbank market. That is, government debt does not have an immutable, special role in the economy.

We introduce a government into this environment that faces exogenous expenditure shocks, can raise taxes, and can manipulate asset markets by imposing restrictions on financial sector portfolios. This introduces additional asset-specific regulatory pricing distortions, which creates direct regulatory demand for government debt (a *forced holding effect*) and also changes the equilibrium co-movement between government debt prices and aggregate shocks (a *hedging effect*). In particular, the regulatory portfolio restrictions induce crowding into the secondary government debt market in crisis times. This makes government debt an effective hedging asset. We show that both the forced holding effect and the hedging effect generate “non-pecuniary” or “inelastic” demand for Treasuries which opens up a funding advantage for the government.

The implied regulatory wedges function as a form of “financial tax” that banks seek to minimize. A key feature of our model is that banks have the ability to mitigate the

impact of these wedges, though only by incurring a cost (either through costly equity raising in the morning market or by holding low return storage assets with the same duration as their liabilities). When this cost is high, regulation generates strong captive demand for government debt, producing a sizable funding advantage. However, unless the cost is prohibitive, the banking sector’s profit-maximizing behavior renders such captive demand inherently fragile. In Section 4 we show how this fragility gets activated by inflation risk.

3.1 Environment

Setting: The economy is in discrete time with infinite horizon: $t = 0, 1, 2, \dots$. Each period has morning and afternoon sub-periods. We interpret the afternoon sub-period as a primary asset market and the morning sub-period as a secondary (inter-bank) asset market. There is a final good that is used for consumption and investment and there is an input good that is combined with capital to produce final goods. There is a unit continuum of households, a unit continuum of banks, and a unit continuum of firms. There is a government that purchases consumption goods, issues debt, B_t , in the primary asset market, raises taxes from the family in the afternoon, and imposes regulatory constraints on bank portfolios. There is an exogenous state ϵ_t that is realized at the start of the morning market and follows a Markov Chain with transition matrix Π . We denote variables in the morning market with a breve, \check{v} , and in the afternoon market without a breve, v . We use a lower case letter v to denote a variable for an individual agent and a capital letter V to denote the aggregate variable.

Firms and production: Firms control an “afternoon” Cobb-Douglas production technology that transforms k_{t-1} units of capital and l_t units of “input goods” into $y_t = z(\epsilon_t)k_{t-1}^\alpha l_t^{1-\alpha}$ units of final goods in the afternoon of t . Capital depreciates at rate $\delta > 0$, so firms—who own the capital stock—enter each period t with $(1 - \delta)k_{t-1}$ units. In the afternoon of t , prior to production, they can convert final goods to capital goods 1 – 1. This implies a law of motion for capital stock $k_t = (1 - \delta)k_{t-1} + i_t$, where i_t is investment of final goods. Firms can issue long-term bonds, h_t , and equity, n_t in the afternoon market. Long term bonds repay a fraction ω of the outstanding balance in final goods each afternoon. Following the macro-finance literature (e.g. [Holmstrom and Tirole \(1997\)](#), [Kiyotaki and Moore \(2019\)](#)), firms face a collateral constraint that debt must be backed by capital to ensure no-default: $q_t^h h_t \leq \theta q_t^k k_t$, where q_t^h and q_t^k are the price of bonds and capital respectively.

Households: The economy is populated by a continuum of infinitely lived households with discount factor β . In the morning, households produce input goods according to a produc-

tion function $l_t = \check{l}_t$ where \check{l}_t is morning labor supply. However, only a fraction $1 - \lambda(\varepsilon_t)$ of households can use their own labor to produce input goods. The remaining fraction $\lambda(\varepsilon_t)$ need to use the labor of other households but lack the commitment required to issue credit and so must pay for labor inputs using deposits when purchasing labor (i.e. they face a “deposit-in-advance” constraint). All households supply labor at linear disutility ζ_t that is chosen to ensure that all households enter the afternoon with the same portfolio.⁶ Households can store the goods from morning to afternoon and sell them to firms. In the afternoon, all households get flow utility $u(c_t)$ from consuming c_t final goods in the afternoon.

Banks: In the afternoon of period $t - 1$, a one-period-lived representative bank issues demand deposits, d_{t-1} , and equity, e_{t-1} , to the households. In the morning of period t , a fraction $\lambda(\varepsilon_t)$ of households transfer deposits to other households, generating a gross outflow of $\lambda(\varepsilon_t)d_{t-1}$ deposits at the opening of the morning market and an offsetting gross inflow of the same magnitude at its close. Each deposit promises 1 unit of input goods if withdrawn in the morning and 1 unit of final goods if withdrawn in the afternoon. Banks may also carry m_{t-1} units of afternoon good into the morning via an inefficient storage technology that yields ϕm_{t-1} units of morning input good ($\phi < 1$) in the morning of period t , so excess deposit withdrawal that requires settlement is $\lambda(\varepsilon_t)d_{t-1} - \phi m_{t-1}$. Settlement is frictional. The interbank morning markets preclude free netting of outflows and future inflows: banks cannot write uncollateralized IOUs with each other and must settle gross outflows by either raising costly funds from households, or selling long-term assets. Specifically, banks may issue “non-preferred equity” \check{e}_t at morning price \check{q}_t^e , each share returning one final good in the afternoon, but doing so incurs a quadratic resource cost $\Psi(\check{q}_t^e \check{e}_t) = \frac{\psi}{2}(\max\{\check{q}_t^e \check{e}_t, 0\})^2$ borne in the afternoon.⁷ In the afternoon, banks sell their remaining assets to newly formed banks, pay out dividends x_t^e per share, and exit.⁸

Markets: We use the input good as the numeraire in the morning market and final goods as the numeraire in the afternoon market. In the afternoon, government bonds, corporate bonds, firm equity, bank deposits, bank equity, capital, and input goods are traded in competitive markets at prices $(q_t^b, q^h, q_t^n, q_t^d, q_t^e, q_t^k, w_t)$ respectively. In the morning, after the shocks are realized, banks can trade government bonds, at price \check{q}_t^b , and corporate bonds, at price \check{q}_t^h , in the secondary asset markets, and issue non-preferred equity at price \check{q}^e . However,

⁶We specify the details in Appendix C.2.

⁷Non-preferred equity \check{e}_t could equivalently be interpreted as an intra-period loan from households to banks because there is no risk between morning and afternoon.

⁸We use bank exit for expositional simplicity. Because there are no equity raising frictions in the afternoon market, we could equivalently build the model with infinitely lived banks that recapitalize each afternoon.

they cannot short-sell during the morning market.

Government: In the afternoon of period t , the government purchases consumption goods $g(\varepsilon_t)Y_t$, raises lump-sum taxes $\tau_t Y_t$ on the household, and issues long-term bonds in the primary asset market that repay a fraction ω of the outstanding balance in consumption goods at time t . The government's one-period budget constraint in the afternoon is:

$$(\omega + (1 - \omega)q_t^b)B_{t-1} \leq (\tau_t - g(\varepsilon_t))Y_t + q_t^b B_t. \quad (3.1)$$

where B_t denote government bonds outstanding at t . In addition, the government follows an exogenously determined fiscal rule, stipulating that taxes follow:

$$\tau_t = \bar{\tau} + \eta \left(\frac{B_{t-1}}{Y_t} - \bar{b} \right) \quad (3.2)$$

where \bar{b} is a “target level” of debt-to-output ratio (evaluated at par) and $\eta \geq 0$ measures the sensitivity of tax-to-output to deviations from the target level of outstanding debt-to-output. In Section 4 we extend the model to consider nominal debt and inflation.

The government can also impose restrictions on banks' portfolios after re-trading in the secondary asset markets, modeled as a stylized minimum capital requirement:⁹

$$\underbrace{\check{q}_t^b b_{t-1} + \check{q}_t^h h_{t-1} - (d_{t-1} - \phi m_{t-1}) + (1 - \kappa^e) \check{q}_t^e \check{e}_t}_{\text{regulatory capital}} \geq \varrho \underbrace{(\kappa^b \check{q}_t^b \check{b}_t + \kappa^h \check{q}_t^h \check{h}_t)}_{\text{weighted assets}} \quad (3.3)$$

where the left-hand side is regulatory capital defined as the bank's morning net worth adjusted for the equity raised during settlement and $(\check{b}_t, \check{h}_t)$ are post-trade holdings of government debt and corporate debt. The regulatory parameters $(\varrho, \kappa^b, \kappa^h, \kappa^e)$ have the following interpretations. $\varrho \in [0, 1]$ is the minimum capital ratio. $\kappa^b, \kappa^h \in [0, 1]$ are “penalty weights” on government and corporate debt: lower weights make an asset more favorable for regulatory capital purposes.¹⁰ $\kappa^e \in [0, 1]$ penalizes non-preferred equity as regulatory capital; at $\kappa^e = 1$ it is excluded entirely. Purchasing bond j tightens the regulatory constraint via penalty-weighted assets but relaxes it through the required equity buffer; the net regulatory effect is $\mathcal{K}^j := 1 - \kappa^e - \varrho \kappa^j$ for $j \in \{b, h\}$. The gap $\mathcal{K}^b - \mathcal{K}^h$ measures the relative regulatory privilege of government debt over corporate bonds. Although (3.3) is deliberately stylized, its parameters map directly onto historical regulatory regimes. However, our goal is not to

⁹Alternatively, we can think of this as a leverage constraint that restricts the bank's ability to back its deposit with long term assets.

¹⁰These can be mapped to Basel III risk weights or National Banking Era asset restrictions.

model any specific regulatory regime in detail, but to show that regulatory parameters have both financial and fiscal consequences.

3.2 Interbank Asset Market Equilibrium

We characterize partial equilibrium in the interbank morning market at time t for a given homogeneous bank portfolio $(m_{t-1}, b_{t-1}, h_{t-1}, d_{t-1})$ of storage assets, government bonds, corporate bonds, and deposits chosen in the preceding afternoon sub-period¹¹, a given realization λ_t , and a given set of afternoon payoffs from government bonds and corporate bonds, respectively:

$$x_t^b := (\omega + (1 - \omega)q_t^b), \quad x_t^h := (\omega + (1 - \omega)q_t^h).$$

We show how the regulatory and equity raising constraints discount the afternoon bond payoffs to the morning and how the co-movement between the two constraint can make government or corporate bonds a good hedge for banks in the morning market. This gives a tractable model for connecting financial regulations, risk, and financial frictions within the banking sector.

Bank problem in the morning: In the morning market, banks choose a new portfolio of government bonds, corporate bonds, deposits, and non-preferred equity $(\check{b}_t, \check{h}_t, \check{d}_t, \check{e}_t)$ subject to three constraints. The first is the gross withdrawal constraint:

$$\lambda_t d_{t-1} \leq \phi m_{t-1} + \check{q}_t^b (b_{t-1} - \check{b}_t) + \check{q}_t^h (h_{t-1} - \check{h}_t) + \check{q}_t^e \check{e}_t, \quad (3.4)$$

which states that, to finance morning deposit withdrawals, $\lambda_t d_{t-1}$, the bank must use its storage asset, sell its bonds amounting to $(b_{t-1} - \check{b}_t, h_{t-1} - \check{h}_t)$, or raise additional funds equal to $\check{q}_t^e \check{e}_t$ from households with idle deposits. The second is the morning budget constraint:

$$\check{q}_t^b \check{b}_t + \check{q}_t^h \check{h}_t - \check{d}_t - \check{q}_t^e \check{e}_t \leq \phi m_{t-1} + \check{q}_t^b b_{t-1} + \check{q}_t^h h_{t-1} - d_{t-1}, \quad (3.5)$$

which states that the market value of the bank's new portfolio is the market value of the assets they bring into the period. The final constraint is the regulatory constraint, which can be rewritten as an upper bound on the bank's regulatory debt obligations:

$$\check{d}_t + \kappa^e \check{q}_t^e \check{e}_t \leq (1 - \varrho \kappa^b) \check{q}_t^b \check{b}_t + (1 - \varrho \kappa^h) \check{q}_t^h \check{h}_t, \quad (3.6)$$

¹¹In equilibrium, all banks choose the same portfolio in the each afternoon market.

For future reference, let $\widehat{d}_t := \check{d}_t + \kappa^e \check{q}_t^e \check{e}_t$ denote the bank's regulatory obligations equal to the market value of long-term assets net of regulatory capital.¹²

In the following afternoon, banks pay their equity and deposit holders subject to the budget constraint:

$$x_t^e + \check{e}_t + \check{d}_t \leq x_t^b \check{b}_t + x_t^h \check{h}_t - \Psi(\check{q}_t^e \check{e}_t), \quad (3.7)$$

where $\Psi(\cdot)$ is the cost of raising morning equity.

Taking initial individual portfolios $(m_{t-1}, b_{t-1}, h_{t-1}, d_{t-1})$, morning asset prices $(\check{q}_t^b, \check{q}_t^h, \check{q}_t^e)$, and the afternoon payoffs (x_t^b, x_t^h) as given, banks solve the following problem each morning:

$$\begin{aligned} W_t(m_{t-1}, b_{t-1}, h_{t-1}, d_{t-1}) &:= \max_{\check{d}, \check{e}, \check{b}, \check{h}, x^e} \{ x_t^e \} \\ \text{s.t.} \quad &(3.4), (3.5), (3.6), (3.7), \\ &0 \leq \check{d}_t, \check{e}_t, \check{b}_t, \check{h}_t, \end{aligned} \quad (3.8)$$

where $W_t(\cdot)$ is the value of the bank in the morning (where we have subsumed the dependence on the aggregate state into the time subscript). We show in Section 3.5 that $\check{q}_t^e = 1$ and use this result throughout this Section.

Interbank equilibrium: is formalized in Definition 1 below. The conditions are standard, except for the deposit market clearing condition, which needs to account for both withdrawals and redepositing. Deposit demand falls in the morning due to withdrawals for trade and deposits swapped for equity, partially offset by constrained households redepositing after trade:

$$\check{d}_t = d_{t-1} \underbrace{- \lambda_t d_{t-1}}_{\text{withdrawal}} \underbrace{- \max\{\check{q}_t^e \check{e}_t, 0\}}_{\text{equity swap}} \underbrace{+ \max\{\lambda_t d_{t-1} - \phi m_{t-1}, 0\}}_{\text{redeposit}}$$

In equilibrium, the last two terms either cancel or both vanish, depending on whether the settlement constraint (3.4) binds. The presence of redepositing makes the model fundamentally different to [Diamond and Dybvig \(1983\)](#) where agents withdraw goods for consumption in the morning and never redeposit back into the banking sector.

Definition 1 (Competitive Interbank Asset Market Equilibrium). Given initial homogeneous bank portfolios $(m_{t-1}, b_{t-1}, h_{t-1}, d_{t-1})$ and afternoon payouts (x_t^b, x_t^h) , a competitive interbank market equilibrium is a collection of morning prices $(\check{q}_t^b, \check{q}_t^h, \check{q}_t^e)$ and bank choices

¹²From (3.5), we have $\widehat{d}_t := \check{d}_t + \kappa^e \check{q}_t^e \check{e}_t = \check{q}_t^b \check{b}_t + \check{q}_t^h \check{h}_t - (\phi m_{t-1} + \check{q}_t^b b_{t-1} + \check{q}_t^h h_{t-1} - d_{t-1} + (1 - \kappa^e) \check{q}_t^e \check{e}_t)$.

$(\check{b}_t, \check{h}_t, \check{e}_t, \check{d}_t, x_t^e)$, such that

- Taking prices as given, banks solve (3.8),
- Morning markets clear:

$$\check{b}_t = b_{t-1}, \quad \check{h}_t = h_{t-1}, \quad \check{d}_t = (1 - \lambda_t)d_{t-1}.$$

The morning market is sufficiently simple that equilibrium can be characterized in closed form, which we do in Proposition 1.

Proposition 1. *In a competitive interbank market equilibrium with $\kappa^e < 1$ and $\mathcal{K}^j > 0$ for $j \in \{b, h\}$, the morning prices $(\check{q}_t^b, \check{q}_t^h)$ satisfy:*

$$\check{q}_t^j = \frac{x_t^j}{(1 + \psi\check{e}_t)(1 - \mathcal{K}^j\check{\nu}_t)} \approx \frac{x_t^j}{(1 + \psi\check{e}_t - \mathcal{K}^j\check{\nu}_t)}, \quad j \in \{b, h\} \quad (3.9)$$

where the amount of equity raising, \check{e}_t , satisfies:

$$\check{e}_t = \max\{\lambda_t d_{t-1} - \phi m_{t-1}, 0\}$$

and the Lagrange multiplier on the regulatory constraint can be expressed as a function $\check{\nu}_t = \nu(\Gamma_t^b, \Gamma_t^h)$ of regulatory coverage ratios (assets' contribution toward regulatory requirement):

$$\Gamma_t^b := \frac{(1 - \rho\kappa^b)x_t^b b_{t-1}}{(1 + \psi\check{e}_t)(\check{d}_{t-1} + \kappa^e\check{e}_t)} \quad \Gamma_t^h := \frac{(1 - \rho\kappa^h)x_t^h h_{t-1}}{(1 + \psi\check{e}_t)(\check{d}_{t-1} + \kappa^e\check{e}_t)}$$

The regulatory constraint binds ($\check{\nu}_t > 0$) iff $\check{e} > 0$ and $\Gamma_t^b + \Gamma_t^h < 1$. In this case

$$\check{\nu}_t = \frac{\mathcal{K}^h(1 - \Gamma_t^b) + \mathcal{K}^b(1 - \Gamma_t^h)}{2\mathcal{K}^b\mathcal{K}^h} - \sqrt{\left(\frac{\mathcal{K}^h(1 - \Gamma_t^b) - \mathcal{K}^b(1 - \Gamma_t^h)}{2\mathcal{K}^b\mathcal{K}^h}\right)^2 + \frac{\Gamma_t^b\Gamma_t^h}{\mathcal{K}^b\mathcal{K}^h}}.$$

Proof. See Appendix C.1 □

Equation (3.9) in Proposition 1 says that the morning price of bond j is the afternoon payoff discounted by (approximately) $(1 + \psi\check{e}_t - \mathcal{K}^j\check{\nu}_t)$. The first distortion in the discount factor, $\psi\check{e}_t$, is the marginal cost of raising equity while the second distortion, $\mathcal{K}^j\check{\nu}_t$, is the marginal impact of bond j on the morning regulatory constraint.

To understand morning discounting, we start by focusing on the marginal cost of equity raising in an environment without a binding regulatory constraint, $\check{\nu}_t = 0$. In our model,

settlement frictions may force banks to raise costly equity in the morning, leading them to place a premium on morning resources relative to the afternoon, when equity raising is frictionless. This is the central friction in much of the macro-finance literature (Gertler and Kiyotaki, 2010). In our setup, the severity of this friction depends on banks' holdings of maturity-matched storage assets m_{t-1} . When storage suffices ($\phi m_{t-1} \geq \lambda_t d_{t-1}$) morning markets are frictionless. Banks issue no equity ($\check{e}_t = 0$) and morning prices equal afternoon payoffs ($\check{q}_t^j = x_t^j$). When storage falls short ($\phi m_{t-1} < \lambda_t d_{t-1}$), settlement constraint binds and banks must raise equity at marginal cost ψ . This drives morning prices below afternoon payoffs x_t^j for both bonds with larger withdrawal shocks λ_t intensifying the downward pressure on asset prices. Figure 2 illustrates this point: black lines show the ex-regulation prices that would prevail with settlement frictions and without regulation.

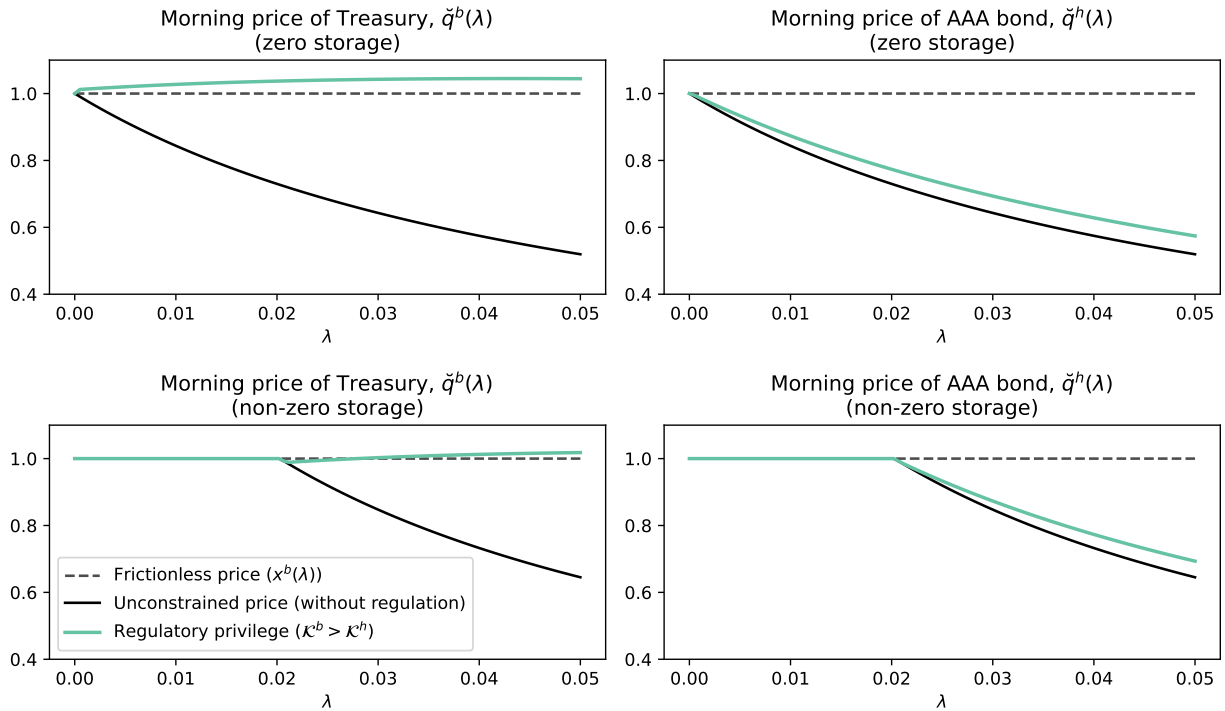


Figure 2: Morning Asset Prices With Regulatory Privilege

Black lines show the unconstrained prices, i.e., the price that would prevail without regulation. The teal lines show the morning market asset prices in an environment with regulations that privilege government debt. We used $b = 3.0$, $h = 3.5$, $d = 3.7$, $\psi = 5$, $\kappa^b = 1$, $\kappa^h = 0.19$.

Now consider regulation. The regulatory constraint (3.6) binds when the total regulatory coverage ratio $\Gamma_t^b + \Gamma_t^h$ falls below one. This creates an additional wedge $\check{\nu}_t > 0$ between morning and afternoon valuations, but with an opposite effect than equity frictions. Two forces determine the equilibrium value of $\check{\nu}$: scarcity and privilege. Scarcity operates through

the regulatory coverage ratios Γ^j . When withdrawals spike, equity \check{e}_t increases, depressing prices, eroding the regulatory coverage ratios and tightening the regulatory constraint. This mechanism ties \check{e}_t and $\check{\nu}_t$ together: bad states feature both high withdrawal pressure and tight regulatory constraints. Privilege enters through the gap $\mathcal{K}^b - \mathcal{K}^h$. When government debt is privileged, regulation tilts morning asset demand toward Treasuries.

We summarize key comparative statics in Corollary 1:

Corollary 1. *The regulatory wedge $\check{\nu}_t$ satisfies (see Appendix D.1):*

- (i) $\frac{\partial \check{\nu}_t}{\partial \Gamma_t^j} < 0$: *More regulatory coverage lowers regulatory tightness.*
- (ii) $\frac{\partial \check{\nu}_t}{\partial \check{e}_t} > 0$: *Equity issuance and regulatory tightness move together.*
- (iii) $\frac{\partial \check{\nu}_t}{\partial m_{t-1}} < 0$: *Storage mitigates both settlement and regulatory frictions.*

The relative morning price satisfies:

$$\frac{\check{q}_t^b}{\check{q}_t^h} \approx \left(\frac{x_t^b}{x_t^h} \right) \left(1 + (\mathcal{K}^b - \mathcal{K}^h) \check{\nu}_t \right). \quad (3.10)$$

When government debt is privileged ($\mathcal{K}^b > \mathcal{K}^h$), a tighter regulatory constraint raises the relative morning price of government debt.

Corollary 1 delivers a key insight: regulatory privilege creates a hedge, i.e., state-dependent “captive demand”, whereby periods of high withdrawals force equity issuance, which tilts interbank demand toward government debt. In effect, regulatory privilege exploits morning interbank market frictions to create a “safe asset” whose price appreciates precisely when banks need liquidity most. Figure 2 illustrates this point: teal lines show actual morning prices when government debt receives regulatory privilege ($\mathcal{K}^b > \mathcal{K}^h$). Government bond prices (left panel) appreciates as withdrawal λ increases. Corporate bond prices (right panel) fall below their afternoon payoff.

Storage plays a dual role. It relieves settlement pressure directly by reducing equity issuance \check{e}_t . It also relaxes regulatory constraints by improving coverage ratios Γ^j for $j \in \{b, h\}$. Both effects lower $\check{\nu}$, moderating the regulatory wedge. The bottom panels in Figure 2 illustrate the equilibrium price effects of bringing more storage asset into the morning market. The black lines are uniformly higher, the teal lines are uniformly lower than in the case with no storage asset (top panels). Clearly, holding more storage asset reduces the severity of morning interbank market frictions.

3.3 Bank Portfolio Choice and Inelastic Treasury Demand

Subsection 3.2 characterized the morning interbank market equilibrium for a given bank portfolio chosen the previous afternoon. We now turn to the bank's afternoon problem to determine that portfolio. We show that the morning equilibrium provides a micro-foundation for the non-pecuniary Treasury demand that appears in workhorse macro-finance models. This is ultimately what will generate a general equilibrium funding advantage for the government.

In the afternoon of period t , after production and dividend payouts, banks choose portfolios (m_t, b_t, h_t, d_t) of storage asset, government bonds, corporate bonds, and deposits. Taking prices and the household's one-period stochastic discount factor, $\xi_{t,t+1}$, as given, the representative bank solves:

$$\begin{aligned} \max_{m_t, b_t, h_t, d_t} \quad & \mathbb{E}_t \left[\xi_{t,t+1} W_{t+1}(m_t, b_t, h_t, d_t) \right] + q_t^d d_t - m_t - q_t^b b_t - q_t^h h_t \\ \text{s.t.} \quad & 0 \leq m_t, b_t, h_t, d_t \end{aligned} \quad (3.11)$$

The first order conditions for the portfolio choice in the afternoon market are (dropping the short selling constraints for (b_t, h_t, d_t) which don't bind):

$$[m_t] : \quad 1 \geq \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 + \kappa^e \check{\nu}_{t+1}) \right] \phi =: q_t^m = 1 \quad \text{if } m_t > 0$$

$$[b_t] : \quad q_t^b = \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 + \kappa^e \check{\nu}_{t+1}) \check{q}_{t+1}^b \right] \quad (3.12)$$

$$[h_t] : \quad q_t^h = \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 + \kappa^e \check{\nu}_{t+1}) \check{q}_{t+1}^h \right] \quad (3.13)$$

$$[d_t] : \quad q_t^d = \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) \left(\frac{1 + \lambda_{t+1} \psi \check{e}_{t+1}}{1 + \psi \check{e}_{t+1}} + (1 - \lambda_{t+1} (1 - \kappa^e)) \check{\nu}_{t+1} \right) \right] \quad (3.14)$$

where we used the Envelope conditions. We define q_t^m as the shadow value of storage in the afternoon of period t : if this value falls below the price of one, the bank sets $m_t = 0$. otherwise $q_t^m = 1$. Equations (3.12), and (3.13) are standard portfolio Euler equations augmented with the wedges $(\check{e}_{t+1}, \check{\nu}_{t+1})$ which capture how interbank-market frictions distort the bank's portfolio allocation. Equation (3.14) equates the deposit price to the risk-weighted average marginal cost of servicing a unit of deposits across the morning and afternoon.

The elasticity matrix: To build intuition for the bank's afternoon portfolio problem, we characterize the bank's asset demand functions conditional on beliefs about the law of motion of aggregate states through the matrix of log-price elasticities:

Proposition 2. *The bank's afternoon portfolio choices can be characterized by total price*

derivatives, defined by partial elasticities between the vectors of log-prices and asset demands:

$$\begin{pmatrix} d \log q_t^b \\ d \log q_t^h \\ d \log q_t^d \\ d \log q_t^m \end{pmatrix} = \begin{bmatrix} \eta_{t,b}^b & \eta_{t,h}^b & \eta_{t,d}^b & \eta_{t,m}^b \\ \eta_{t,b}^h & \eta_{t,h}^h & \eta_{t,d}^h & \eta_{t,m}^h \\ \eta_{t,b}^d & \eta_{t,h}^d & \eta_{t,d}^d & \eta_{t,m}^d \\ \eta_{t,b}^m & \eta_{t,h}^m & \eta_{t,d}^m & \eta_{t,m}^m \end{bmatrix} \begin{pmatrix} d \log b_t \\ d \log h_t \\ d \log d_t \\ d \log m_t \end{pmatrix}$$

where for the two bonds $j \in \{b, h\}$ and assets $i \in \{b, h, d, m\}$ we have (see Appendix D.1):

$$\eta_{t,i}^j = -\mathbb{E}_t \left[\frac{\xi_{t,t+1} \frac{\partial \log \Gamma_{t+1}}{\partial \log i} \left(\frac{(1-\varrho\kappa^j)}{(1-\mathcal{K}^j \check{\nu}_{t+1})^2} \frac{x_{t+1}^j}{q_t^j} \right)}{\sqrt{\left(\mathcal{K}^h (1 - \Gamma_{t+1}^b) - \mathcal{K}^b (1 - \Gamma_{t+1}^h) \right)^2 + \mathcal{K}^b \mathcal{K}^h \Gamma_{t+1}^b \Gamma_{t+1}^h}} \right]$$

with

$$\eta_{t,b}^b < 0 \quad \eta_{t,h}^b < 0 \quad \eta_{t,m}^b \leq 0 \quad \eta_{t,d}^b > 0 \quad \eta_{t,b}^h < 0 \quad \eta_{t,h}^h < 0 \quad \eta_{t,m}^h \leq 0 \quad \eta_{t,d}^h > 0$$

Proof. See Appendix D.1 □

Each elasticity $\eta_{t,i}^j$ factors into three components: a common denominator term capturing the overall tightness of the morning regulatory constraint, a bond-specific term (in parentheses) reflecting asset j 's regulatory weight and pecuniary return, and the sensitivity of total regulatory coverage ratio Γ_{t+1} to holdings of asset i . Signs follow directly. In particular, own-price elasticities are negative, indicating downward-sloping asset demands. Evidently, the elasticities are non-linear functions of regulatory parameters as well as the co-movement between the household SDF, $\xi_{t,t+1}$, asset returns x_{t+1}^j/q_t^j and the tightness of the regulatory constraint through the regulatory coverage ratios. The model's structure for η is sufficiently flexible to match the log-price elasticities identified in the applied-micro and financial-IO literatures using exogenous variation in bond supply.

Pecuniary vs Non-pecuniary returns: Substituting the morning equilibrium prices from Proposition 1 into the government bond Euler equation gives:

$$q_t^b = \underbrace{\mathbb{E}_t \left[\xi_{t,t+1} x_{t+1}^b \right]}_{\text{pecuniary benefit}} + \underbrace{(1 - \varrho\kappa^b) \mathbb{E}_t \left[\xi_{t,t+1} x_{t+1}^b \frac{\check{\nu}_{t+1}}{1 - \mathcal{K}^b \check{\nu}_{t+1}} \right]}_{=:\nu_t^b \quad \text{non-pecuniary benefit}}$$

The first term is the bond's "fundamental value". The second, ν_t^b , is what the banks are

willing to pay *above* fundamental value: it is the morning regulatory wedge $\check{\nu}_{t+1}$ translated into afternoon dollars. Corporate bonds carry an analogous premium ν_t^h determined by their regulatory weight. To measure the substitutability of government and corporate bonds, we study how relative demand $(q_t^b b_t)/(q_t^h h_t)$ responds to changes in a relative price—but not just any price. From the bond Euler equation, the convenience yield on asset j is:

$$cy_t^j := 1 - \mathbb{E}_t \left[\xi_{t,t+1} x_{t+1}^j \right] / q_t^j \quad j \in \{b, h\}$$

the share of q_t^b unexplained by discounted cash-flows—equivalently, the portion of the price attributable to non-pecuniary benefits. In a frictionless environment $\nu_t^b = cy_t^b = \nu_t^h = cy_t^h = 0$, the two bonds are perfect substitutes and their prices are pinned down entirely by fundamentals.

When the two bonds carry different non-pecuniary benefits, they become imperfect substitutes. To measure this, we ask how readily one bond can be swapped for the other in the bank portfolio as the relative (shadow) price of their non-pecuniary benefits changes, holding bank shareholder value fixed and taking other asset prices as given. Proposition 3 gives the resulting elasticity of substitution between b_t and h_t :

Proposition 3. *The elasticity of substitution between b_t and h_t is*

$$\sigma_t(\bar{q}^e, \bar{q}^d) := - \left. \frac{\partial \log((q_t^b b_t)/(q_t^h h_t))}{\partial \log(cy_t^b/cy_t^h)} \right|_{(q_t^e = \bar{q}^e, q_t^d = \bar{q}^d, q_t^m = 1)}$$

yielding the point-by-point decomposition:

$$\log((q_t^b b_t)/(q_t^h h_t)) = \zeta_t(\bar{q}^e, \bar{q}^d) + \sigma_t(\bar{q}^e, \bar{q}^d) \log(cy_t^b/cy_t^h)$$

where ζ_t is the “demand shifter”, i.e., the σ_t -implied relative demand that would prevail under equal non-pecuniary benefits ($\nu_t^b = \nu_t^h$). The elasticity takes the form:

$$\sigma_t = - \frac{\mathbb{E}_t \left[\xi_{t,t+1} \Omega_{t,t+1} \left(\frac{(1-\kappa^b \varrho)}{(1-\mathcal{K}^b \check{\nu}_{t+1})^2} \frac{x_{t+1}^b}{q_t^b} - \frac{(1-\kappa^h \varrho)}{(1-\mathcal{K}^h \check{\nu}_{t+1})^2} \frac{x_{t+1}^h}{q_t^h} \right) \right]}{\mathbb{E}_t \left[\xi_{t,t+1} \Omega_{t,t+1} \left(\frac{1-cy_t^b}{cy_t^b} \frac{(1-\kappa^b \varrho)}{(1-\mathcal{K}^b \check{\nu}_{t+1})^2} \frac{x_{t+1}^b}{q_t^b} - \frac{1-cy_t^h}{cy_t^h} \frac{(1-\kappa^h \varrho)}{(1-\mathcal{K}^h \check{\nu}_{t+1})^2} \frac{x_{t+1}^h}{q_t^h} \right) \right]}$$

where

$$\Omega_{t,t+1} := \frac{\left(\sum_{i \in \{b, h, d, m\}} \frac{\partial \log \Gamma_{t+1}}{\partial \log i} \beta_{t,b}^i \right)}{\sqrt{\left(\mathcal{K}^h (1 - \Gamma_{t+1}^b) - \mathcal{K}^b (1 - \Gamma_{t+1}^h) \right)^2 + \mathcal{K}^b \mathcal{K}^h \Gamma_{t+1}^b \Gamma_{t+1}^h}}$$

with $\beta_{t,b}^b = 1$ and

$$\begin{pmatrix} \beta_{t,b}^h \\ \beta_{t,b}^m \\ \beta_{t,b}^d \end{pmatrix} := - \begin{bmatrix} \eta_{t,h}^e & \eta_{t,m}^e & \eta_{t,d}^e \\ \eta_{t,h}^m & \eta_{t,m}^m & \eta_{t,d}^m \\ \eta_{t,h}^d & \eta_{t,m}^d & \eta_{t,d}^d \end{bmatrix}^{-1} \begin{bmatrix} \eta_{t,b}^e \\ \eta_{t,b}^m \\ \eta_{t,b}^d \end{bmatrix}$$

Proof. See Appendix D.2 □

The elasticity of substitution σ_t and the demand shifter ζ_t are not deep parameters—they vary with the regulatory environment. This dependence comes from two sources: regulatory scarcity, $1/\mathbb{E}_t[\Gamma_{t+1}]$, which measures how tightly the regulatory constraint binds on average, and regulatory privilege, $\mathcal{K}^b - \mathcal{K}^h$, which measures the gap in regulatory treatment between government and corporate bonds. Indeed, the latter is the very source of imperfect substitutability: were the two assets treated identically by the regulator, no wedge between their convenience yields would arise and they would be perfect substitutes. Figure 3 plots σ_t against both dimensions and delivers a clear message: the tighter the regulation or the wider the regulatory privilege of government bonds, the less substitutable the two assets become. Substitutability is thus endogenous to policy—each turn of the regulatory screw makes government and corporate bonds harder to swap, amplifying the price impact of any shock that shifts their relative supply.

Proposition 3 provides a micro-foundation for the elasticity of substitution—and show it is anything but constant. Substitutability moves with the regulatory environment, echoing the policy-sensitivity we documented in Section 2. This stands in sharp contrast to a large macro-finance literature (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016) that models imperfect substitutability between government and corporate bonds via a bond-in-utility (BIU) specification with constant elasticity of substitution (CES). By hard-wiring σ as a preference parameter, this approach assumes away the inherent policy-dependence: the funding advantage of government bonds becomes invariant to the regulatory regime, leading to misguided conclusions about the equilibrium behavior of funding advantage. We return to this point in Section 5.

3.4 General Equilibrium

Sections 3.2 and 3.3 solved for the interbank market equilibrium and the bank’s portfolio choice problem, taking the household stochastic discount factor, deposit demand, and corporate bond supply as given. We now close the model by characterizing general equilibrium across the morning and afternoon markets, which lets us trace how the household and firm

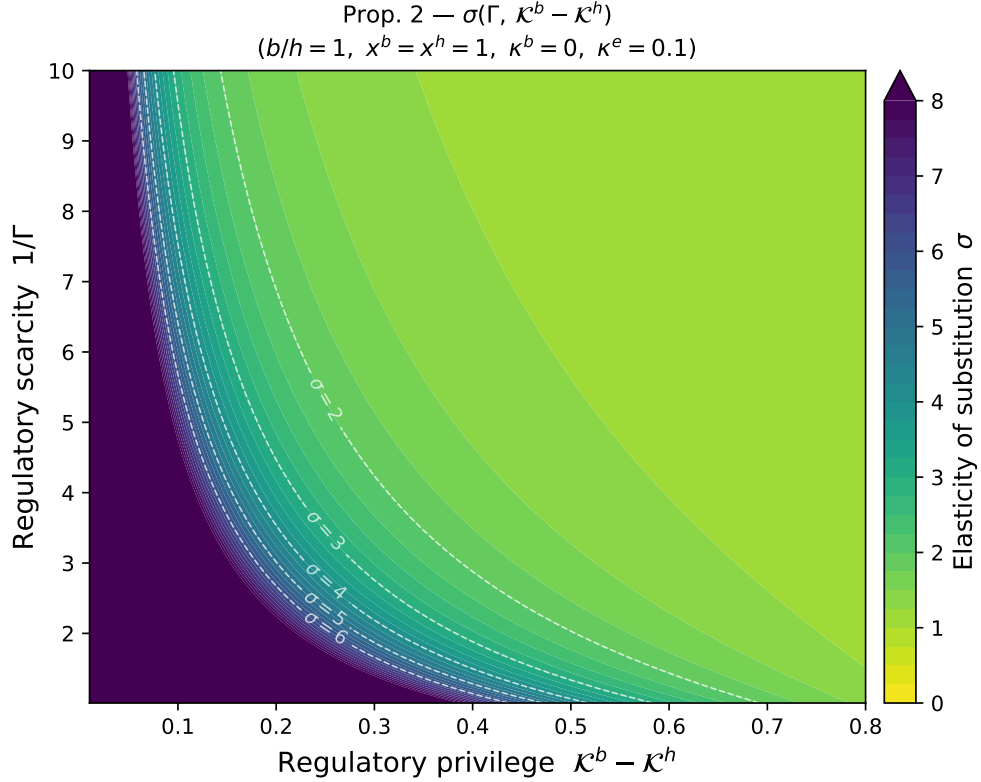


Figure 3: Elasticity of substitution

sectors interact with banks.

Household problem: At the start of the afternoon sub-period, a household has unspent wealth a_t , input goods l_t and tax obligations $\tau_t Y_t$. The household's budget constraint in the afternoon sub-period at time t is:

$$c_t + q_t^d d_t + q_t^e e_t + q_t^n n_t \leq a_t + w_t l_t - \tau_t Y_t \quad (3.15)$$

where c_t denotes goods consumed by the household in the afternoon sub-period, w_t is the afternoon price of the input good, l_t is the amount of input good produced by the household in the morning and stored between the morning and afternoon, and (d_t, e_t, n_t) denote the new portfolio of bank deposits, bank and firm equity, respectively, and τ_t denotes the tax-to-output ratio in the afternoon sub-period.

In the following morning sub-period, the new exogenous aggregate states ϵ_{t+1} and the households' idiosyncratic shock for their morning trading needs are realized with probability $\lambda_{t+1} = \lambda(\epsilon_{t+1})$. Unconstrained households (indexed by u) that can self-produce choose their own labor effort $\check{l}_{t+1,u}$ and potentially contribute preferred equity \check{e}_{t+1} that is repaid in the

afternoon. Constrained households (indexed by c) that need to purchase inputs choose how much labor to sell to the market \check{l}_{t+1} and how much labor to purchase from the market $\check{l}_{t+1,c}$ subject to the deposit in advance constraint:

$$\check{w}_{t+1}\check{l}_{t+1,c} \leq d_t \quad (3.16)$$

So a household of type $j \in \{u, c\}$ enter the afternoon sub-period with input goods $\check{l}_{t+1,j}$ and financial wealth:

$$a_{t+1,j} = d_t - \tilde{\Delta}_{t+1,j} + x_{t+1}^e e_t + (x_{t+1}^n + q_{t+1}^n)n_t \quad (3.17)$$

where $\tilde{\Delta}_{t-1,j}$ is the financial profit of a type j household in the morning. Specifically, $\tilde{\Delta}_{t+1,u} = -\check{q}_t^e \check{e}_{t+1} + \check{e}_{t+1}$ for unconstrained households and $\tilde{\Delta}_{t+1,c} = -\check{w}_{t+1}\check{l}_{t+1,c} + \check{w}_{t+1}\check{l}_{t+1}$ for constrained households. In equilibrium, we will have $\tilde{\Delta}_{t+1,u} = \tilde{\Delta}_{t+1,c} = 0$.

Let $V_t(a, l)$ denote the value of the household with unspent wealth a and input goods l at the start of the afternoon (where the dependence on aggregate state variables has been subsumed into the t subscript). Then, taking the prices as given, the value function $V_t(\cdot)$ satisfies the Bellman equation (3.18) below:

$$\begin{aligned} V_t(a, l) = & \max_{\left\{ \begin{array}{l} (\check{c}, \check{l}^u, \check{l}, \check{e})_{t+1}, \\ (c, e, d, n)_t \end{array} \right\}} \left\{ u(c_t) + \beta \mathbb{E}_t \left[\lambda_{t+1} (-\zeta_{t+1,c} \check{l}_{t+1,c} + V_{t+1}(a_{t+1,c}, l_{t+1,c})) \right. \right. \\ & \left. \left. + (1 - \lambda_{t+1}) (-\zeta_{t+1,u} \check{l}_{t+1,u} + V_{t+1}(a_{t+1,u}, l_{t+1,u})) \right] \right\} \quad (3.18) \\ & s.t. \quad (3.15), (3.16), (3.17) \end{aligned}$$

For the morning choices, we set $\zeta_{t+1}^u = \zeta_{t+1}^c + \check{w}_{t+1}\check{\mu}_{t+1}^d$ which equates $l_{t+1,u}$ and $l_{t+1,c}$ and thereby permits aggregation across households. We also have that $\check{q}^e = 1$. Imposing these morning choices and applying the envelope theorem, the first-order conditions for the afternoon choices yield:

$$\begin{aligned} [d_t] : & \quad q_t^d = \mathbb{E}_t \left[\xi_{t,t+1} \left(1 + \lambda_{t+1} \check{L}_{t+1} \right) \right] \\ [e_t] : & \quad q_t^e = \mathbb{E}_s \left[\xi_{t,t+1} x_{t+1}^e \right] \\ [n_t] : & \quad q_t^n = \mathbb{E}_s \left[\xi_{t,t+1} x_{t+1}^n \right] \end{aligned}$$

where the stochastic discount factor (SDF) and the “liquidity wedge” are defined by:

$$\xi_{t,t+1} := \beta \frac{\partial_c u(c_{t+1})}{\partial_c u(c_t)}, \quad \check{L}_{t+1} := \frac{\nu \check{\mu}_{t+1}^d}{\partial_c u(c_t)}.$$

The liquidity wedge, \check{L}_{t+1} , appears because demand deposits provide liquidity services to the households by allowing them to insure trading shocks in the morning sub-period. The presence of this asset-specific wedge implies that households are willing to hold demand deposits at a discount.

Firm problem: Taking prices and the household’s SDF as given, a representative firm entering the afternoon with (k, h) solves:

$$\begin{aligned} V_t^f(k, h) = \max_{l_t, k_t, h_t} & \left\{ z(\varepsilon_t) k^\alpha l_t^{1-\alpha} - w_t l_t + q_t^h h_t - q_t^k k_t - x_t^h h_t + \right. \\ & \left. + q_t^k (1 - \delta) k + \mathbb{E}_t [\xi_{t,t+1} V_{t+1}^f(k_t, h_t)] \right\} \quad (3.19) \\ \text{s.t.} & \quad q_t^h h_t \leq \theta q_t^k k_t \end{aligned}$$

The first order conditions are:

$$\begin{aligned} [l_t] : & \quad w_t = (1 - \alpha) z(\varepsilon_t) k^\alpha l_t^{-\alpha} \\ [k_t] : & \quad q_t^k (1 - \theta \mu_t^h) = \mathbb{E}_t [\xi_{t,t+1} x_{t+1}^n] \\ [h_t] : & \quad q_t^h (1 - \mu_t^h) = \mathbb{E}_t [\xi_{t,t+1} x_{t+1}^h] \end{aligned}$$

where μ_t^h is the Lagrange multiplier on the collateral constraint and the afternoon payoff from firm equity is

$$x_t^n := \alpha y_t / k + (1 - \delta) q_t^k$$

In equilibrium, the multiplier μ_t^h is linked to the banking frictions. Greater regulatory privilege on corporate debt leads to a higher μ_t^h . We can also see that $\mu_t^h > 0$ leads to a distortion in capital choice (capital price is above “fundamental value”) because capital is needed to issue bonds when the collateral constraint binds.

We can now set up a competitive equilibrium. Given a fiscal rule (3.2) and bond price process $(q_t^b)_{t \geq 0}$, a budget-feasible government issuance rule $(B_t)_{t \geq 0}$ satisfies (3.1).

Definition 2 (Budget-feasible Competitive Equilibrium). For given regulation parameters $(\varrho, \kappa^e, \kappa^b, \kappa^h)$, and a budget-feasible government policy processes $(\tau_t, g_t, B_t)_{t \geq 0}$, a competitive

equilibrium is a collection of functions for price processes $(q_t^d, q_t^e, q_t^n, q_t^h, q_t^k, q_t^b, \check{q}_t^h, \check{q}_t^b, \check{q}_t^e, \check{w}_t)_{t \geq 0}$, payoffs $(x_t^e, x_t^n)_{t \geq 0}$, household policies $(d_{t,h}, e_t, n_t, c_t, \check{\ell}_t, \check{l}_{t,c}, \check{l}_{t,u}, i_{t,h})_{t \geq 0}$, firm policies $(k_t, l_{t,f}, h_{t,f})_{t \geq 0}$, and bank policies $(\check{b}_t, \check{h}_t, \check{e}_t, d_t, h_t, b_t, m_t, x_t^e)_{t \geq 0}$ such that

- Taking prices as given, households solve (3.18), banks solve (3.11), and firms solve (C.21),
- The morning interbank market $(\check{q}_t^h, \check{q}_t^b, \check{q}_t^e, \check{b}_t, \check{h}_t, \check{e}_t)$ variables satisfy the morning market equilibrium (Definition 1) and the morning labor market clears: $\check{l}_{t,c} = \check{\ell}_t$.
- The afternoon input good, capital, and goods markets clear:

$$m_{t,f} = m_t, \quad i_{t,h} = k_t - (1 - \delta)k_{t-1}, \quad c_t + i_t + m_t + g_t = y_t - \Psi_t,$$

and afternoon asset markets clear:

$$d_{t,h} = d_t, \quad e_t = 1, \quad n_t = 1, \quad b_t = B_t, \quad h_{t,f} = h_t.$$

3.5 Equilibrium Government Funding Advantage

We now return to the government's funding advantage, defined as the yield spread between government and corporate bonds—neither of which carries default risk in our model. Because the two securities deliver identical cash-flow streams (the same ω), any observed spread must reflect the non-pecuniary benefits arising from their differential regulatory treatment. Formally, we define the government funding advantage as:

$$\begin{aligned} \chi_t &:= -\omega \log(q_t^h) - (-\omega \log(q_t^b)) \\ &= \omega \log \left(\mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 + \kappa^e \check{\nu}_{t+1}) \frac{\check{q}_{t+1}^b}{\check{q}_{t+1}^h} q_{t+1}^h \right] \right) \\ &\quad - \omega \log \left(\mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 + \kappa^e \check{\nu}_{t+1}) \check{q}_{t+1}^h \right] \right) \end{aligned} \quad (3.20)$$

where we have expanded the terms using the bank first order conditions. We interpret χ_t as the model counterpart to our empirical measure of government funding advantage from Section 2.1. Our χ_t is also approximately equal to the relative convenience yield we used in Subsection 3.3 when computing elasticities, $\log(cy_t^b/cy_t^h) = (1 - \mathbb{E}_t[\xi_{t,t+1}(\omega + (1 - \omega)q_{t+1}^b)]/q_t^b)/(1 - \mathbb{E}_t[\xi_{t,t+1}(\omega + (1 - \omega)q_{t+1}^h)]/q_t^h)$.

In our model, government funding advantage arises from the special role that government debt plays in the financial sector in the morning market. We can see this by expanding (3.20)

to get the approximate expression:

$$\chi_t \approx \underbrace{\omega \log \left(\mathbb{E}_t \left[\frac{\check{q}_{t+1}^b}{\check{q}_{t+1}^h} \right] \right)}_{\text{forced holdings}} + \underbrace{\omega \text{Cov}_t \left(\frac{\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 + \kappa^e \check{\nu}_{t+1}) \check{q}_{t+1}^h}{q_t^h}, \frac{\check{q}_{t+1}^b / \check{q}_{t+1}^h}{\mathbb{E}_t [\check{q}_{t+1}^b / \check{q}_{t+1}^h]} \right)}_{\text{hedging motive}}$$

So, the government’s funding advantage arises from the average appreciation of government debt in the next period’s morning markets and the covariance between government debt appreciation and the bank’s marginal valuation of additional resources. By introducing regulation that ensures that re-trading government debt is valuable in bad times, the government introduces a positive covariance and so introduces a government borrowing cost advantage. That is, regulation makes government debt a particularly “good-hedge” for mitigating the banking sector’s frictions in the morning market and so earns a premium.

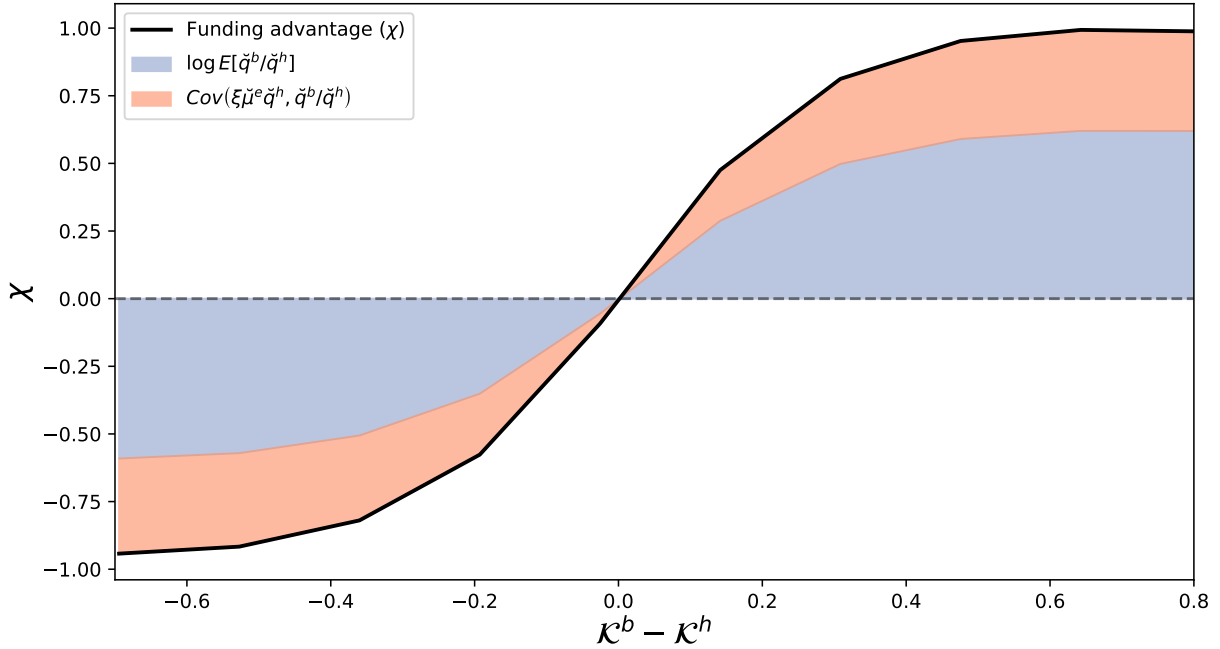


Figure 4: Decomposition of funding advantage as a function of regulatory privilege.

We used $b = 3.0$, $h = 3.5$, $d = 3.7$, $\psi = 5$.

Self-confirming regulatory “risk weights”: Figure 4 illustrates the government funding advantage—along with its two primary components—relative to the regulatory privilege of Treasuries. We can see that regulation can dictate which asset becomes a good hedge against the banking sector’s settlement risks. In this sense, there is self-confirming nature of regulatory “risk

weights”. Specifically, when corporate debt holds the regulatory advantage ($\mathcal{K}^b - \mathcal{K}^h < 0$), the covariance term is negative, meaning government debt prices are low precisely when banks need them to be high. Conversely, when government debt is privileged ($\mathcal{K}^b - \mathcal{K}^h > 0$), the covariance term turns positive (government debt becomes a good hedge), contributing a non-trivial increase in the government funding advantage.

3.6 Asset Scarcity and Government Funding Advantage

We close this section by showing that our model is sufficiently flexible to generate patterns resembling those in the data from Section 2. Figure 5 presents scatter plots correlating the funding advantage with debt-to-GDP ratios across various policy combinations. Evidently, both regulations and fiscal rules influence the government funding advantage. For a fixed debt-to-GDP target (left two and right two clusters), increasing regulatory privilege, ($\mathcal{K}^b - \mathcal{K}^h$) \uparrow , raises both the average funding advantage and its sensitivity (slope) to the debt-to-GDP ratio. Conversely, for a given level of regulatory privilege (top two and bottom two clusters), higher debt-to-GDP targets result in a lower average funding advantage and a flatter slope. This highlights that the “safe-asset” curve that has frequently been targeted by macro-finance modeling is ultimately policy variant.

4 Inflation Risk and Funding Advantage

Section 3 showed how financial regulation generates a government funding advantage by turning government debt into the banking sector’s “safe asset” for hedging risk. But this advantage does not exist in a vacuum—it interacts with monetary-fiscal policy and the government budget constraint. We now consider devaluation risk: a stagflation scenario in which high inflation strikes in bad states of the world. This policy regime flips the sign of regulation’s support for government debt. The regulatory privilege $\mathcal{K}^b - \mathcal{K}^h$ still pushes banks toward Treasuries in the morning for regulatory reasons, but inflation now turns those holdings into losses precisely when banks can least afford them. Banks respond by raising equity and exiting the deposit market, and the morning hedging demand that once propped up government bond prices in bad states evaporates.

The afternoon bank portfolio problem is affected by this loss of a “safe asset”. Banks now need to manage settlement risk and inflation risk simultaneously, and the cleanest way to mitigate them is to hold more of the storage asset and sidestep the regulated sector altogether. The banking sector contracts, the captive market shrinks, and as regulation’s grip on relative bond demand loosens, government and corporate bonds become more substitutable—the

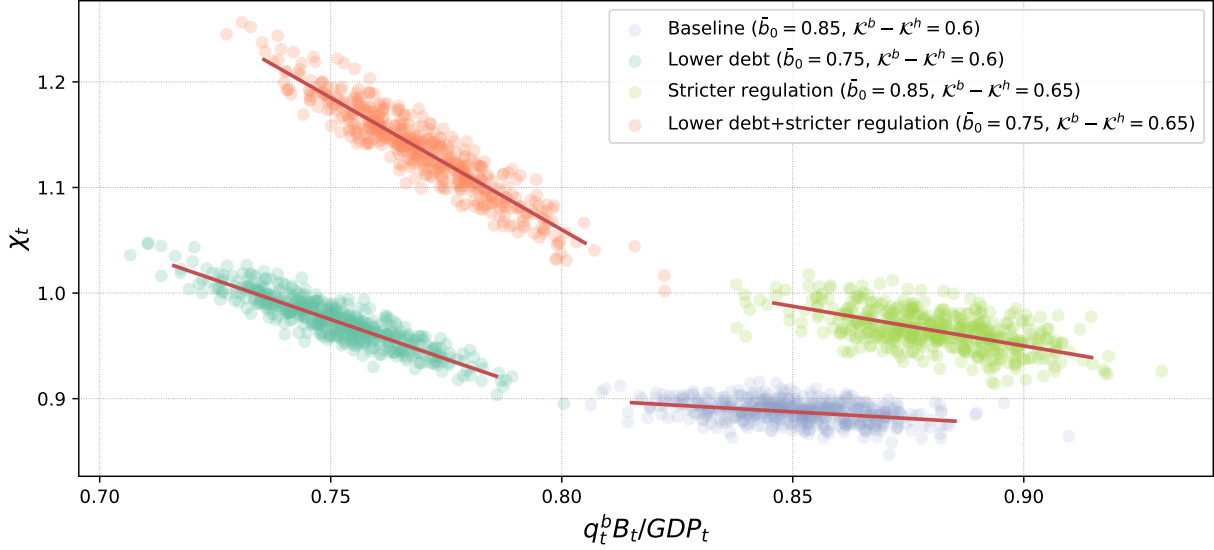


Figure 5: Funding advantage vs debt-to-GDP under different policy mixes.

Different colors represent different policy mixes: blue cluster (bottom right) shows the baseline with regulatory privilege $\mathcal{K}^b - \mathcal{K}^h = 0.6$ and debt-to-GDP target $\bar{b} = 0.85$; green cluster (bottom left) shows regulatory privilege $\mathcal{K}^b - \mathcal{K}^h = 0.6$ and a lower debt-to-GDP target of $\bar{b} = 0.75$; yellow cluster (top right) shows higher regulatory privilege $\mathcal{K}^b - \mathcal{K}^h = 0.66$ and debt-to-GDP target of $\bar{b} = 0.85$; orange cluster (top left) shows higher regulatory privilege $\mathcal{K}^b - \mathcal{K}^h = 0.65$ and lower debt-to-GDP target $\bar{b} = 0.75$ than the baseline.

special status of Treasuries fades. The result is a contraction on both margins: less deposit creation overall (extensive) and less government debt used to back what remains (intensive).

The government's funding advantage is therefore fragile. Inflation risk makes the banking sector substitute away from the very debt regulation was meant to privilege. Ultimately, the government faces a domestic financing trilemma: it cannot simultaneously sustain (i) a high funding advantage, (ii) a stable financial sector, and (iii) a monetary-fiscal regime that destabilizes Treasury prices through inflation.

4.1 Environment Changes

The setup is as in Section 3.1 but we introduce nominal debt and inflation risk.

Government: We now assume that the government issues money and that government bonds, corporate bonds, and deposits pay in units of money in the afternoon. We impose that the government chooses supply so that gross inflation $1 + \pi(\varepsilon_t)$ is a lognormal random variable with mean one, standard deviation σ_π , and is negatively correlated with output growth. We

refer to this shock as inflation risk or devaluation risk. We follow [Woodford and Walsh \(2005\)](#) and take the cashless limit so that money quantities do not affect the equilibrium.¹³ We continue to use morning and afternoon goods as the numeraire. The government's one-period real budget constraint in the afternoon now becomes:

$$\left(\omega + (1 - \omega)q_t^b\right) \frac{B_{t-1}}{(1 + \pi(\boldsymbol{\varepsilon}_t))Y_t} \leq \tau_t - g(\boldsymbol{\varepsilon}_t) + \frac{q_t^b B_t}{Y_t}$$

and the politically determined fiscal rule becomes:

$$\tau_t = \bar{\tau} + \eta \left(\frac{B_{t-1}}{(1 + \pi(\boldsymbol{\varepsilon}_t))Y_t} - \bar{b} \right).$$

where B_t denotes the total real principle promised. So, we can see that the inflation of the nominal government debt appears like a haircut on the real value of government debt.

Asset pricing: The real afternoon payoffs of government and corporate bonds become:

$$x_t^j := \frac{1}{(1 + \pi(\boldsymbol{\varepsilon}_t))} \left(\omega + (1 - \omega)q_t^j \right), \quad j \in \{b, h\}, \quad \forall \mathbf{s} \quad (4.1)$$

Otherwise, the equilibrium asset pricing conditions remain the same.

4.2 Inflation Risk and Government Funding Advantage

Figure 6 plots the morning price of Treasuries, the morning price of corporate bonds, and the funding advantage as a function of inflation risk. The black lines show the economy with regulatory privilege but without debt devaluation through inflation while the teal line shows the economy with stagflation risk. Evidently, the introduction of inflation risk changes both the forced holding and hedging components of χ . The introduction of inflation risk turns Treasuries into assets that devalue during bad times and so reverses the sign of the hedging motive. At the same time, holding Treasuries to satisfy regulatory constraint leads to losses, which imply more costly equity raising in the morning, the forced holding component of the funding advantage also reduces. As a result, higher values of σ_π end up leading to a lower value of χ .

To understand how the introduction of inflation risk changes the role of government debt in the financial sector, we can return to the characterization of the interbank market.

¹³There is a large literature pointing out the issues with taking a cashless limit (e.g. see [Lagos \(2024\)](#)). We abstract from these issues to focus on long-term debt pricing rather than monetary economics.

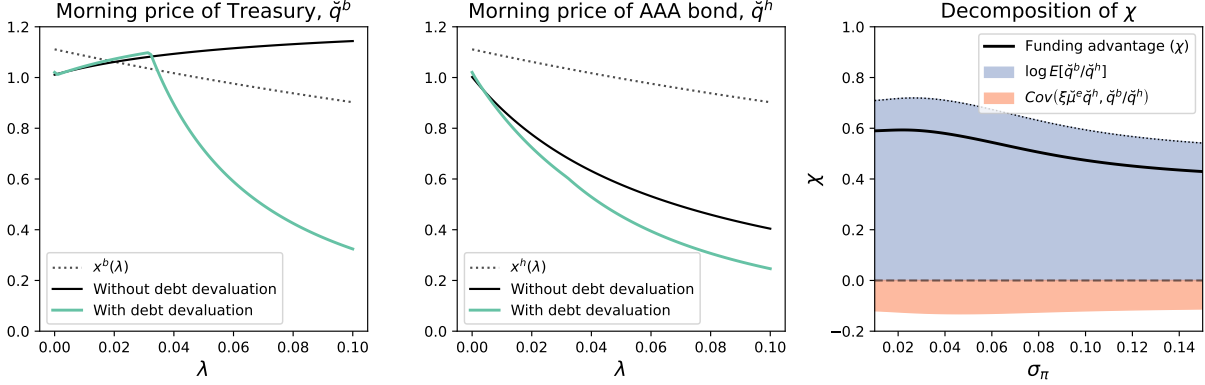


Figure 6: Morning Asset Prices With and Without Debt Devaluation.

Black lines shows the morning market prices in an environment with regulatory privilege without debt devaluation through inflation. Green lines show the morning market prices for an economy with regulatory privilege and inflation. Dashed lines show the afternoon payoff. The final subplot decomposes the funding advantage χ .

Revisiting the expression for the relative price ratio in the morning market, (3.10), we can see that the direct impact of $\pi(\varepsilon_t)$ on payouts from equation (4.1) does not show up in the funding advantage because both government and corporate bonds are nominal so the direct impact of inflation is differenced out. Instead, higher inflation risk changes the magnitude and sign of $\check{\nu}$.

Conceptually, the combination of regulatory privilege and government devaluation through inflation leads to these outcomes because the combination puts the banking sector in a difficult position. If banks don't purchase government debt in the morning market, then they violate the regulatory constraint. However, if they purchase government debt, then the government's monetary-fiscal policy devalues their debt in the afternoon and forces losses onto the equity holders. The banks respond to this lose-lose situation by effectively "exiting" the deposit market (i.e. inflation activates the "extensive margin" to push banks out of the market).

Taken together, our results show that our model gives the government both the strength to create a funding advantage but also makes the funding advantage fragile and sensitive to monetary-fiscal policy—the government must supports the longer term value of government debt in order to keep its funding advantage.

Comparison to other risk premia: To understand these results, it is helpful to distinguish between our funding advantage and other spreads. We can decompose the difference

between the household required rate of return and the treasury yield as:

$$\begin{aligned}
& -\omega \log(\mathbb{E}_t[\xi_{t,t+1}]) - (-\omega \log(q_t^b)) & (4.2) \\
& = \underbrace{\omega \log \left(\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{1 + \kappa^e \check{\nu}_{t+1}}{1 - \mathcal{K}^b \check{\nu}_{t+1}} \right) x_{t+1}^b \right] \right) - \omega \log \left(\mathbb{E}_t \left[\xi_{t+1} \left(\frac{1 + \kappa^e \check{\nu}_{t+1}}{1 - \mathcal{K}^b \check{\nu}_{t+1}} \right) x_{t+1}^h \right] \right)}_{\text{Government funding advantage} =: \chi_t} \\
& \quad + \underbrace{\omega \log \left(\mathbb{E}_t \left[\xi_{t+1} \left(\frac{1 + \kappa^e \check{\nu}_{t+1}}{1 - \mathcal{K}^b \check{\nu}_{t+1}} \right) x_{t+1}^h \right] \right) - \omega \log \left(\mathbb{E}_t \left[\xi_{t,t+1} x_{t+1}^h \right] \right)}_{\text{Market "segmentation" premium}} \\
& \quad + \underbrace{\omega \log \left(\mathbb{E}_t \left[\xi_{t+1} x_{t+1}^h \right] \right) - \omega \log(\mathbb{E}_t[\xi_{t,t+1}])}_{\text{Standard risk premium (using the household SDF)}}
\end{aligned}$$

The first component is the government funding advantage compared to corporate bonds, both of which are traded by the financial sector in the morning market (expressed by substituting in the morning market price formulas). The second component is the difference between the banking sector's valuation of the corporate bond (which they can use to trade in the morning market) and the household's valuation of the same bond (which they cannot use to trade in the morning market). This can be interpreted as the wedge from the market segmentation that prevents households from directly trading with bonds in the morning market and the different discount rate faced by the banking sector. In this sense, it could be interpreted as a type of common liquidity premium on government bonds. The final component is the risk premium on government debt, as valued by the households in the economy.

The decomposition in equation (4.2) highlights that our funding advantage is not a standard risk premium and so the reason why inflation affected the funding advantage is not because an inflation risk premium arises. Both government and corporate bonds are nominal and so the inflation risk premium (and other parts of the standard risk premium) difference out when we compare them.

4.3 A Domestic Financing Trilemma

We close this section by bringing our results together and returning to the feasible policy combinations of the government. We focus on two government policy parameters: the degree of regulatory privilege for government debt ($\mathcal{K}^b - \mathcal{K}^h$) and inflation volatility (σ_π). Figure 7 shows two equilibrium variables as these variables are changed: the frequency of forced equity raising (or bank default) (on the LHS) and the government funding advantage, χ , (on the RHS). The x-axis shows σ_π and the y-axis shows $\mathcal{K}^b - \mathcal{K}^h$. The ergodic mean magnitudes of the morning equity raised and the funding advantage are indicated by a heat map. Evi-

dently, increasing financial repression can increase the private-public borrowing cost spread. However, when accompanied by a devaluation of US debt, increasing financial repression also leads to higher distress in the financial sector. As discussed, this is because banks are being forced to hold debt with a negative return and so they start to force losses onto depositors and exit the deposit market.

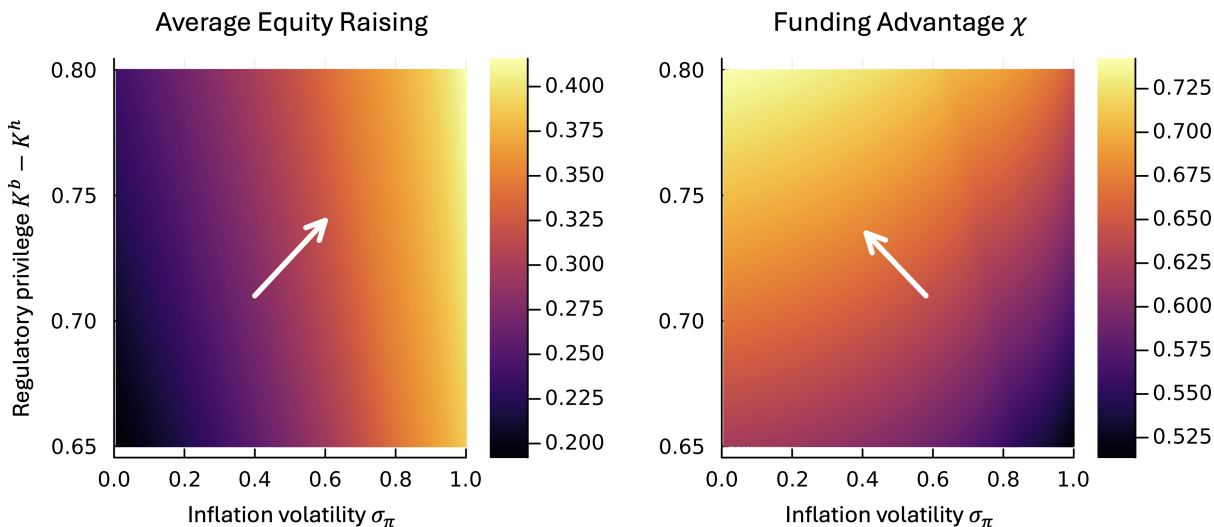


Figure 7: Government Financing Trade-offs

The left subplot shows a heat map with level of government bond regulatory privilege on the y-axis, the inflation volatility on the x-axis, and average forced equity raising (or default) in the financial sector as the color. The right subplot shows a heat map with the same x and y-axes but with the private-public borrowing cost spread as the color.

We summarize these observations as a stylized “trilemma” for the government. In our model, by varying $\mathcal{K}^b - \mathcal{K}^h$ and σ_π , the government cannot choose all three of:

1. High government funding advantage (a high χ),
2. A well-functioning, profitable, and stable financial sector (low costly equity raising or default, \check{e}^r), and
3. Monetary-fiscal policy that generates high inflation risk (a high σ_π).

Intuitively, this Trilemma says that when a government uses the financial sector to generate funding advantage, then it intertwines the balances sheets of the banking sector and the government, which constrains the range of feasible government policies. If the government sets a high $\mathcal{K}^b - \mathcal{K}^h$ (choosing part 1 of the trilemma), then it either needs to run monetary-fiscal policy that targets bond price stability (giving up part 3 of the trilemma) or it will

put the financial sector into distress (giving up part 2 of the trilemma). In this sense, our model of endogenous funding advantage both gives the government more freedom to expand funding advantage but also highlights that this freedom rests on a stable monetary-fiscal environment.

This trilemma can also be interpreted as introducing a notion of financial dominance to complement existing notions of fiscal and monetary dominance. There are many macroeconomic theories and models of monetary-fiscal interactions (e.g. Keynes (1924), Friedman (1995), Hansen (1949), Tobin (1969), Sargent and Wallace (1981), Wallace (1981), Aiyagari and Gertler (1985), Leeper (1991), Sims (1994), Woodford (1994), Cochrane (2023)). These papers often present a type of dichotomy between monetary dominance on the one hand and fiscal dominance on the other hand. In the former case, monetary policy is actively chosen by the government and fiscal policy has to accommodate to deliver the desired nominal interest rate path. In the latter case, fiscal policy is chosen by the government and monetary policy accommodates to deliver the required monetary policy. However, very few of these papers consider the role of the financial sector in assessing government debt sustainability even though historically much US federal debt has been held by financial intermediaries. Our trilemma suggests that introducing a financial sector leads to the possibility of financial dominance. If the government chooses a high funding advantage and a stable financial sector, then it must run combined monetary-fiscal policy that stabilizes long-term debt prices and protects the balance sheet of the financial sector.

5 Revisiting Funding Advantage in the 1970s

In this section, we revisit the 1970s-80s in the United States, which is one of the periods in US history with the highest inflation volatility and lowest government funding advantage. We use this historical episode to highlight how our model explains the loss of funding advantage during this period. We contrast this to other models (namely bond-in-the-utility and bond-in-advance), which give the counterfactual prediction that a higher inflation volatility should coincide with higher funding advantage.

5.1 An Episode of High Inflation

Figure 8 depicts our historical estimates (from Lehner et al. (2025)) for the ratio of the market value of government debt to GDP, the 15 year government advantage, and the comovement between Treasury returns and stock returns (which we compute as the bond-stock

beta) during the the period from 1971 to 1980.¹⁴ As we have discussed in previous work, this is one of the periods in US history with the highest inflation volatility and strongest negative correlation between inflation and output growth (e.g. see Payne et al. (2023b)). Figure 13 in Appendix B summarizes the key results and show that the only period since 1790 with higher inflation volatility is the Civil War.

During 1971 to 1980, the bond-stock beta goes from approximately zero to approximately 0.8 during the early 1980s. This is the largest increase in our overall sample from 1860-2025. We interpret the bond-stock beta as a data counterpart to the hedging role of Treasuries in our model, with a positive β indicating that government debt is a poor hedge and a negative beta indicating that government debt is a good hedge. So, this indicates holding treasuries provided almost no hedge against aggregate risk during this period. The increase in the bond-stock beta corresponds to a sustained decline in the funding spread even though the Debt-to-GDP ratio moves very little during the period. This means that 1970s-80s are a key period during which there are large changes in US government funding advantage that are not associated with changes to the market value of debt.

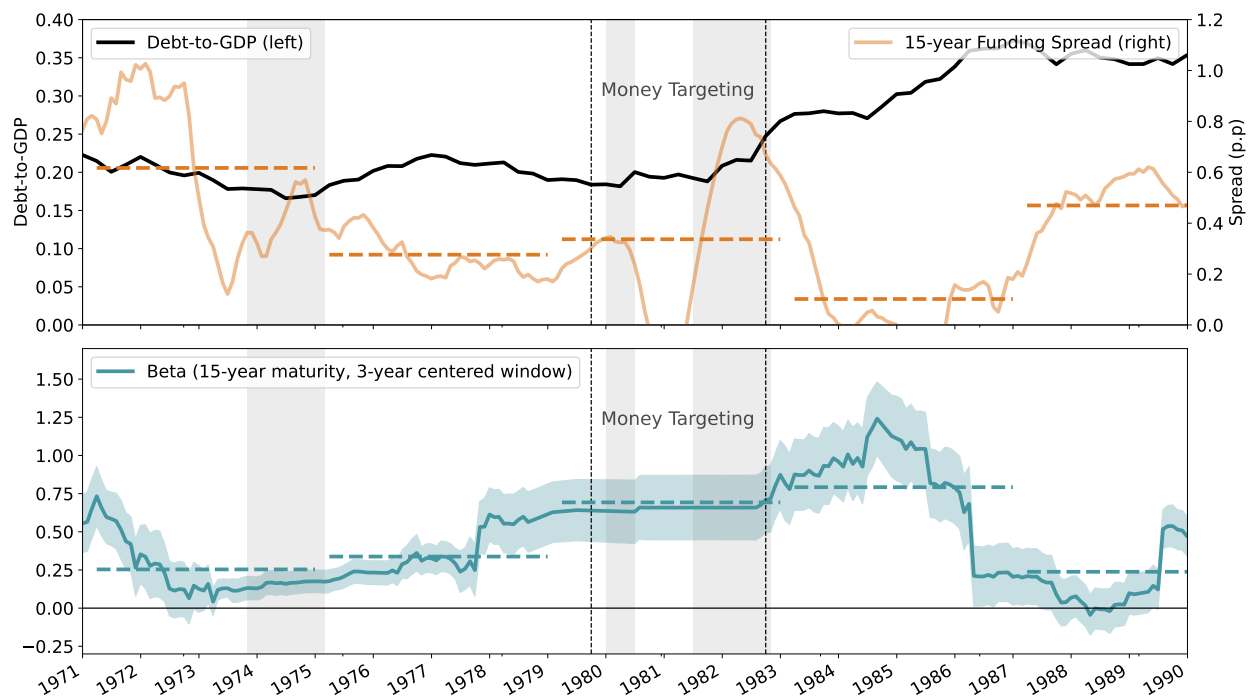


Figure 8: Sub-period From 1971 to 1990.

¹⁴Technically, the bond-stock “beta” is the coefficient when excess holding returns are regressed against excess stock returns (which we refer to as the bond-stock beta). Formally, for each maturity j , we regress the monthly excess holding return $rx_{t+1}^{j-1} = \log(q_{t+1}^{(j-1)}) - \log(q_t^{(j)}) - r_t$ against the monthly percentage change in the GFD historical total return index on US equities.

5.2 Interpretation Through Our Model

At a stylized level, we can interpret the 1970s-80s through the lens of our results in Section 4. During this period, the US government ran systematically high and volatile inflation leading to the real devaluation of US debt. According to the trilemma, this meant the government had to choose between maintaining its funding advantage by forcing the financial sector to hold more government debt and maintaining financial stability by allowing the financial sector to substitute away from government debt at the expense of the government's funding advantage. So our model interprets Figure 8 as the government losing its funding advantage from 1975 to 1985 rather than forcing losses onto the banking sector. Indeed, if we look at evidence on banking sector government debt holdings, we see that the balance sheet of the banking sector went from being approximately 90% treasuries and reserves during World War I to approximately 20% treasuries in the 1980 (see [Chari, Dovis and Kehoe \(2020\)](#)).

5.3 Interpretation Through Bond-in-Utility and Bond-in-Advance

When presented through our model and the data, it may seem natural that the government lost its funding advantage during the 1970s when inflation volatility emergence and holding government debt become more risky. However, many other commonly used models end up generating the opposite prediction. We explore this in two commonly used models of government funding advantage and convenience yields: bond-in-utility (BIU) and bond-in-advance (BIA).

5.3.1 Bond-in-Utility (BIU)

Environment: We remove the morning market, regulatory constraints, and banking sector and instead follow [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and [Nagel \(2016\)](#) by introducing a household utility benefit of holding government debt and corporate bonds in the afternoon market. Formally, suppose that now the representative household seeks to maximize the objective:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \bar{\zeta}_t l_t + \alpha \log(\mathcal{B}_t) \right) \right]$$

where $u(\cdot) = \log(\cdot)$, $\mathcal{B}_t = ((1 - \bar{\lambda}_t)(q_t^b b_t)^\rho + \bar{\lambda}_t(q_t^h h_t)^\rho)^{1/\rho}$ is the composite bond index, and $\bar{\lambda}_t$ is a potentially stochastic weight.

Household optimization problem: The household Bellman equation for a household entering

period t with wealth a becomes:

$$V_t(a) = \max_{c_t, l_t, b_t, h_t, n_t} \left\{ u(c_t) - \bar{\zeta}_t l_t + \frac{\alpha}{\rho} \log \left((1 - \bar{\lambda}_t)(q_t^b b_t)^\rho + \bar{\lambda}_t (q_t^h h_t)^\rho \right) + \beta \mathbb{E}_s[V_{t+1}(a_{t+1})] \right\}$$

$$s.t. \quad c_t + q_t^b b_t + q_t^h h_t + q_t^n n_t \leq a - w_t l_t - \tau_t Y_t$$

$$a_{t+1} = x_{t+1}^b b_t + x_{t+1}^h h_t + x_{t+1}^n n_t,$$

Taking FOCs and imposing the envelope theorem $\partial_a V_t(a) = u'(c)$ leads to the conditions:

$$u'(c_t) \left(1 - \beta \mathbb{E}_t \left[\frac{(\omega + (1 - \omega)q_{t+1}^b) u'(c_{t+1})}{q_t^b} \right] \right) = \frac{\alpha(1 - \lambda_t)(q_t^b b_t)^{\rho-1}}{(1 - \lambda_t)(q_t^b b_t)^\rho + \lambda_t (q_t^h h_t)^\rho}$$

$$u'(c_t) \left(1 - \beta \mathbb{E}_t \left[\frac{(\omega + (1 - \omega)q_{t+1}^h) u'(c_{t+1})}{q_t^h} \right] \right) = \frac{\alpha \lambda_t (q_t^h h_t)^{\rho-1}}{(1 - \lambda_t)(q_t^b b_t)^\rho + \lambda_t (q_t^h h_t)^\rho}$$

Government funding advantage: After rearranging the FOCs and imposing equilibrium, we get:

$$\chi_t = \log \left(\frac{(\omega + (1 - \omega)q_{t+1}^b) \frac{u'(c_{t+1})}{u'(c_t)}}{(\omega + (1 - \omega)q_{t+1}^h) \frac{u'(c_{t+1})}{u'(c_t)}} \right) + \log \left(\frac{1 - \frac{\alpha}{u'(c_t)} \frac{(1 - \lambda_t)(q_t^b b_t)^{\rho-1}}{\mathcal{B}_t^\rho}}{1 - \frac{\alpha}{u'(c_t)} \frac{\lambda_t (q_t^h h_t)^{\rho-1}}{\mathcal{B}_t^\rho}} \right)$$

There are two special cases that have been much studied in the literature:

1. *Special case:* $\omega = 1$: For short term bonds, the first term drops out and so the funding advantage becomes the relative non-pecuniary benefit:

$$\chi_t = \log \left(\frac{1 - \frac{\alpha}{u'(c_t)} \frac{(1 - \lambda_t)(q_t^b b_t)^{\rho-1}}{\mathcal{B}_t^\rho}}{1 - \frac{\alpha}{u'(c_t)} \frac{\lambda_t (q_t^h h_t)^{\rho-1}}{\mathcal{B}_t^\rho}} \right) \approx \frac{\alpha}{u'(c_t)} \left(\frac{(1 - \lambda_t)(q_t^b b_t)^{\rho-1} - \lambda_t (q_t^h h_t)^{\rho-1}}{\mathcal{B}_t^\rho} \right)$$

2. *Special case:* $\omega = 1$, $\lambda_t = 0$, and $c_t = y_t$: In this case, we recover one of the expressions that motivated the regressions in the original [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) paper:

$$\chi_t = \log \left(1 - \frac{\alpha}{q_t^b b_t / y_t} \right) \approx \frac{\alpha}{q_t^b b_t / y_t} \quad (5.1)$$

These special cases show that, for one-period bonds, the BIU model has the particularly stark prediction that government policies only impact the funding advantage by changing $q^b b$ rather than changing the elasticity parameters. For long-term bonds, this is only approximately true. As discussed in Section 3.3, relative to the BIU formulation, our model endogenizes

how the shifter and elasticity parameters in the BIU functional form relate to the government financial policy parameters and their monetary-fiscal rule.

5.3.2 Bond-in-Advance (BIA)

Environment: We remove the banking sector and instead follow [Svensson \(1985\)](#) and [Lucas Jr and Stokey \(1985\)](#) and introduce a “bond-in-advance constraint” that households must use money or bonds to trade for consumption goods in the morning market and then can only trade financial assets in the afternoon market. For tractability, we consider an endowment economy where a “firm” operates a Lucas tree that produces Y_t each period. The firm issues equity and bonds (N_t, H_t) to households in a frictionless competitive market. The government issues money and government bonds (M_t, B_t) and raises taxes. We restrict attention to one-period nominal bonds purchased in the afternoon of period t , usable as payment instruments in the morning of period $t+1$, and redeemed for one dollar in the afternoon of period $t+1$. The process for money and government bonds (M_t, B_t) is

Household Problem: At the start of the afternoon sub-period, suppose the family has unspent nominal wealth a_t . The household’s budget constraint in the afternoon sub-period at time t is:

$$m_t + P_t^d b_t + P_t^h h_t + P_t^n n_t \leq a_t + (1 - \tau_t) Y_t \quad (5.2)$$

where P_t^j is the nominal afternoon price of asset j , and (m_t, b_t, h_t, n_t) denote the household portfolio of money, government bonds, corporate bonds, and firm equity, respectively.

In the following morning sub-period, the households need to purchase consumption goods using a bond index $\mathcal{B}_{t+1} = \left((1 - \bar{\lambda}_{t+1})(b_t)^\rho + \bar{\lambda}_{t+1}(h_t)^\rho \right)^{1/\rho}$ where $\bar{\lambda}_{t+1}$ is potentially stochastic. Formally, we have:

$$\check{P}_{t+1} c_{t+1} \leq m_t + \alpha \mathcal{B}_{t+1} =: l_{t+1} \quad (5.3)$$

where \check{P}_{t+1} is morning price of consumption goods in terms of money and l_t is aggregate liquidity available in the morning market. So the wealth that households bring into the afternoon is:

$$a_{t+1} = m_t + b_t + h_t - \check{P}_{t+1} c_{t+1} \quad (5.4)$$

Then the household Bellman equation becomes:

$$V_t(a) = \max_{m_t, b_t, h_t, c_{t+1}} \left\{ \beta \mathbb{E}_t \left[u(c_{t+1}) + V_{t+1} \left(m_t + b_t + h_t - \check{P}_{t+1} c_{t+1} \right) \right] \right\} \quad (5.2), (5.3), (5.4)$$

and FOCs for consumption and bonds are:

$$\begin{aligned} [c_{t+1}] : \quad & 0 = u'(c_{t+1}) - \check{P}_{t+1} \partial_a V_{t+1}(a) - \mu_{t+1}^r \check{P}_{t+1} \\ [b_t] : \quad & 0 = -P_t^b \mu_t^c + \mathbb{E}_t \left[\beta \partial_a V_{t+1}(a_{t+1}) + \mu_{t+1}^r \alpha (1 - \lambda_{t+1}) (b_t)^{\rho-1} \mathcal{B}_{t+1}^{1-\rho} \right] \\ [h_t] : \quad & 0 = -P_t^h \mu_t^c + \mathbb{E}_t \left[\beta \partial_a V_{t+1}(a_{t+1}) + \mu_{t+1}^r \alpha \lambda_{t+1} (h_t)^{\rho-1} \mathcal{B}_{t+1}^{1-\rho} \right] \end{aligned}$$

where μ_t^c is the Lagrange multiplier on the afternoon budget constraint (5.2) and μ_t^r is the Lagrange multiplier on the liquidity-in-advance constraint (5.3). We have that $\mu_t^c = \partial_a V_t(a)$.

So we have:

$$\begin{aligned} \frac{P_t^b}{\check{P}_t} &= \mathbb{E}_t \left[\beta \left(\frac{u'(c_{t+1}) - \mu_{t+1}^r \check{P}_{t+1}}{u'(c_t) - \mu_t^r \check{P}_t} \right) \left(\frac{1}{\check{P}_{t+1}} \right) + \mu_{t+1}^r \left(\frac{\alpha (1 - \bar{\lambda}_{t+1}) (b_t)^{\rho-1} \mathcal{B}_{t+1}^{1-\rho}}{u'(c_t) - \mu_t^r \check{P}_t} \right) \right] \\ \frac{P_t^h}{\check{P}_t} &= \mathbb{E}_t \left[\beta \left(\frac{u'(c_{t+1}) - \mu_{t+1}^r \check{P}_{t+1}}{u'(c_t) - \mu_t^r \check{P}_t} \right) \left(\frac{1}{\check{P}_{t+1}} \right) + \mu_{t+1}^r \left(\frac{\alpha \bar{\lambda}_{t+1} (h_t)^{\rho-1} \mathcal{B}_{t+1}^{1-\rho}}{u'(c_t) - \mu_t^r \check{P}_t} \right) \right] \end{aligned}$$

Equilibrium price level: In equilibrium, we have $c_t = Y_t$, $b_t = B_t$, $h_t = H_t$, $m_t = M_t$. If the liquidity in advance constraint binds, then the price level is given by:

$$\check{P}_{t+1} = \frac{M_t + \alpha \mathcal{B}_{t+1}}{Y_t} =: \frac{L_t}{Y_t}$$

Government Funding Advantage: Define $q_t^j = P_t^j / \check{P}_t$. Impose market equilibrium. Then the funding advantage $\log(q_t^b) - \log(q_t^h)$ is (taking a first order Taylor approximation around $(u'(c_{t+1}) - \mu_{t+1}^r \check{P}_{t+1}) / \check{P}_{t+1}$):

$$\chi_t \approx \frac{\mathbb{E}_t \left[\mu_{t+1}^r \alpha \mathcal{B}_{t+1}^{1-\rho} \left((1 - \lambda_{t+1}) (B_t)^{\rho-1} - \lambda_{t+1} (H_t)^{\rho-1} \right) \right]}{\mathbb{E}_t \left[\beta (u'(Y_{t+1}) / \check{P}_{t+1} - \mu_{t+1}^r) \right]}$$

The key object in this expression is the multiplier μ_{t+1}^r , which can be approximated

$$\mu_{t+1}^r = \mathcal{M} \left(\frac{\check{P}_{t+1} Y_{t+1}}{M_t + \alpha \mathcal{B}_{t+1}} \right),$$

A reasonable approximation to the hard constraint should make $\mathcal{M}(\cdot)$ increasing and convex. Then a mean preserving increase in the volatility of \mathcal{B}_{t+1} raises $\mathbb{E}_t[\mu_{t+1}^r]$. That is, more volatile

bond liquidity raises the expected shadow value of relaxing the bond-in-advance constraint because agents have an additional precautionary motive for holding bonds. By contrast, if $\mathcal{M}(\cdot)$ is linear in \mathcal{B}_{t+1} , then a mean preserving increase in the volatility of \mathcal{B}_{t+1} would leave $\mathbb{E}_t[\mu_{t+1}^r]$ unchanged.

Once again, we can consider some special cases:

1. *Special case: $\lambda_t = \lambda$:* If the relative bond weight is constant, then we can take relative marginal bond benefit outside the expectation:

$$\chi_t \approx \frac{\alpha \mathcal{B}_t^{1-\rho} ((1-\lambda)(B_t)^{\rho-1} - \lambda(H_t)^{\rho-1}) \mathbb{E}_t[\mu_{t+1}^r]}{\mathbb{E}_t[u'(Y_{t+1})/\check{P}_{t+1}] - \mathbb{E}_t[\mu_{t+1}^r]}$$

So the responsiveness of funding advantage to inflation risk depends upon how the bond-in-advance multiplier relates to relative scarcity, which is entirely driven by the output process. In this sense, inflation risk does not show up in the in the funding advantage.

2. *Special case: $Cov(\mu_{t+1}^r, \lambda_{t+1}) > 0$:* If the relative liquidity demand for corporate bonds increases sufficiently much when the liquidity-in-advance constraint binds, then the model can overcome the result from the first special case and generate the result that greater inflation volatility can decrease the government funding advantage. Our model can be thought of as providing one explanation for when this can happen.

5.3.3 Returning to the Government Budget Constraint

Models that generate non-pecuniary demand for government debt that is unrelated (or even enhanced) by inflation risk have strong implications for the government budget constraint. To make this concrete, we examine analytically how inflation risk impacts impact the government budget constraint in the BIU model. Under BIU with $\lambda_t = 0$, we have:

$$\chi_t \approx \omega \log \left(\Omega \left(\frac{q_t^b b_t}{y_t}; \zeta_t \right) \right), \quad \frac{(q_t^b - q_t^h) b_t}{y_t} \approx \frac{q_t^b b_t}{y_t} \left(1 + \Omega \left(\frac{q_t^b b_t}{y_t}; \zeta_t \right) \right) \quad (5.5)$$

where for short-term debt the formulas are precise (see equation (5.1)). This implies that the convenience revenue maximizing debt-to-GDP ratio is independent of other government policies. We illustrate this with the red arrows in the top subplot in Figure 9, which show how an increase in the market value of government debt-to-GDP lowers the spread and moves the economy along the convenience revenue curve. In this sense, in these models the government faces a ‘‘Laffer curve’’ style revenue maximization challenge reminiscent of the monetary literature.

We can then consider how devaluation risk changes government funding advantage in a BIU model. Suppose the government introduces a monetary-fiscal policy that makes government debt a worse hedge (e.g. in our model this would be an increase in σ_π) and so devalues the total government debt portfolio (a decrease in q_t^b). Figure 9 shows the impact on the private-public borrowing spread and convenience revenue under the BIU specification (5.5) (the red arrows) and compares it a model like ours where higher inflation volatility ends up decreasing demand (the blue arrows). Evidently, under the BIU model, increasing return risk moves the economy up along the private-public borrowing spread curve and the convenience revenue curve. In this sense, return risk does not change the convenience revenue trade-off but rather provides another way of moving to the convenience revenue maximizing value of $q_t^b b_t / y_t$. By contrast, in our model, as we saw in Section 4, introducing return risk shifts the private-public borrowing spread curve down. This contracts the convenience revenue curve and so the government budget constraint.

We can interpret these difference in terms of decreases in the quantity (b_t) and quality (β_t) of government debt. In the BIU model, changes to quantity and quality both enter the private-public borrowing spread formula in the same way by decreasing $q_t^b b_t$ and increasing private-public borrowing spread. By contrast, in the data and our more general model, decreases in quality shift the private-public borrowing spread to Debt-to-GDP relationship, which leads to a decrease in the private-public borrowing spread.

6 Counterfactual: Financing Large Debt Increases

We conclude the paper by examining the macroeconomic implications of a government that seeks to finance large spending increases by imposing restrictions on financial sector portfolios. We first consider the four major debt expansions in the US history: the Civil War, World War I, World War II, and the Global Financial Crisis. We then compute counterfactual experiments if the government had financed any of these major debt expansions with alternative policies.

6.1 Historical Episodes of Large Debt-to-GDP Increases

The top four subplots of Figure 10 show the Debt-to-GDP ratio and the private-public borrowing spread during the Civil War, the Global Financial Crisis, World War I, and World War II, which are the periods in our historical sample with large increases in the market value of government debt-to-GDP (ranging from 25 to 60 percentage points).

Evidently, during both the Civil War and the Global Financial Crisis the US government

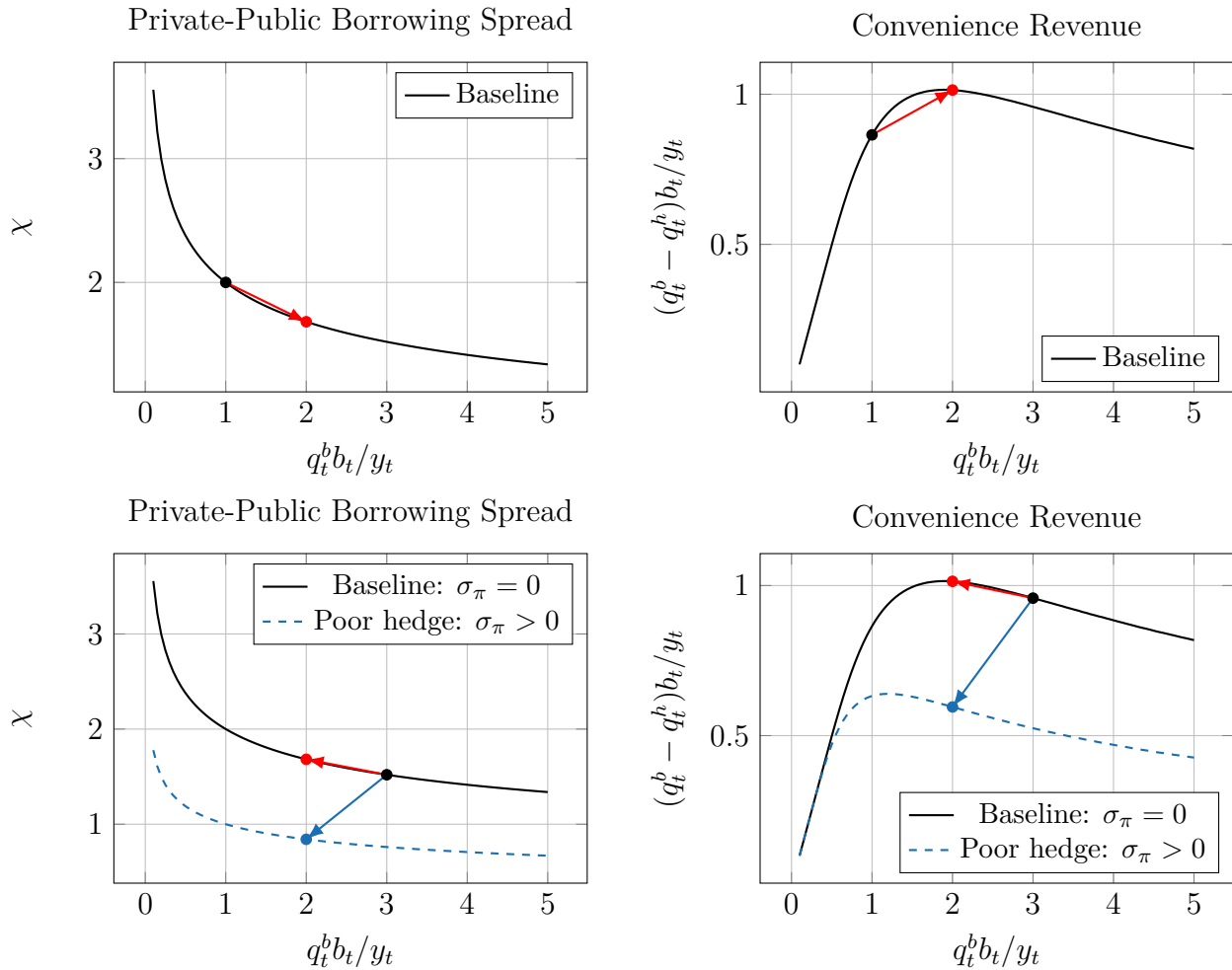


Figure 9: Top plot: shows the impact of a debt-to-GDP increase in a BIU model. Middle plot: show the impact of regulation that increases government debt demand. Bottom plot: shows the impact of an increase in risk on holding government debt.

was able to simultaneously increase debt-to-GDP and its funding advantage. These are both episodes when government debt issuance coincided with large changes to financial sector regulation that increased financial sector incentives to hold treasuries. For the Civil War episode, between 1862-6, Congress passed a collection of National Banking Acts, which established a system of nationally chartered banks that were allowed to issue bank notes up to 90% of the minimum of par and market value of qualifying US federal bonds¹⁵ and could only issue a narrow range of loans¹⁶. For the Global Financial Crisis, Congress enacted the Dodd-Frank Act in July 2010 and the Basel III rules were adopted in 2012. Both of these regulatory changes introduced a large collection of portfolio constraints on the banking sector that penalized bank leverage ratios and encouraged bank government debt holding. Interpreted through the lens of our model, neither period is consistent with the commonly used stable equilibrium relationship outlined in equation (5.5). Instead, both periods are consistent with regulation shifting the government debt demand curve up at the same time as supply increased.

The episode with a clear negative relationship between debt supply and the government funding spread is World War I, although identification is complicated by the gradual retiring of the national banking system after the introduction of the Fed in 1913. The story during World War II is more complicated. On average, the very large debt-to-GDP increase during the war does coincide with lower government funding advantage, although the decrease is small and the relationship is less clear after the war, particularly once the Treasury-Fed Accord is agreed and the Fed becomes independent.

The bottom four plots in Figure 10 show the rolling bond-stock beta calculated over a 3-year centered window during the four large debt-to-GDP increases in our sample.¹⁷ We can see a striking difference between outcomes during the Global Financial Crisis when government funding advantage went up and World War I when government funding advantage went down. Following the introduction of the Dodd-Frank Act in 2012, the bond-stock beta dropped sharply indicating that holding government debt became a good hedge against

¹⁵Technically, national banks could issue bank notes for circulation according to the following rules. Banks had to deposit certain classes of US Treasury bonds as collateral for note issuance. Permissible bonds were US federal registered bonds bearing coupons of 5% or more. Deposited bonds had to be at least one-third of the bank's capital (not less than \$30,000). Banks could issue bank notes up to an amount of 90% of the maximum of the market value of the bonds and the par value of the bonds. The 90% value was changed to 100% in 1900.

¹⁶National banks could only operate one branch. They were restricted from making mortgages unless they were operating in rural areas, where they could make a limited range of loans collateralized by agricultural land.

¹⁷The bond-stock "beta" is the coefficient when excess holding returns are regressed against excess stock returns (which we refer to as the bond-stock beta). Formally, for each maturity j , we regress the monthly excess holding return $rx_{t+1}^{j-1} = \log(q_{t+1}^{(j-1)}) - \log(q_t^{(j)}) - r_t$ against the monthly percentage change in the GFD historical total return index on US equities.

aggregate risk. By contrast, during World War I, the bond-stock beta went from zero to weakly positive. For World War II, the bond-stock beta was never significantly different to zero during the period of yield curve control (1942-1951) when the Fed and treasury worked together to stabilize the yield curve. For the Civil War, the short maturity Greenback yield curve estimates are too noisy during the greenback period (1862-1879) to get a clear estimate. However, we can see that the beta for gold-denominated government debt is negative during the war. While it becomes temporarily positive after the war, once convertibility is restored, the bond-stock beta is essentially zero indicating that throughout the 19th century when government funding advantage was high government debt was a “safe-asset” compared to equities.

6.2 Refinancing WWI

We use our model to revisit the financing of World War I. We calibrate the real business cycle parameters $(\beta, \gamma, \alpha, \delta, \omega, \phi, \theta, g, \rho_z)$ externally to match standard parameters in the RBC model. We calibrate the parameters $(\lambda, \nu, \psi, \zeta_c, \rho\kappa^h, \bar{\mathbf{b}}, \bar{\tau})$ internally to target average spreads and bank leverage (see Appendix ?? additional details) for the period.

Figure 11 show a counterfactual path if World War I were financed with similar level of government debt regulatory privilege to the Global Financial Crisis to make government debt a safe asset during the period, like it was during the Global Financial crisis. We feed in a sequence of government spending shocks to match spending-to-GDP during World War I. We then show Debt-to-GDP, the funding advantage, taxes-to-GDP, Investment/Capital, and Deposits-to-GDP under a counterfactual economy with higher regulatory privilege. We can see the basic trade-offs for the economy. Under our baseline calibration, government funding advantage decreases dramatically from around 1% to 0.5% as the debt-to-GDP ratio expands from close to zero to around 0.3 (as it does in the data in Figure 10). By contrast, when we tighten regulatory privilege at the outset of the War, then the funding advantage increases (as it does in the data for the Civil War and Financial Crisis). This decreases the average tax burden in the economy but also decreases both investment/capital and deposits/capital. This comes from the contract of the financial sector in the face of the tighter regulatory privilege.

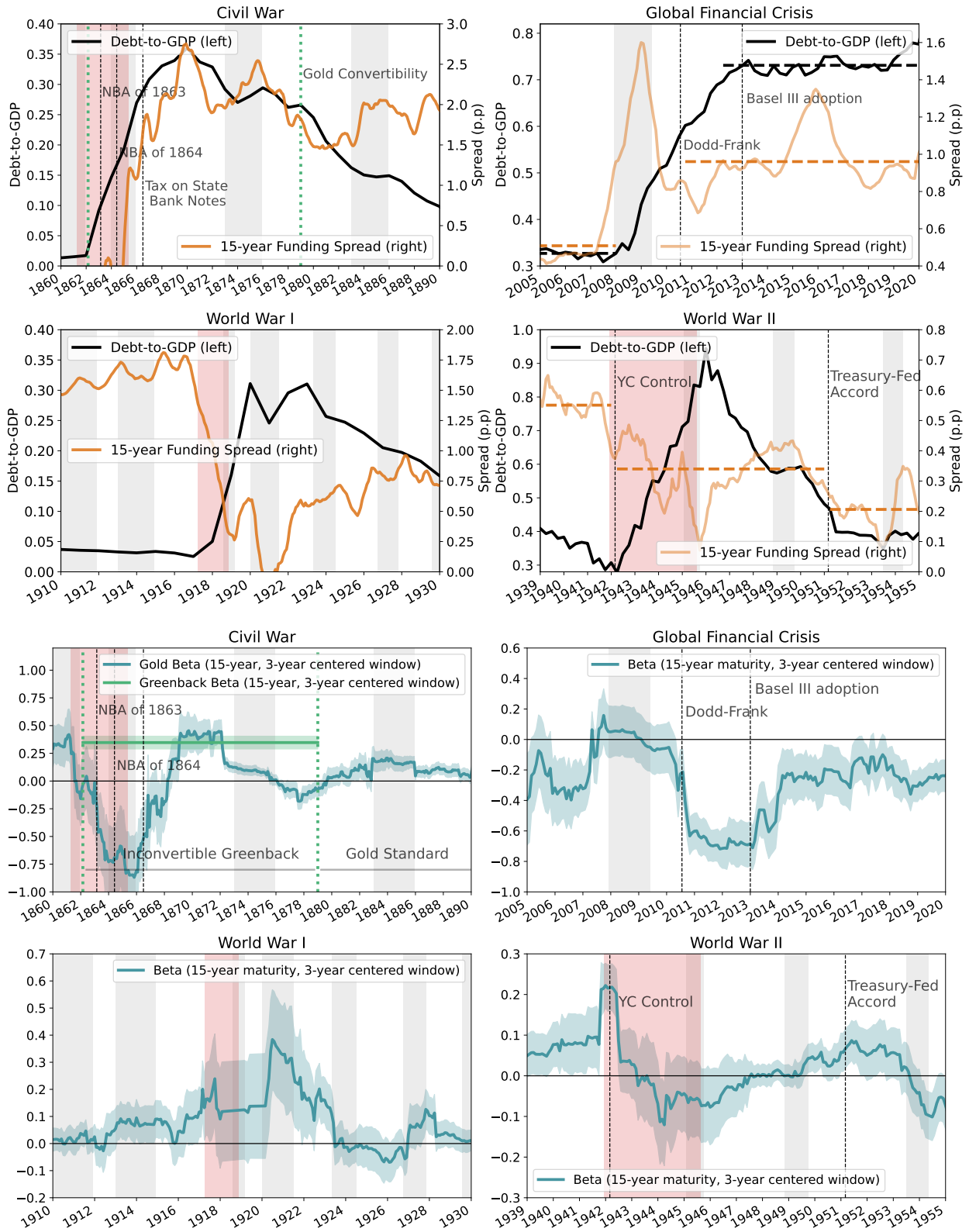


Figure 10: Four Large Debt Expansions: The top four plots show the market value of debt-to-GDP and 15 year private-public borrowing spread during the Civil War, Global Financial Crisis, World War I, and World War II. The bottom four plots show the rolling 36 month bond-stock beta during the same period. The shaded areas show 95% confidence intervals.

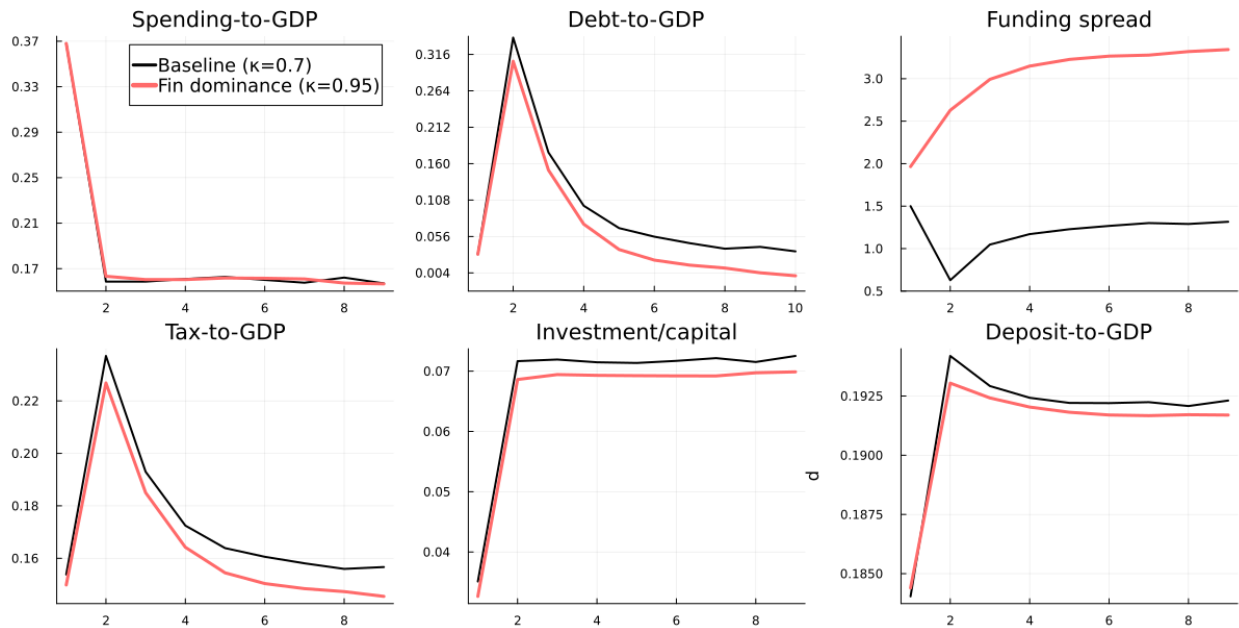


Figure 11: Counterfactual Expansion. The baseline sets $\kappa = \mathcal{K}^b - \mathcal{K}^h = 0.7$ and the counterfactual increases this to $\kappa = \mathcal{K}^b - \mathcal{K}^h = 0.95$.

7 Conclusion

In this paper, we show how the government can generate a funding advantage through restrictions on the financial sector that make government debt a “safe-asset” for the economy. Endogenizing government funding advantage in this way allows us to characterize how it is related to financial and fiscal policy. We show that inflation erodes its funding advantage because it changes the role that government debt plays in the financial sector and so changes the debt demand function. This is very different to bond-in-utility and bond-in-advance models where bond demand is exogenous and the funding advantage increases when the government starts to default (because the real value of government debt becomes scarce). Our results suggest that macroeconomists should be very cautious about modeling government funding advantage using exogenous, immutable demand functions that fit empirical “safe-asset” curves. Like for the Phillips Curve, these relationships break down once the government attempts to exploit them.

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A Budget Constraint Arithmetic

Let $b_t^{(n)}$ denote the amount of n -period zero-coupon bonds issued at time t . Let the total (face value of) outstanding debt in period t be $b_t := \sum_{n=1}^{\infty} b_t^{(n)}$ and the “portfolio shares” are $b_t^{(n)}/b_t$. In any period t , the government enters with a stock of promised payments $\{b_{t-1}^{(n)}\}_{n \geq 1}$ and issues new (zero-coupon) bonds $\{b_t^{(n)}\}_{n \geq 1}$, where $b_t^{(n)}$ is the amount of bond of maturity n issued in period t .¹⁸ The government budget constraint can be written as

$$\begin{aligned} \sum_{n=1}^{\infty} q_t^{(n)} b_t^{(n)} &= \sum_{n=1}^{\infty} q_t^{(n-1)} b_{t-1}^{(n)} + g_t - \tau_t \\ &= b_{t-1}^{(1)} + \sum_{n=1}^{\infty} q_t^{(n)} b_{t-1}^{(n+1)} + g_t - \tau_t \end{aligned}$$

where g_t is government spending and τ_t is tax revenues. Let Δ_t be the net amount of dollars that the government raises in period t from “refinancing” its debt:

$$\Delta_t := \sum_{n=1}^{\infty} q_t^{(n)} \left[b_t^{(n)} - b_{t-1}^{(n+1)} \right]$$

so that the budget constraint becomes

$$g_t + b_{t-1}^{(1)} = \tau_t + \Delta_t.$$

The role of the yield curve for government financing can be summarized by the Δ_t term. The government’s total deficit (including interest payments) is $g_t + b_{t-1}^{(1)} - \tau_t$, while its primary deficit is $def_t := g_t - \tau_t$. As a result, the difference between Δ_t and $\tilde{\Delta}_t$ can be viewed as the contribution of the borrowing cost spread to period t surplus.

Iterating the budget constraint forward gives the lifetime budget constraint:

$$\begin{aligned} \sum_{j=1}^{\infty} q_t^{(j-1)} b_{t-1}^{(j)} &= \mathbb{E}_t \left[\underbrace{\sum_{s=0}^{\infty} \xi_{t,t+s} (\tau_{t+s} - g_{t+s})}_{(i)} \right] + \underbrace{\sum_{j=1}^{\infty} (q_t^{(j)} - \tilde{q}_t^{(j)}) b_{t-1}^{(j+1)}}_{(ii)} \\ &\quad + \underbrace{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \xi_{t,t+s} \left\{ \sum_{j=1}^{\infty} (q_{t+s}^{(j)} - \tilde{q}_{t+s}^{(j)}) (b_{t+s}^{(j)} - b_{t-1+s}^{(j+1)}) \right\} \right]}_{(iii)}. \end{aligned}$$

where $q_t^{(j)}$ and $\tilde{q}_t^{(j)}$ denote the zero-coupon prices of the government and public sector’s

¹⁸For instance, one period bond issued in period t and maturing in $t + 1$ is $b_t^{(1)}$. Similarly, $b_{t-1}^{(n)}$ is the amount of n -period bond issued in period $t - 1$ coming due in period $t - 1 + n$.

j -period zero-coupon bonds, respectively.

A.1 Models with representative long-term debt

The *admissible set of portfolios* is restricted to follow an exponential rule, i.e.

$$b_t^{(n)} = b_t \omega (1 - \omega)^{n-1}$$

In other words, the assumption is that we can summarize/proxy the portfolio $\{b_t^{(n)}\}_{n=1}^{\infty}$ with a pair of scalars (b_t, ω) . The variable Δ_t can be written as:

$$\Delta(b_t, \omega_t; b_{t-1}, \omega_{t-1}) := \sum_{n=1}^{\infty} q_t^{(n)} \left[\underbrace{(1 - \omega_t)^{n-1} \omega_t b_t}_{=: b_t^{(n)}} - \underbrace{(1 - \omega_{t-1})^n \omega_{t-1} b_{t-1}}_{=: b_{t-1}^{(n+1)}} \right]$$

In the above expression, if the government enters the period with a portfolio (b_{t-1}, ω_{t-1}) and wants to exit it with a portfolio (b_t, ω_t) , then for each maturity $n \geq 1$ it must issue/buy back $b_t^{(n)} - b_{t-1}^{(n+1)}$ many bonds at price $q_t^{(n)}$.

Suppose for now that ω_t is fixed over time, i.e. $\omega_t = \omega$. We can then write

$$\Delta_t := \underbrace{\left(\sum_{n=1}^{\infty} q_t^{(n)} (1 - \omega)^{n-1} \omega \right)}_{=: q_t^b(\omega)} \left(b_t - (1 - \omega) b_{t-1} \right) = q_t^b \left(b_t - (1 - \omega) b_{t-1} \right)$$

where q_t^b denotes the market price of a “unit” of government debt portfolio (at face value) with average maturity $1/\omega$. From the definition of Δ_t we can write the law of motion of (the face value of) debt as

$$b_t = (1 - \omega) b_{t-1} + \frac{\Delta_t}{q_t^b}$$

so if $\omega < 1$ and q_t^b depends on (b_t, b_{t-1}) , q_t^b will behave as an (endogenous) debt adjustment cost. In this case, the government budget constraint is

$$g_t + \omega b_{t-1} = \tau_t + q_t^b \left(b_t - (1 - \omega) b_{t-1} \right).$$

The intertemporal budget constraint can be written as

$$\begin{aligned}
b_{t-1}^{(1)} + \underbrace{\left\{ \sum_{j=1}^{\infty} q_t^{(j)} (1-\omega)^{j-1} \omega \right\}}_{=: q_t^b(\omega)} (1-\omega) b_{t-1} &= \mathbb{E}_t \left[\underbrace{\sum_{s=0}^{\infty} \xi_{t,t+s} (\tau_{t+s} - g_{t+s})}_{(i)} \right] + \\
&+ \underbrace{\left(q_t^b - \tilde{q}_t^b \right) (1-\omega) b_{t-1}}_{(ii)} + \underbrace{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \xi_{t,t+s} \left(b_{t+s} - (1-\omega) b_{t+s-1} \right) \left(q_{t+s}^b - \tilde{q}_{t+s}^b \right) \right]}_{(iii)}.
\end{aligned}$$

or using the definition of the “weighted price” q_t^b :

$$\begin{aligned}
\left(\omega + q_t^b (1-\omega) \right) b_{t-1} &= \mathbb{E}_t \left[\underbrace{\sum_{s=0}^{\infty} \xi_{t,t+s} (\tau_{t+s} - g_{t+s})}_{(i)} \right] + \underbrace{\left(q_t^b - \tilde{q}_t^b \right) (1-\omega) b_{t-1}}_{(ii)} \\
&+ \underbrace{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \xi_{t,t+s} \left(q_{t+s}^b - \tilde{q}_{t+s}^b \right) \left(b_{t+s} - (1-\omega) b_{t+s-1} \right) \right]}_{(iii)}.
\end{aligned}$$

The “yield” embedded in the representative price of the government debt portfolio is

$$q_t^b = \sum_{j=1}^{\infty} \left(\frac{1}{1+y_t} \right)^j (1-\omega)^{j-1} \omega = \frac{\omega}{1+y_t} \left(\frac{1}{1-\frac{1-\omega}{1+y_t}} \right) = \frac{1}{1+y_t/\omega} \approx \exp(-y_t/\omega)$$

which makes sense given that the government portfolio’s average maturity is $1/\omega$.

B Additional Empirical Evidence

To help see these descriptive patterns more clearly, Figure 12 plots times series for our entire sample from 1860-2024. The top subplot show the 15-year spread between corporate and treasury yields (the orange line) and the market value of debt-to-GDP (the black line). The bottom subplot shows the rolling bond-stock beta (calculated as the coefficient when treasury returns are regressed against stock returns) over a 3-year centered window. We treat the bond-stock beta as a proxy for how effectively treasuries can be used to hedge risk. The spread is systematically higher during the National Banking Era (approximately 1862-1920), lower during the rest of the twentieth century, and higher again over the last 15 years. The co-movement between debt-to-GDP and the spread varies greatly throughout the sample, while there is a noisy, but consistently negative relationship between the bond-stock beta

and the spread.

To make these observations about co-movements more formal, in Table 2, we regress the spread against Debt-to-GDP, the bond-stock beta, stock market volatility, and controls for regulatory eras. In column (1) we show that, unconditionally, dummies for the National Banking Era, and the Post GFC period explain the majority of the variation in the long sample (an adjusted R^2 of 0.70). Introducing the log-debt-to-GDP ratio and stock market volatility in column (2) does help explain additional variation (the adjusted R^2 increases to 0.77) but there is no significantly negative relationship to log-debt-to-GDP. By contrast, introducing the bond-stock beta and stock market volatility in column (3) leads to greater forecastability (the adjusted R^2 increases to 0.86) and the relationship to the bond-stock beta is significant at the 5% threshold. In the final column we include all variables. This does recover a negative relationship between spreads and log-debt-to-GDP during main time period (1917-2010). However, the relationship is actually positive during the National Banking Era and insignificant after the GFC. Furthermore, the introducing log-debt-to-GDP only increases the adjusted R^2 from 0.855 to 0.864, which is a negligible improvement. This reinforces what we saw in the historical episodes—the historical equilibrium relationship between quantities and spreads is not mechanically negative and quantities changes are not sufficient for explaining the historical movements. None of these regressions identify demand functions and so are not inconsistent with the large applied microeconomic literature that use a collection of different instruments to identify that demand for Treasuries is downward sloping in market value of Treasury holdings. Indeed, all the modeling in our paper will generate demand functions that match the microeconomic literature. Instead, our results emphasize that the macro-finance practice of targeting stable equilibrium relationships between quantities and spreads is problematic.

In this section of the appendix, we include additional empirical results. Table 3 shows the regression for the full sample with different controls. Table 4 shows moments for different subperiods.

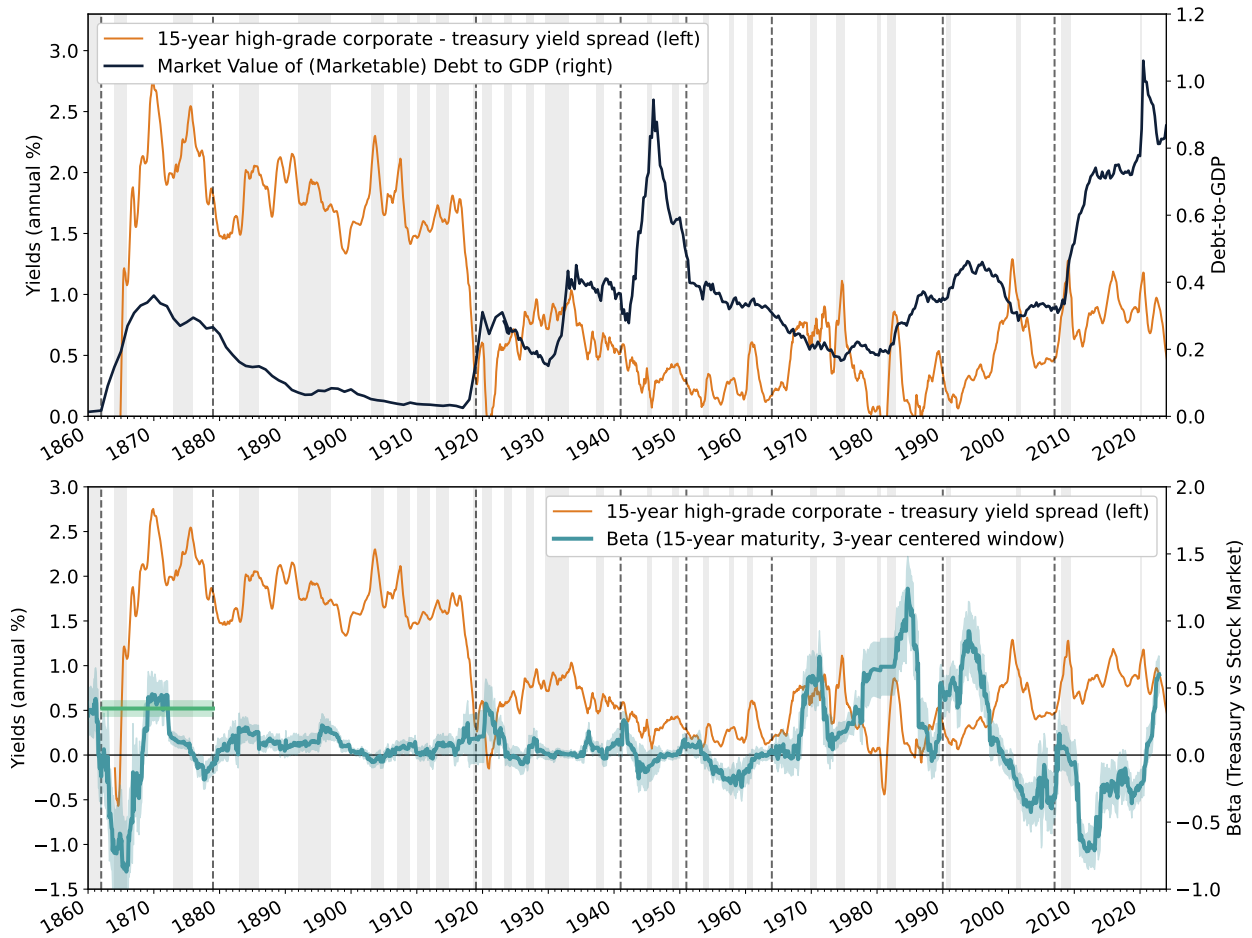


Figure 12: Full Sample: 1860-2024

Table 1: Regression Results: Funding Advantage

	<i>Dependent variable: Funding Advantage (20-Year)</i>			
	(1)	(2)	(3)	(4)
log(Debt/GDP)[All]		-0.143 (0.105)		-0.211*** (0.073)
Beta (36M)			-0.178** (0.082)	-0.238*** (0.082)
Volatility		1.902*** (0.493)	1.906*** (0.340)	1.725*** (0.336)
Slope		-0.012 (0.037)	-0.028 (0.024)	-0.003 (0.025)
Pre-1920 Dummy	1.271*** (0.065)	1.848*** (0.254)	1.127*** (0.131)	1.720*** (0.398)
Post-2010 Dummy	0.448*** (0.115)	1.138*** (0.373)	0.791*** (0.302)	1.571*** (0.514)
log(Debt/GDP) × Pre-1920 Dummy		0.225* (0.120)		0.308** (0.122)
log(Debt/GDP) × Post-2010 Dummy		0.350 (0.938)		1.667 (1.104)
Volatility × Pre-1920 Dummy		-1.722*** (0.634)	-0.473 (0.614)	-0.430 (0.615)
Volatility × Post-2010 Dummy		-2.587*** (0.920)	-1.914 (1.421)	-3.127* (1.737)
Slope × Pre-1920 Dummy		0.109** (0.043)	0.068 (0.047)	0.012 (0.056)
Slope × Post-2010 Dummy		0.000 (0.125)	0.003 (0.093)	0.062 (0.111)
Beta × Pre-1920 Dummy			0.732 (0.713)	0.222 (0.904)
Beta × Post-2010 Dummy			0.176 (0.296)	0.170 (0.292)
Constant	0.473*** (0.040)	0.008 (0.147)	0.218*** (0.059)	-0.014 (0.098)
Significance:	* $p < 0.1$	** $p < 0.05$	*** $p < 0.01$	
Period:	1860-2025	1860-2025	1880-2025	1880-2025
Observations	163	163	138	138
Adjusted R^2	0.704	0.767	0.855	0.864

Table 2: Regression Analysis

Dependent variable: 10y AAA Corporate Bond Yield - 10y Treasury Yield

	(1)	(2)	(3)
const	1.064*** (0.200)	1.957*** (0.318)	1.946*** (0.315)
debt-to-GDP	-0.331*** (0.037)	0.052 (0.061)	0.046 (0.061)
sigma(R)	-0.379*** (0.085)	-0.012 (0.136)	-0.022 (0.135)
slope	0.011 (0.030)	-0.034 (0.027)	-0.024 (0.028)
volatility	1.451*** (0.334)	-0.237 (0.438)	-0.221 (0.435)
1920-2024		-1.047*** (0.379)	
1920-2024*debt-to-GDP		-0.261*** (0.090)	
1920-2024*sigma(R)		-0.290* (0.169)	
1920-2024*vol		2.101*** (0.639)	
1920-2007			-1.006*** (0.381)
1920-2007*debt-to-GDP			-0.335*** (0.108)
1920-2007*sigma(R)			-0.344** (0.173)
1920-2007*vol			2.172*** (0.663)
2009-2024			-0.488 (1.647)
2009-2024*debt-to-GDP			0.516 (1.144)
2009-2024*sigma(R)			-0.264 (0.789)
2009-2024*vol			-0.884 (1.755)
Observations	154	154	154
R^2	0.598	0.725	0.736
Adjusted R^2	0.587	0.709	0.714
Residual Std. Error	0.398 (df=149)	0.334 (df=145)	0.332 (df=141)
F Statistic	55.402*** (df=4; 149)	47.668*** (df=8; 145)	32.758*** (df=12; 141)

	1870-1919	1920-1951	1952-1993	1994-2007	2008-2025
Private-public borrowing cost spread: (χ_t)					
mean	1.734	1.066	0.431	0.663	0.717
vol	0.277	0.302	0.269	0.228	0.247
corr(\cdot , Δy)	-0.142	-0.121	-0.128	-0.557	-0.529
Debt-to-GDP: $(q_t^b b_t / y_t)$					
mean	0.126	0.512	0.477	0.655	1.07
vol	0.065	0.174	0.137	0.047	0.084
corr(\cdot , Δy)	0.044	-0.24	-0.049	-0.374	-0.079
Real return: $((\omega + (1 - \omega)q_{t+1}^b) / q_t^b - 1)$					
mean	2.338	2.262	1.818	3.004	1.057
vol	4.71	7.68	8.61	9.57	10.09
corr(\cdot , Δy)	-0.206	-0.295	-0.14	-0.414	0.131
Surplus-to-GDP: $(g_t - \tau_t) / y_t$					
mean	0.071	-3.794	-1.956	-1.168	-6.153
vol	2.611	7.298	1.067	1.763	3.554
corr(\cdot , Δy)	-0.083	-0.221	-0.261	0.137	0.356

Table 4: Summary statistics for different policy eras.

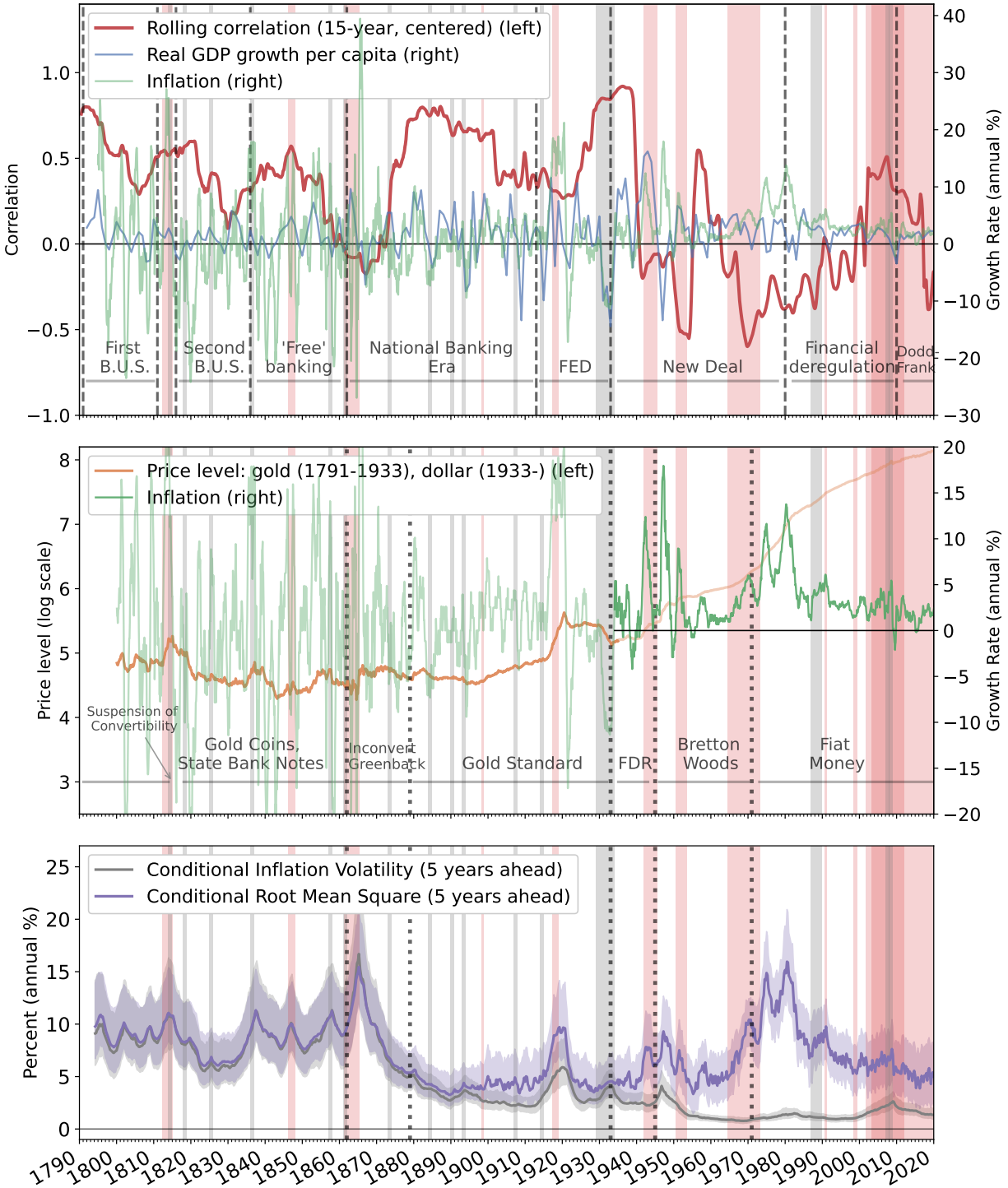


Figure 13: Top plot: The pale blue line depicts annual real GDP growth per capita, the pale green line depicts our annual inflation series, both measured on the right axis. The red thick line shows the 15-year (centered) rolling correlation between the blue and green series, measured on the left axis. Middle plot: The orange line depicts our combined log price index (left axis). It measures the gold price of goods before 1933 and the price of goods in dollar after 1933. The green line shows the annual growth rates (inflation) of the orange line (right axis). Bottom plot: The solid grey line depicts the posterior median estimate for the 5-year-ahead smoothed, annualized conditional inflation volatility. The solid purple line depicts the posterior median estimate for the 5-year-ahead smoothed conditional root mean square statistic. The light gray intervals depict banking crises from Reinhart and Rogoff (2009).

C Recursive Characterization of Equilibrium

In this Appendix, we set up the equilibrium recursively. The morning aggregate state vector is $\check{\mathbf{s}} := (\varepsilon, K, B, M, D, H)$ and the afternoon state vector is $\mathbf{s} = (\varepsilon, K, B) \subset \check{\mathbf{s}}$. We guess and verify that afternoon prices are functions of the form $(q^d(\mathbf{s}), q^e(\mathbf{s}), q^k(\mathbf{s}), q^b(\mathbf{s}), q^h(\mathbf{s}), q^n(\mathbf{s}))$ and morning prices are functions of the form $(\check{q}^b(\check{\mathbf{s}}), \check{q}^h(\check{\mathbf{s}}), \check{q}^e(\check{\mathbf{s}}))$.

C.1 Bank Problem

In the morning, each bank enters with a portfolio (m, b, h, d) of storage asset, government bonds, corporate bonds, and deposits chosen in the preceding afternoon sub-period. They then face the budget constraint:

$$\check{q}^b(\check{\mathbf{s}})\check{b}(\check{\mathbf{s}}) + \check{q}^h(\check{\mathbf{s}})\check{h}(\check{\mathbf{s}}) - \check{d}(\check{\mathbf{s}}) - \check{q}^e(\check{\mathbf{s}})\check{e}(\check{\mathbf{s}}) \leq \phi m + \check{q}^b(\check{\mathbf{s}})b + \check{q}^h(\check{\mathbf{s}})h - d, \quad \forall \check{\mathbf{s}} \quad (\text{C.1})$$

along with the net settlement restriction:

$$\lambda(\check{\mathbf{s}})d \leq \phi m + \check{q}^b(\check{\mathbf{s}})(b - \check{b}(\check{\mathbf{s}})) + \check{q}^h(\check{\mathbf{s}})(h - \check{h}(\check{\mathbf{s}})) + \check{q}^e(\check{\mathbf{s}})\check{e}(\check{\mathbf{s}}), \quad \forall \check{\mathbf{s}} \quad (\text{C.2})$$

where $(\check{b}, \check{h}, \check{d})(\cdot)$ denote the remaining portfolio of government bond, corporate bond, and deposits after servicing depositors. Constraint (C.2) states that, to finance morning deposit withdrawals, $\lambda(\cdot)d$, the bank may sell its bonds amounting to $(b - \check{b}(\cdot), h - \check{h}(\cdot))$, or raise additional funds equal to $\check{q}^e(\cdot)\check{e}(\cdot)$ from households with idle deposits, where each unit of \check{e} denotes a unit of “non-preferred equity” that returns one final good in the afternoon. The regulatory constraint can be rewritten as

$$\check{d}(\check{\mathbf{s}}) + \kappa^e \check{q}^e(\check{\mathbf{s}})\check{e}(\check{\mathbf{s}}) \leq \mathcal{K}^b \check{q}^b(\check{\mathbf{s}})\check{b}(\check{\mathbf{s}}) + \mathcal{K}^h \check{q}^h(\check{\mathbf{s}})\check{h}(\check{\mathbf{s}}), \quad \forall \check{\mathbf{s}} \quad (\text{C.3})$$

In the afternoon, banks pay their equity and deposit holders subject to the budget constraint:

$$x^e(\mathbf{s})e + \check{d}(\mathbf{s}) \leq x^b(\mathbf{s})\check{b}(\mathbf{s}) + x^h(\mathbf{s})\check{h}(\mathbf{s}) - \check{e}(\mathbf{s}) - \Psi(\check{q}^e(\mathbf{s})\check{e}(\mathbf{s})), \quad \forall \mathbf{s} \quad (\text{C.4})$$

where $\Psi(\cdot)$ is the cost of raising morning equity, and $x^b(\cdot)$ and $x^h(\cdot)$ are the afternoon payoffs from government bonds and corporate bonds, respectively:

$$\begin{aligned} x^b(\mathbf{s}) &:= \left(\omega + (1 - \omega)q^b(\mathbf{s}) \right), & \forall \mathbf{s} \\ x^h(\mathbf{s}) &:= \left(\omega + (1 - \omega)q^h(\mathbf{s}) \right), & \forall \mathbf{s}. \end{aligned}$$

Taking the predetermined individual portfolios (m, b, h, d, e) , morning asset prices $(\check{q}^b, \check{q}^h, \check{q}^e)(\cdot)$, and the afternoon payoffs $(x^b, x^h)(\cdot)$ as given, banks solve the following problem each morning for all \mathbf{s} :

$$\begin{aligned}
W(m, b, h, d, e; \mathbf{s}) &:= \max_{(\check{e}, \check{d}, \check{b}, \check{h}, x^e)(\cdot)} \left\{ x^e(\mathbf{s})e \right\} \\
s.t. \quad & \text{(C.1), (C.2), (C.3), (C.4),} \\
& 0 \leq \check{d}(\check{\mathbf{s}}), \check{e}(\check{\mathbf{s}}), \check{b}(\check{\mathbf{s}}), \check{h}(\check{\mathbf{s}}), \quad \forall \mathbf{s}
\end{aligned}$$

where $W(\cdot)$ is the value of the bank in the morning. The necessary first order conditions are:

$$[\check{d}(\cdot)] : \quad \check{\mu}(\check{\mathbf{s}}) - \check{\mu}^r(\check{\mathbf{s}}) = 1 \quad (\text{C.5})$$

$$[\check{e}(\cdot)] : \quad \check{q}^e(\check{\mathbf{s}}) \left(\check{\mu}(\check{\mathbf{s}}) + \check{\mu}^e(\check{\mathbf{s}}) \right) \leq 1 + \partial \Psi \left(\check{q}^e(\check{\mathbf{s}}) \check{e}(\check{\mathbf{s}}) \right) \check{q}^e(\check{\mathbf{s}}) + \kappa^e \check{\mu}^r(\check{\mathbf{s}}) \check{q}^e(\check{\mathbf{s}}) \quad (\text{C.6})$$

$$[\check{b}(\cdot)] : \quad \check{q}^b(\check{\mathbf{s}}) \left(\check{\mu}(\check{\mathbf{s}}) + \check{\mu}^e(\check{\mathbf{s}}) \right) = x^b(\mathbf{s}) + (1 - \varrho \kappa^b) \check{\mu}^r(\check{\mathbf{s}}) \check{q}^b(\check{\mathbf{s}}) \quad (\text{C.7})$$

$$[\check{h}(\cdot)] : \quad \check{q}^h(\check{\mathbf{s}}) \left(\check{\mu}(\check{\mathbf{s}}) + \check{\mu}^e(\check{\mathbf{s}}) \right) = x^h(\mathbf{s}) + (1 - \varrho \kappa^h) \check{\mu}^r(\check{\mathbf{s}}) \check{q}^h(\check{\mathbf{s}}) \quad (\text{C.8})$$

where $\check{\mu}(\cdot) \geq 0$, $\check{\mu}^e(\cdot) \geq 0$ and $\check{\mu}^r(\cdot) \geq 0$ are the Lagrange multipliers on the morning budget constraint (C.1), the settlement constraint (C.2) and the regulatory constraint (C.3) respectively and we have dropped the short selling constraints since they will not bind. Equation (C.6) equates the marginal cost of issuing non-preferred stocks to the marginal benefit of relaxing the morning budget through additional resources from the households. Equations (C.7) and (C.8) equate the marginal cost of purchasing each asset in the secondary asset markets with its corresponding pecuniary payoff and non-pecuniary benefit arising from a relaxed regulatory constraint.

Our morning market is sufficiently simple that equilibrium can be characterized in closed form, which we do in Proposition 4 below.

Proposition 4. *In a competitive interbank market equilibrium with $\kappa^e < 1$ and $\mathcal{K}^j > 0$ for $j \in \{b, h\}$, functions $(\check{q}^b, \check{q}^h, \check{e}, \check{\nu})(\cdot)$ satisfy the following for all λ :*

$$\begin{aligned}
\check{e}(\mathbf{s}) &= \max\{\lambda(\mathbf{s})d_{t-1} - \phi m_{t-1}, 0\}, \\
\check{q}^j(\mathbf{s}) &= \frac{x^j(\mathbf{s})}{(1 + \psi \check{e}(\mathbf{s}))(1 - \mathcal{K}^j \check{\nu}(\mathbf{s}))}, \quad j \in \{b, h\}, \\
\check{\nu}(\mathbf{s}) &= \begin{cases} (\mathfrak{B}(\mathbf{s})/2) - \sqrt{(\mathfrak{B}(\mathbf{s})/2)^2 - \mathfrak{C}(\mathbf{s})}, & \text{iff } \check{e}(\mathbf{s}) > 0, \text{ and } \mathfrak{C}(\mathbf{s}) > 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

where: $\mathfrak{B}(\mathbf{s}) := \frac{1}{\kappa^b} (1 - \Gamma^b(\mathbf{s})) + \frac{1}{\kappa^h} (1 - \Gamma^h(\mathbf{s}))$ and $\mathfrak{C}(\mathbf{s}) := \frac{1}{\kappa^b \kappa^h} (1 - \Gamma^b(\mathbf{s}) - \Gamma^h(\mathbf{s}))$ and

$$\Gamma^b(\mathbf{s}) := \frac{(1 - \varrho \kappa^b) x^b(\mathbf{s}) b_{t-1}}{(1 + \psi \check{e}(\mathbf{s})) (\check{d}_{t-1} + \kappa^e \check{e}(\mathbf{s}))} \quad \Gamma^h(\mathbf{s}) := \frac{(1 - \varrho \kappa^h) x^h(\mathbf{s}) h_{t-1}}{(1 + \psi \check{e}(\mathbf{s})) (\check{d}_{t-1} + \kappa^e \check{e}(\mathbf{s}))}$$

Proof. Plugging market clearing conditions and $\check{q}^e(\mathbf{s}) = 0$ into budget constraint (C.1):

$$\check{e}(\mathbf{s}) = \max\{\lambda(\mathbf{s})d - \phi m, 0\}$$

with $\check{e} = 0$ iff the settlement constraint (C.2) does not bind. We prove the rest of the proposition in four steps, establishing the conditions under which the regulatory constraint binds and does not bind and deriving the equilibrium multiplier $\check{\nu}(\mathbf{s})$.

Claim 1: If $\check{e}(\check{\mathbf{s}}) = 0$, the regulatory constraint cannot bind.

From the first-order condition for equity issuance (C.6) with $\check{q}^e(\check{\mathbf{s}}) = 1$:

$$\check{\mu}(\check{\mathbf{s}}) + \check{\mu}^e(\check{\mathbf{s}}) - \kappa^e \check{\mu}^r(\check{\mathbf{s}}) \leq 1 + \psi \check{e}(\check{\mathbf{s}}), \quad (= \text{ if } \check{e}(\check{\mathbf{s}}) > 0).$$

If $\check{e}(\check{\mathbf{s}}) = 0$, this becomes:

$$\check{\mu}(\check{\mathbf{s}}) + \check{\mu}^e(\check{\mathbf{s}}) - \kappa^e \check{\mu}^r(\check{\mathbf{s}}) \leq 1. \quad (\text{C.9})$$

From the first-order condition for deposits (C.5):

$$\check{\mu}(\check{\mathbf{s}}) - \check{\mu}^r(\check{\mathbf{s}}) = 1. \quad (\text{C.10})$$

Combining (C.9) and (C.10):

$$\begin{aligned} 1 + \check{\mu}^r(\check{\mathbf{s}}) + \check{\mu}^e(\check{\mathbf{s}}) - \kappa^e \check{\mu}^r(\check{\mathbf{s}}) &\leq 1 \\ \check{\mu}^e(\check{\mathbf{s}}) + (1 - \kappa^e) \check{\mu}^r(\check{\mathbf{s}}) &\leq 0. \end{aligned}$$

Since $\check{\mu}^e(\check{\mathbf{s}}) \geq 0$ and $\check{\mu}^r(\check{\mathbf{s}}) \geq 0$ by the complementary slackness conditions, and $(1 - \kappa^e) > 0$, this inequality can only hold if both multipliers are zero: $\check{\mu}^e(\check{\mathbf{s}}) = \check{\mu}^r(\check{\mathbf{s}}) = 0$.

Therefore, when $\check{e}(\check{\mathbf{s}}) = 0$, the regulatory constraint must be slack ($\check{\mu}^r(\check{\mathbf{s}}) = 0$). Equivalently, if the regulatory constraint binds ($\check{\mu}^r(\check{\mathbf{s}}) > 0$), then the settlement constraint must also bind, forcing $\check{e}(\check{\mathbf{s}}) > 0$. This establishes that equity issuance and binding regulatory constraints are intrinsically linked in equilibrium.

The expression for morning prices comes from rearranging the FOCs (C.7) and (C.8):

$$\check{q}^j(\mathbf{s}) = \frac{x^j(\mathbf{s})}{\check{\mu}(\mathbf{s}) + \check{\mu}^e(\mathbf{s}) - (1 - \varrho\kappa^j)\check{\mu}^r(\mathbf{s})} = \frac{x^j(\mathbf{s})}{(1 + \psi\check{e}(\mathbf{s}))(1 - \mathcal{K}^j\check{\nu}(\mathbf{s}))}$$

where

$$\check{\nu} := \frac{\check{\mu}^r(\mathbf{s})}{1 + \psi\check{e}(\mathbf{s})}$$

From Claim 1, if $\check{e} = 0$, morning prices equal to the afternoon payoffs ($\check{q}^j = x^j$). Now consider the regulatory constraint with equality

$$\frac{(1 - \varrho\kappa^b)x_t^b b_{t-1}}{(1 + \psi\check{e}(\mathbf{s}))(1 - \mathcal{K}^b\check{\nu}(\mathbf{s}))} + \frac{(1 - \varrho\kappa^h)x_t^h h_{t-1}}{(1 + \psi\check{e}(\mathbf{s}))(1 - \mathcal{K}^h\check{\nu}(\mathbf{s}))} = (\check{d}_t + \kappa^e \check{e}_t)$$

and use the introduced notation to write

$$\frac{\Gamma^b(\mathbf{s})}{(1 - \mathcal{K}^b\check{\nu}(\mathbf{s}))} + \frac{\Gamma^h(\mathbf{s})}{(1 - \mathcal{K}^h\check{\nu}(\mathbf{s}))} = 1$$

Rearranging gives the quadratic

$$\begin{aligned} 0 &= \check{\nu}(\mathbf{s})^2 - \frac{\mathcal{K}^b(1 - \Gamma^h(\mathbf{s})) + \mathcal{K}^h(1 - \Gamma^b(\mathbf{s}))}{\mathcal{K}^b\mathcal{K}^h}\check{\nu}(\mathbf{s}) + \frac{1 - \Gamma^b(\mathbf{s}) - \Gamma^h(\mathbf{s})}{\mathcal{K}^b\mathcal{K}^h} \\ 0 &= \check{\nu}(\mathbf{s})^2 - \mathfrak{B}(\mathbf{s})\check{\nu}(\mathbf{s}) + \mathfrak{C}(\mathbf{s}) \end{aligned}$$

with the definitions:

$$\begin{aligned} \mathfrak{B}(\check{\mathbf{s}}) &:= \frac{1}{\mathcal{K}^b}(1 - \Gamma^b(\check{\mathbf{s}})) + \frac{1}{\mathcal{K}^h}(1 - \Gamma^h(\check{\mathbf{s}})), \\ \mathfrak{C}(\check{\mathbf{s}}) &:= \frac{1}{\mathcal{K}^b\mathcal{K}^h}(1 - \Gamma^b(\check{\mathbf{s}}) - \Gamma^h(\check{\mathbf{s}})), \end{aligned}$$

Claim 2: The discriminant is positive whenever $\mathcal{K}^j > 0$ and $\Gamma^j(\check{\mathbf{s}}) > 0$ for $j \in \{b, h\}$.

To ensure real solutions of $\check{\nu}(\check{\mathbf{s}})$ exist, we must verify that the discriminant $(\mathfrak{B}(\check{\mathbf{s}})/2)^2 - \mathfrak{C}(\check{\mathbf{s}}) \geq 0$. Expanding the squared term:

$$\begin{aligned} &(\mathcal{K}^h(1 - \Gamma^b(\check{\mathbf{s}})) + \mathcal{K}^b(1 - \Gamma^h(\check{\mathbf{s}})))^2 \\ &= (\mathcal{K}^h)^2(1 - \Gamma^b(\check{\mathbf{s}}))^2 + 2\mathcal{K}^b\mathcal{K}^h(1 - \Gamma^b(\check{\mathbf{s}}))(1 - \Gamma^h(\check{\mathbf{s}})) + (\mathcal{K}^b)^2(1 - \Gamma^h(\check{\mathbf{s}}))^2. \end{aligned}$$

Subtracting $4\mathcal{K}^b\mathcal{K}^h(1 - \Gamma^b(\check{s}) - \Gamma^h(\check{s}))$:

$$\begin{aligned} & (\mathcal{K}^h)^2(1 - \Gamma^b(\check{s}))^2 + 2\mathcal{K}^b\mathcal{K}^h(1 - \Gamma^b(\check{s}))(1 - \Gamma^h(\check{s})) + (\mathcal{K}^b)^2(1 - \Gamma^h(\check{s}))^2 \\ & \quad - 4\mathcal{K}^b\mathcal{K}^h(1 - \Gamma^b(\check{s})) - 4\mathcal{K}^b\mathcal{K}^h(1 - \Gamma^h(\check{s})) + 4\mathcal{K}^b\mathcal{K}^h \\ & = \left[\mathcal{K}^h(1 - \Gamma^b(\check{s})) - \mathcal{K}^b(1 - \Gamma^h(\check{s})) \right]^2 + 4\mathcal{K}^b\mathcal{K}^h\Gamma^b(\check{s})\Gamma^h(\check{s}). \end{aligned}$$

Therefore:

$$(\mathfrak{B}(\check{s})/2)^2 - \mathfrak{C}(\check{s}) = \left(\frac{\mathcal{K}^h(1 - \Gamma^b(\check{s})) - \mathcal{K}^b(1 - \Gamma^h(\check{s}))}{2\mathcal{K}^b\mathcal{K}^h} \right)^2 + \frac{\Gamma^b(\check{s})\Gamma^h(\check{s})}{\mathcal{K}^b\mathcal{K}^h} \geq 0,$$

with strict inequality whenever $\Gamma^b(\check{s}), \Gamma^h(\check{s}) > 0$. This ensures real solutions exist for $\check{\nu}(\check{s})$.

Claim 3: The regulatory constraint binds iff $\check{\epsilon} > 0$ and $\mathfrak{C} > 0$.

First, we show that $\mathfrak{C}(\check{s}) > 0$ implies $\mathfrak{B}(\check{s}) > 0$. Suppose $\mathfrak{C}(\check{s}) > 0$. By definition:

$$\mathfrak{C}(\check{s}) = \frac{1}{\mathcal{K}^b\mathcal{K}^h} (1 - \Gamma^b(\check{s}) - \Gamma^h(\check{s})) > 0 \implies \Gamma^b(\check{s}) + \Gamma^h(\check{s}) < 1.$$

Since $\Gamma^b(\check{s}), \Gamma^h(\check{s}) \geq 0$ by construction (they are ratios of positive quantities), we have:

$$\Gamma^b(\check{s}) < 1 \quad \text{and} \quad \Gamma^h(\check{s}) < 1 \implies 1 - \Gamma^b(\check{s}) > 0 \quad \text{and} \quad 1 - \Gamma^h(\check{s}) > 0.$$

Therefore:

$$\mathfrak{B}(\check{s}) = \frac{1}{\mathcal{K}^b} (1 - \Gamma^b(\check{s})) + \frac{1}{\mathcal{K}^h} (1 - \Gamma^h(\check{s})) > 0,$$

since both terms are positive given $\mathcal{K}^b, \mathcal{K}^h > 0$. Moreover, we can establish that $\mathfrak{B}(\check{s})^2 \geq 4\mathfrak{C}(\check{s})$ is always satisfied when the discriminant is non-negative (Claim 2), ensuring that the quadratic formula yields real solutions. When $\mathfrak{C}(\check{s}) > 0$ and $\mathfrak{B}(\check{s}) > 0$, both roots of the quadratic are real and positive, so the regulatory constraint must bind.

When $\mathfrak{C}(\check{s}) \leq 0$ (equivalently, $\Gamma^b(\check{s}) + \Gamma^h(\check{s}) \geq 1$), banks' asset holdings already exceed regulatory requirements, so the constraint cannot bind. In these cases, $\check{\nu}(\check{s}) = 0$, corresponding to a slack regulatory constraint.

Claim 4: The lower root is the economically relevant solution when $\mathfrak{C}(\check{s}) > 0$.

The quadratic equation for $\check{\nu}(\check{s})$ has two roots:

$$\check{\nu}^\pm(\check{s}) = \frac{\mathfrak{B}(\check{s})}{2} \pm \sqrt{\left(\frac{\mathfrak{B}(\check{s})}{2} \right)^2 - \mathfrak{C}(\check{s})}.$$

Both roots satisfy the regulatory constraint with equality, yet only the lower root represents a valid equilibrium. To establish this, we analyze the feasibility and stability properties of each root. For equilibrium morning prices to be positive and finite, we require:

$$1 - \mathcal{K}^j \check{\nu}(\check{\mathbf{s}}) > 0 \quad \text{for all } j \in \{b, h\} \quad \implies \quad \check{\nu}(\check{\mathbf{s}}) < \min \left\{ \frac{1}{\mathcal{K}^b}, \frac{1}{\mathcal{K}^h} \right\}. \quad (\text{C.11})$$

(i) *Upper root:* The upper root satisfies:

$$\check{\nu}^+(\check{\mathbf{s}}) = \frac{\mathfrak{B}(\check{\mathbf{s}})}{2} + \sqrt{\left(\frac{\mathfrak{B}(\check{\mathbf{s}})}{2}\right)^2 - \mathfrak{C}(\check{\mathbf{s}})} > \frac{\mathfrak{B}(\check{\mathbf{s}})}{2}.$$

By definition, $\mathfrak{B}(\check{\mathbf{s}}) = \frac{1}{\mathcal{K}^b}(1 - \Gamma^b(\check{\mathbf{s}})) + \frac{1}{\mathcal{K}^h}(1 - \Gamma^h(\check{\mathbf{s}}))$. Consider the case where $\Gamma^b(\check{\mathbf{s}}), \Gamma^h(\check{\mathbf{s}})$ are small (banks hold limited regulatory assets relative to their obligations, i.e. the regulatory constraint is tight). Then $\mathfrak{B}(\check{\mathbf{s}}) \approx \frac{1}{\mathcal{K}^b} + \frac{1}{\mathcal{K}^h}$, and:

$$\check{\nu}^+(\check{\mathbf{s}}) > \frac{1}{2} \left(\frac{1}{\mathcal{K}^b} + \frac{1}{\mathcal{K}^h} \right).$$

Without loss of generality, assume $\mathcal{K}^b \leq \mathcal{K}^h$. If the regulatory constraint binds at the upper root, we would have $\check{\nu}^+(\check{\mathbf{s}}) > \frac{1}{2\mathcal{K}^b}$. For $\Gamma^b(\check{\mathbf{s}})$ sufficiently small (which occurs precisely when the regulatory constraint is tight), the upper root can exceed $\frac{1}{\mathcal{K}^b}$, violating (C.11) and causing $\check{q}^b(\check{\mathbf{s}}) \leq 0$. This renders the upper root economically infeasible.

(iii) *The lower root as the unique equilibrium:* In contrast, the lower root satisfies:

$$0 < \check{\nu}^-(\check{\mathbf{s}}) = \frac{\mathfrak{B}(\check{\mathbf{s}})}{2} - \sqrt{\left(\frac{\mathfrak{B}(\check{\mathbf{s}})}{2}\right)^2 - \mathfrak{C}(\check{\mathbf{s}})} < \frac{\mathfrak{B}(\check{\mathbf{s}})}{2}.$$

We can verify that the lower root always satisfies the feasibility constraint (C.11). When the regulatory constraint binds with $\Gamma^b(\check{\mathbf{s}}) + \Gamma^h(\check{\mathbf{s}}) < 1$, the lower root represents the minimal shadow cost of satisfying the regulatory constraint. This is the natural equilibrium selection: competitive banks operate at the point where the marginal cost of meeting regulatory requirements equals its marginal benefit, which corresponds to the smallest multiplier consistent with the constraint binding. □

C.1.1 Afternoon problem

In the afternoon new banks choose portfolios (m', b', h', d') of storage asset, government bonds, corporate bonds, and deposits. Taking prices and the household's SDF as given, the

representative bank solves:

$$\begin{aligned} \max_{m', b', h', d'} \quad & \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) W(m', b', h', d'; \mathbf{s}') \right] + q^d(\mathbf{s})d' - m' - q^b(\mathbf{s})b' - q^h(\mathbf{s})h' \\ \text{s.t.} \quad & 0 \leq m', b', h', d' \end{aligned} \quad (\text{C.12})$$

where ξ is the household's stochastic discount factor. The first order conditions for the portfolio choice in the afternoon market are (dropping the short selling constraints for (b', h', d') which don't bind):

$$\begin{aligned} [m'] : \quad & 1 \geq \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) \left(1 + \psi \check{e}(\mathbf{s}') \right) \left(1 + \kappa^e \check{\nu}(\mathbf{s}') \right) \phi \right] = \quad \text{if } m' > 0 \\ [b'] : \quad & q^b(\mathbf{s}) = \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) \left(1 + \psi \check{e}(\mathbf{s}') \right) \left(1 + \kappa^e \check{\nu}(\mathbf{s}') \right) \check{q}^b(\mathbf{s}') \right] \end{aligned} \quad (\text{C.13})$$

$$[h'] : \quad q^h(\mathbf{s}) = \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) \left(1 + \psi \check{e}(\mathbf{s}') \right) \left(1 + \kappa^e \check{\nu}(\mathbf{s}') \right) \check{q}^h(\mathbf{s}') \right] \quad (\text{C.14})$$

$$[d'] : \quad q^d(\mathbf{s}) = \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) \left(1 + \psi \check{e}(\mathbf{s}') \right) \left(\frac{1 + \lambda(\mathbf{s}') \psi \check{e}(\mathbf{s}')}{1 + \psi \check{e}(\mathbf{s}')} + (1 - \lambda(\mathbf{s}')(1 - \kappa^e)) \check{\nu}(\mathbf{s}') \right) \right] \quad (\text{C.15})$$

where we used the Envelope conditions. We can see that equations (C.13), and (C.14), are the standard portfolio choice equations augmented with the wedges $(\check{e}, \check{\nu})(\cdot)$ reflecting how the interbank asset market frictions distort the bank's portfolio. Equation (C.15) equates the deposit price to the risk-weighted average marginal cost of servicing a unit of deposits in the morning and afternoon.

C.2 Household Problem

We now complete the model by recursively characterizing general equilibrium across the morning and afternoon markets. This allows us to study how the price dynamics in the morning market generate a government funding advantage in the afternoon market.

Household problem: At the start of the afternoon sub-period, suppose the family has unspent wealth a and input goods m . The family's budget constraint in the afternoon sub-period at time t is:

$$c + q^d(\mathbf{s})d' + q^e(\mathbf{s})e' + q^n(\mathbf{s})n' \leq a + q^m m - \tau(\mathbf{s})Y(\mathbf{s}) \quad (\text{C.16})$$

where c denotes goods consumed by the household in the afternoon sub-period, q^m is the afternoon price of the input good, m is the amount of input good produced by the household in the morning and stored between the morning and afternoon, and (d', e', n') denote the

family portfolio of bank deposits, bank and firm equity, respectively, i denotes consumption goods used for capital production, and $\tau(\mathbf{s})$ denotes the tax-to-output ratio in the afternoon sub-period.

In the following morning sub-period, the new exogenous aggregate states $\boldsymbol{\varepsilon}'$ and the households' idiosyncratic shock for their morning trading needs are realized with probability λ . Unconstrained households that can self-produce choose their own labor effort \check{l}_u and potentially contribute additional equity \check{e} that is repaid in the afternoon. So the financial wealth and input goods that unconstrained households bring into the following afternoon are:

$$\begin{aligned} a'_u &= (d - \check{q}^e \check{e}' + \check{e}') + x^e(\mathbf{s}')e + (x^n(\mathbf{s}') + q^n(\mathbf{s}'))n \\ m'_u &= \check{l}'_u \end{aligned} \quad (\text{C.17})$$

Constrained households that need to purchase inputs choose how much labor to sell to the market \check{l}_c and how much labor to purchase from the market $\check{\ell}$ subject to the deposit in advance constraint:

$$\check{w}(\mathbf{s}')\check{\ell}' \leq \nu d \quad (\text{C.18})$$

So the wealth and input goods that they bring into the afternoon are:

$$\begin{aligned} a'_c &= d - \check{w}(\mathbf{s}')\check{\ell}' + \check{w}(\mathbf{s}')\check{l}'_c + x^e(\mathbf{s}')e + (x^n(\mathbf{s}') + q^n(\mathbf{s}'))n \\ m'_c &= \check{\ell}' \end{aligned} \quad (\text{C.19})$$

Let $V(a, m, \mathbf{s})$ denote the value of the household with unspent wealth a and input goods m at the start of the afternoon. Then, taking as given the law of motion for the aggregate states, the value function $V(a, m, \mathbf{s})$ satisfies the Bellman equation (C.20) below:

$$\begin{aligned} V(a, m, \mathbf{s}) &= \max_{\left\{ \begin{array}{l} \check{l}^e, \check{l}^u, \check{\ell}, \check{e} \\ c, m, e, d, n \end{array} \right\}} \left\{ u(c) + \beta \mathbb{E}_{\mathbf{s}} \left[\lambda(\mathbf{s}')(-\zeta^e(\mathbf{s}')\check{l}_c + V(a'_c, m'_c, \mathbf{s}')) + \right. \right. \\ &\quad \left. \left. + (1 - \lambda(\mathbf{s}'))(-\zeta^u(\mathbf{s}')\check{l}_u + V(a'_u, m'_u, \mathbf{s}')) \right] \right\} \quad (\text{C.20}) \\ & \text{s.t.} \quad (\text{C.16}), (\text{C.17}), (\text{C.18}), (\text{C.19}). \end{aligned}$$

The necessary first order conditions and the envelope theorem imply:

$$\begin{aligned}
[\check{l}_u] : & \quad 0 = -\zeta^u(\mathbf{s}') + \partial_m V(a'_u, m'_u, \mathbf{s}') \\
[\check{l}_c] : & \quad 0 = -\zeta^c(\mathbf{s}') + \check{w}(\mathbf{s}') \partial_a V(a'_c, m'_c, \mathbf{s}') \\
[\check{\ell}] : & \quad 0 = -\check{w}(\mathbf{s}') \partial_a V(a'_c, m'_c, \mathbf{s}') - \check{w}(\mathbf{s}') \check{\mu}^d(\mathbf{s}') + \partial_m V(a'_c, m'_c, \mathbf{s}') \phi
\end{aligned}$$

Using the envelope theorem we have:

$$\begin{aligned}
\partial_m V(a, m, \mathbf{s}) &= q^m(\mathbf{s}) \partial_c u(c) \\
\partial_a V(a, m, \mathbf{s}) &= \partial_c u(c)
\end{aligned}$$

so we get:

$$\begin{aligned}
[\check{l}_u] : & \quad q^m(\mathbf{s}) \partial_c u(c) = \zeta^u(\mathbf{s}) \\
[\check{l}_c] : & \quad \check{w}(\mathbf{s}) \partial_c u(c) = \zeta^c(\mathbf{s}) \\
[\check{\ell}] : & \quad q^m(\mathbf{s}) \partial_c u(c) = \check{w}(\mathbf{s}) \left(\partial_c u(c) + \check{\mu}^d(\mathbf{s}) \right)
\end{aligned}$$

along with the condition that $\check{q}^e = 1$. Let $\zeta^u(\mathbf{s}) = \zeta^c(\mathbf{s}) + \check{w}(\mathbf{s}) \check{\mu}^d(\mathbf{s})$ implying $m_u = m_c$ in equilibrium. This leads to the first-order-conditions (FOCs) after imposing the Envelope condition:

$$\begin{aligned}
q^d(\mathbf{s}) &= \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) \left(1 + \lambda(\mathbf{s}') \check{L}(\mathbf{s}') \right) \right] \\
q^e(\mathbf{s}) &= \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) x^e(\mathbf{s}') \right] \\
q^n(\mathbf{s}) &= \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) x^n(\mathbf{s}') \right]
\end{aligned}$$

where the stochastic discount factor (SDF) and the ‘‘liquidity wedge’’ are defined by:

$$\begin{aligned}
\xi(\mathbf{s}'; \mathbf{s}) &:= \beta \frac{\partial_c u(c(\mathbf{s}'))}{\partial_c u(c(\mathbf{s}))}, \\
\check{L}(\mathbf{s}) &:= \frac{\nu \check{\mu}^d(\mathbf{s}')}{\partial_c u(c(\mathbf{s}))}.
\end{aligned}$$

The liquidity wedge, $\check{L}(\mathbf{s}')$, appears because demand deposits provide liquidity services to the households by allowing them to insure trading shocks in the morning sub-period. The presence of this asset-specific wedge implies that households are willing to hold demand deposits at a discount.

C.3 Firm Problem

Firm problem: Taking prices and the household's SDF as given, the representative firm solves:

$$\begin{aligned}
 V^f(k, h, \mathbf{s}) = \max_{m, k', h'} & \left\{ z(\mathbf{s})k^\alpha m^{1-\alpha} - q^m(\mathbf{s})m + q^h(\mathbf{s})h' - q^k(\mathbf{s})k' - \left(\omega + (1 - \omega)q^h(\mathbf{s}) \right) h / \pi(\mathbf{s}) + \right. \\
 & \left. + q^k(\mathbf{s})(1 - \delta)k + \mathbb{E}_{\mathbf{s}} \left[\xi(\mathbf{s}'; \mathbf{s}) V^f(k', h', \mathbf{s}') \right] \right\} \\
 \text{s.t.} \quad & q^h h' \leq \theta q^k k'
 \end{aligned} \tag{C.21}$$

The first order conditions are:

$$\begin{aligned}
 [m] : & \quad q^m(\mathbf{s}) = (1 - \alpha)z(\mathbf{s})k^\alpha m^{-\alpha} \\
 [k'] : & \quad q^k(\mathbf{s}) (1 - \theta \mu^h(\mathbf{s})) = \mathbb{E}_{\mathbf{s}} [\xi(\mathbf{s}'; \mathbf{s}) x^n(\mathbf{s}')] \\
 [h'] : & \quad q^h(\mathbf{s}) (1 - \mu^h(\mathbf{s})) = \mathbb{E}_{\mathbf{s}} [\xi(\mathbf{s}'; \mathbf{s}) x^h(\mathbf{s}')]
 \end{aligned}$$

where the afternoon payoff from firm equity is

$$x^n(\mathbf{s}) := \alpha y(\mathbf{s}) / k + (1 - \delta) q^k(\mathbf{s})$$

In equilibrium, the multiplier μ^h is linked to the banking frictions. The higher μ^h (higher liquidity premium on corporate debt), the more distortion in capital choice (capital price is above “fundamental value”).

C.4 Definition of Equilibrium

We can now set up a competitive equilibrium. Given a fiscal rule

$$\tau(\mathbf{s}) = \bar{\tau} + \eta \left(\frac{B}{Y(\mathbf{s})} - \bar{\mathbf{b}} \right)$$

and bond price function $q^b(\cdot)$, a budget-feasible government issuance rule $B'(\mathbf{s})$ satisfies:

$$\left(\omega + (1 - \omega) q^b(\mathbf{s}) \right) B \leq (\tau_t - g(\varepsilon_t)) Y(\mathbf{s}) + q^b(\mathbf{s}) B'(\mathbf{s}).$$

Definition 3 (Budget-feasible Competitive Equilibrium). For given regulation parameters $(\varrho, \kappa^b, \kappa^h, \kappa^e)$, and a budget-feasible government policy $(\tau, g, B')(\cdot)$, a competitive equilibrium is a collection of functions for prices $(q^d, q^e, q^n, q^h, q^k, q^b, \check{q}^h, \check{q}^b, \check{q}^e, \check{w})(\cdot)$, payoffs

$(x^e, x^n)(\cdot)$, household policies $(d'_h, e', n', c, \check{\ell}, \check{\ell}_c, \check{\ell}_u, i_h)(\cdot)$, bank policies $(\check{b}, \check{h}, \check{e}, \check{f}, d', h', b', x^e)(\cdot)$, and firm policies $(k', l_f, h'_f)(\cdot)$ such that

- Taking prices as given, households solve (C.20), banks solve (C.12), and firms solve (C.21),
- The morning interbank market $(\check{q}^h, \check{q}^b, \check{q}^e, \check{b}, \check{h}, \check{e})(\cdot)$ variables satisfy the morning market equilibrium (Definition 1) and the morning labor market clears: $\check{\ell}_c = \check{\ell}$.
- The afternoon input good, capital, and goods markets clear:

$$m_f(\mathbf{s}) = m(\mathbf{s}), \quad i_h(\mathbf{s}) = k'(\mathbf{s}) - (1 - \delta)k, \quad c(\mathbf{s}) + i(\mathbf{s}) + g(\mathbf{s}) = y(\mathbf{s}) - \Psi(\mathbf{s}) - \Phi(\mathbf{s}),$$

and afternoon asset markets clear:

$$d'_h(\mathbf{s}) = d'(\mathbf{s}), \quad e'(\mathbf{s}) = 1, \quad n'(\mathbf{s}) = 1, \quad b'(\mathbf{s}) = B'(\mathbf{s}), \quad h'_f(\mathbf{s}) = h'(\mathbf{s}).$$

D Derivations

D.1 Partial derivatives

We have the following implicit relationship for the key regulatory wedge $\check{\nu}_t$ derived in Proposition 4 in Appendix C:

$$F(\check{\nu}_t, \theta) = \check{\nu}_t^2 - \mathfrak{B}_t \check{\nu}_t + \mathfrak{C}_t = 0$$

where: $\mathfrak{B}_t := \frac{1}{\bar{\kappa}^b} (1 - \Gamma_t^b) + \frac{1}{\bar{\kappa}^h} (1 - \Gamma_t^h)$ and $\mathfrak{C}_t := \frac{1}{\bar{\kappa}^b \bar{\kappa}^h} (1 - \Gamma_t^b - \Gamma_t^h)$ and

$$\Gamma_t^b := \frac{(1 - \rho \kappa^b) x_t^b b_{t-1}}{(1 + \psi \check{e}_t)((1 - \lambda_t) d_{t-1} + \kappa^e \check{e}_t)} \quad \Gamma_t^h := \frac{(1 - \rho \kappa^h) x_t^h h_{t-1}}{(1 + \psi \check{e}_t)((1 - \lambda_t) d_{t-1} + \kappa^e \check{e}_t)}$$

and $\Gamma_t := \Gamma_t^b + \Gamma_t^h$. We always choose the lower root:

$$\check{\nu}_t^- := \frac{\mathfrak{B}_t - \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}}{2}$$

Using the IFT, we can characterize relationships $\check{\nu}_t(\theta)$ through the partial derivative

$$\frac{\partial \check{\nu}_t}{\partial \theta} = - \frac{\partial F / \partial \theta}{\partial F / \partial \check{\nu}_t} = - \frac{-\frac{\partial \mathfrak{B}_t}{\partial \theta} \check{\nu}_t + \frac{\partial \mathfrak{C}_t}{\partial \theta}}{2\check{\nu}_t^- - \mathfrak{B}_t} = \frac{\frac{\partial \mathfrak{C}_t}{\partial \theta} - \frac{\partial \mathfrak{B}_t}{\partial \theta} \check{\nu}_t}{\sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}}$$

This leads to a general formula for any variable θ

$$\left(\sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t} \right) \left(\frac{\partial \check{\nu}_t}{\partial \theta} \right) = \frac{\partial \mathfrak{C}_t}{\partial \theta} - \frac{\partial \mathfrak{B}_t}{\partial \theta} \check{\nu}_t$$

The partial derivatives of the regulatory wedge with respect to Γ_t^j is:

$$\frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^b} = - \frac{(1 - \mathcal{K}^h \check{\nu}_t) \Gamma_t^b}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} \quad \frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^h} = - \frac{(1 - \mathcal{K}^b \check{\nu}_t) \Gamma_t^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}}$$

The partial derivatives of the regulatory wedge with respect to \check{e}_t :

$$\frac{\partial \check{\nu}_t}{\partial \check{e}_t} = \left(\frac{\psi}{1 + \psi \check{e}_{t+1}} + \frac{\kappa^e}{\check{d}_{t-1} + \kappa^e \check{e}_t} \right) \left(\frac{(1 - \mathcal{K}^h \check{\nu}_t) \Gamma_t^b + (1 - \mathcal{K}^b \check{\nu}_t) \Gamma_t^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} \right) > 0$$

The partial derivatives of the regulatory wedge with respect to parameter κ_t^j :

$$\sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t} \frac{\partial \check{\nu}_t}{\partial \kappa^b} = \frac{\varrho}{\mathcal{K}^b \mathcal{K}^h} \left(\frac{(1 - \Gamma_t - \mathcal{K}^h \check{\nu}_t (1 - \Gamma_t^b))}{\mathcal{K}^b} + \frac{\Gamma_t^b (1 - \mathcal{K}^h \check{\nu}_t)}{(1 - \varrho \kappa^b)} \right)$$

The partial derivatives of the regulatory wedge with respect to the bank portfolios are:

$$\begin{aligned} \frac{\partial \check{\nu}_t}{\partial \log b} &= \frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^b} \frac{\partial \log \Gamma_t^b}{\partial \log b} = - \frac{(1 - \mathcal{K}^h \check{\nu}_t) \Gamma_t^b}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} < 0 \\ \frac{\partial \check{\nu}_t}{\partial \log h} &= \frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^h} \frac{\partial \log \Gamma_t^h}{\partial \log b} = - \frac{(1 - \mathcal{K}^b \check{\nu}_t) \Gamma_t^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} < 0 \\ \frac{\partial \check{\nu}_t}{\partial \log d} &= - \left(\frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^b} + \frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^h} \right) \left(1 + \frac{\psi \check{e}_{t+1} + \psi \phi m_t}{(1 + \psi \check{e}_{t+1})} + \frac{\kappa^e \phi m_t}{\hat{d}_{t+1}} \right) > 0 \\ \frac{\partial \check{\nu}_t}{\partial \log m} &= \left(\frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^b} + \frac{\partial \check{\nu}_t}{\partial \log \Gamma_t^h} \right) \left(\frac{\psi \phi m_t}{1 + \psi \check{e}_{t+1}} + \frac{\kappa^e \phi m_t}{\hat{d}_{t+1}} \right) < 0 \end{aligned}$$

with

$$\begin{aligned} \frac{\partial \log \Gamma_{t+1}^j}{\partial \log d} &= - \left(\frac{\psi \lambda_{t+1} d_t}{(1 + \psi \check{e}_{t+1})} + \frac{(1 - \lambda_{t+1} + \kappa^e \lambda_{t+1}) d_t}{\check{d}_{t+1} + \kappa^e \check{e}_{t+1}} \right) \\ &= - \left(1 + \frac{\psi \check{e}_{t+1} + \psi \phi m_t}{(1 + \psi \check{e}_{t+1})} + \frac{\kappa^e \phi m_t}{\check{d}_{t+1} + \kappa^e \check{e}_{t+1}} \right) \end{aligned}$$

D.1.1 Log-Price Elasticities

Define *log-price elasticities* as

$$\begin{aligned} \eta_{t,k}^j &:= \frac{\partial \log q_t^j}{\partial \log k} \quad \text{for } j, k \in \{b, h, d, m\} \\ &= \frac{1}{q_t^j} \frac{\partial q_t^j}{\partial \log k} = \frac{1}{q_t^j} \mathbb{E}_t \left[\frac{\partial q_t^j}{\partial \check{e}_{t+1}} \frac{\partial \check{e}_{t+1}}{\partial \log k} + \frac{\partial q_t^j}{\partial \check{\nu}_{t+1}} \frac{\partial \check{\nu}_{t+1}}{\partial \log k} \right] \end{aligned}$$

For the two bonds, we have $\frac{\partial \check{e}_{t+1}}{\partial b} = \frac{\partial \check{e}_{t+1}}{\partial h} = 0$ and

$$\frac{\partial q_t^j}{\partial \check{\nu}_{t+1}(\mathbf{s}')} = \pi(\mathbf{s}') \xi_{t,t+1}(\mathbf{s}') x_{t+1}^j(\mathbf{s}') \left(\frac{(1 - \varrho \kappa^j)}{(1 - \mathcal{K}^j \check{\nu}_{t+1}(\mathbf{s}'))^2} \right) \quad j \in \{b, h\}$$

so

$$q_t^j \eta_{t,k}^j = \mathbb{E}_t \left[\left(\frac{\xi_{t,t+1}}{\sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \right) \left(\frac{(1 - \varrho\kappa^j)x_{t+1}^j}{(1 - \mathcal{K}^j\check{\nu}_{t+1})^2} \right) \left(\frac{\partial \mathfrak{C}_{t+1}}{\partial \log k} - \frac{\partial \mathfrak{B}_{t+1}}{\partial \log k} \check{\nu}_{t+1} \right) \right]$$

for $j \in \{b, h\}$ and $k \in \{b, h, d, m\}$.

For the deposit and storage asset, we have

$$\begin{aligned} \frac{\partial q_t^d}{\partial \check{e}_{t+1}(\mathbf{s}')} &= \pi(\mathbf{s}') \xi_{t,t+1}(\mathbf{s}') \psi \left((1 - \lambda_{t+1}) \check{\nu}_{t+1} + \lambda_{t+1} (1 + \kappa^e \check{\nu}_{t+1}) \right) \\ \frac{\partial q_t^m}{\partial \check{e}_{t+1}(\mathbf{s}')} &= \pi(\mathbf{s}') \xi_{t,t+1}(\mathbf{s}') \psi \left(1 + \kappa^e \check{\nu}_{t+1} \right) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial q_t^d}{\partial \check{\nu}_{t+1}(\mathbf{s}')} &= \pi(\mathbf{s}') \xi_{t,t+1}(\mathbf{s}') \left(1 + \psi \check{e}_{t+1}(\mathbf{s}') \right) \left(1 - (1 - \kappa^e) \lambda_{t+1}(\mathbf{s}') \right) \\ \frac{\partial q_t^m}{\partial \check{\nu}_{t+1}(\mathbf{s}')} &= \pi(\mathbf{s}') \xi_{t,t+1}(\mathbf{s}') \left(1 + \psi \check{e}_{t+1}(\mathbf{s}') \right) \kappa^e \end{aligned}$$

so

$$\begin{aligned} q_t^d \eta_{t,k}^d &= \mathbb{E}_t \left[\xi_{t,t+1}(\mathbf{s}') \psi \left((1 - \lambda_{t+1}) \check{\nu}_{t+1} + \lambda_{t+1} (1 + \kappa^e \check{\nu}_{t+1}) \right) \frac{\partial \check{e}_{t+1}}{\partial \log k} \right] \\ &+ \mathbb{E}_t \left[\xi_{t,t+1} \left((1 + \psi \check{e}_{t+1}) (1 - (1 - \kappa^e) \lambda_{t+1}) \right) \left(\frac{\partial \mathfrak{C}_{t+1}}{\partial \log k} - \frac{\partial \mathfrak{B}_{t+1}}{\partial \log k} \check{\nu}_{t+1} \right) \right] \end{aligned}$$

and

$$\begin{aligned} q_t^m \eta_{t,k}^m &= \mathbb{E}_t \left[\xi_{t,t+1}(\mathbf{s}') \psi \left(1 + \kappa^e \check{\nu}_{t+1} \right) \frac{\partial \check{e}_{t+1}}{\partial \log k} \right] \phi \\ &+ \mathbb{E}_t \left[\left(\frac{\xi_{t,t+1}}{\sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \right) \left((1 + \psi \check{e}_{t+1}) \kappa^e \right) \left(\frac{\partial \mathfrak{C}_{t+1}}{\partial \log k} - \frac{\partial \mathfrak{B}_{t+1}}{\partial \log k} \check{\nu}_{t+1} \right) \right] \phi \end{aligned}$$

For completeness, the complete set of log-price elasticities is

$$\begin{aligned}
q_t^b \eta_{t,b}^b &= -\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^b) x_{t+1}^b}{(1 - \mathcal{K}^b \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^b \right] \\
q_t^b \eta_{t,h}^b &= -\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^b) x_{t+1}^b}{(1 - \mathcal{K}^b \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} (1 - \Xi_{t+1}^b) \right] \\
q_t^b \eta_{t,d}^b &= \mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^b) x_{t+1}^b}{(1 - \mathcal{K}^b \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^d \right] \\
q_t^b \eta_{t,m}^b &= -\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^b) x_{t+1}^b}{(1 - \mathcal{K}^b \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^m \right]
\end{aligned}$$

and

$$\begin{aligned}
q_t^h \eta_{t,b}^h &= -\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^h) x_{t+1}^h}{(1 - \mathcal{K}^h \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^b \right] \\
q_t^h \eta_{t,h}^h &= -\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^h) x_{t+1}^h}{(1 - \mathcal{K}^h \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} (1 - \Xi_{t+1}^b) \right] \\
q_t^h \eta_{t,d}^h &= \mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^h) x_{t+1}^h}{(1 - \mathcal{K}^h \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^d \right] \\
q_t^h \eta_{t,m}^h &= -\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \rho \kappa^h) x_{t+1}^h}{(1 - \mathcal{K}^h \check{\nu}_{t+1})^2} \right) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^m \right]
\end{aligned}$$

with

$$\begin{aligned}
\Xi_{t+1}^b &:= \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b}{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h} \\
\Xi_{t+1}^d &:= -\frac{\partial \log \Gamma_{t+1}}{\partial \log d} = \left(1 + \frac{\psi \check{\epsilon}_{t+1}}{(1 + \psi \check{\epsilon}_{t+1})} + \Xi_{t+1}^m \right) \\
\Xi_{t+1}^m &:= -\frac{\partial \log \Gamma_{t+1}}{\partial \log m} = \left(\frac{\psi \phi m_t}{(1 + \psi \check{\epsilon}_{t+1})} + \frac{\kappa^e \phi m_t}{\widehat{d}_{t+1}} \right)
\end{aligned}$$

and

$$\begin{aligned}
q^d \eta_{t,b}^d &= -\mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 - (1 - \kappa^e) \lambda_{t+1}) \frac{(1 - \mathcal{K}^h \check{\nu}_t) \Gamma_t^b}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} \right] \\
q^d \eta_{t,h}^d &= -\mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 - (1 - \kappa^e) \lambda_{t+1}) \frac{(1 - \mathcal{K}^b \check{\nu}_t) \Gamma_t^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} \right] \\
q^d \eta_{t,d}^d &= \mathbb{E}_t \left[\xi_{t,t+1} \left(\psi (\lambda_{t+1} + \check{\nu}_{t+1} (1 - (1 - \kappa^e) \lambda_{t+1})) (\lambda_{t+1} d_t) \right) \right] + \\
&\quad + \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 - (1 - \kappa^e) \lambda_{t+1}) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^d \right] \\
q^d \eta_{t,m}^d &= \mathbb{E}_t \left[\xi_{t,t+1} \left(\psi (\lambda_{t+1} + \check{\nu}_{t+1} (1 - (1 - \kappa^e) \lambda_{t+1})) (-\phi m_t) \right) \right] + \\
&\quad - \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 - (1 - \kappa^e) \lambda_{t+1}) \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^m \right]
\end{aligned}$$

and

$$\begin{aligned}
q_t^m \eta_{t,b}^m &= -\mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) \kappa^e \frac{(1 - \mathcal{K}^h \check{\nu}_t) \Gamma_t^b}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} \right] \phi \\
q_t^m \eta_{t,h}^m &= -\mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) \kappa^e \frac{(1 - \mathcal{K}^b \check{\nu}_t) \Gamma_t^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_t^2 - 4\mathfrak{C}_t}} \right] \phi \\
q_t^m \eta_{t,d}^m &= \mathbb{E}_t \left[\xi_{t,t+1} \left(\psi (\lambda_{t+1} d_t) (1 + \kappa^e \check{\nu}_{t+1}) \right) \right] \phi \\
&\quad + \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) \kappa^e \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^d \right] \phi \\
q_t^m \eta_{t,m}^m &= \mathbb{E}_t \left[\xi_{t,t+1} \left(\psi (-\phi m_t) (1 + \kappa^e \check{\nu}_{t+1}) \right) \right] \phi + \\
&\quad - \mathbb{E}_t \left[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) \kappa^e \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \Xi_{t+1}^m \right] \phi
\end{aligned}$$

and

$$\begin{aligned}\eta_{t,b}^e &= \frac{1}{q_t^e} \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^b] b_t \\ \eta_{t,h}^e &= \frac{1}{q_t^e} \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^h] h_t \\ \eta_{t,d}^e &= -\frac{1}{q_t^e} \mathbb{E}_t[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) \lambda_{t+1} + (1 - \lambda_{t+1})] d_t \\ \eta_{t,m}^e &= \frac{1}{q_t^e} \mathbb{E}_t[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) \phi] m_t\end{aligned}$$

D.2 Elasticity of substitution

The literature cares about the following projection:

- Nagel (2016) uses $u(c) + \alpha \log Q$ and $Q = [(1 - \lambda)(q^b b)^\rho + \lambda(q^h h)^\rho]^{1/\rho}$

$$\log \left(\frac{1 - \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^b / q_t^b]}{1 - \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^h / q_t^h]} \right) = \underbrace{\log \frac{(1 - \lambda)}{\lambda}}_{\text{shifter}} - \underbrace{1/\sigma}_{\text{inverse elasticity}} \log \left((q_t^b b_t) / (q_t^h h_t) \right)$$

where $\sigma = \frac{1}{1-\rho}$ is the intertemporal elasticity of substitution between the two bonds.

- Krishnamurthy and Vissing-Jorgensen (2012) use $\frac{c^{1-\rho}}{1-\rho} + \lambda \frac{(q^b b)^{1-\rho}}{1-\rho}$

$$\log \left(1 - \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^b / q_t^b] \right) = \underbrace{\log \lambda}_{\text{shifter}} - \underbrace{1/\sigma}_{\text{inverse elasticity}} \log \left((q_t^b b_t) / c_t \right)$$

where $\sigma = \frac{1}{\rho}$ is the elasticity of substitution between consumption and government bond.

Elasticity definitions: The macrofinance-literature-consistent elasticity of substitution is

$$\sigma := \frac{d \log \left((q_t^b b_t) / (q_t^h h_t) \right)}{d \log \left(\frac{1 - \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^h / q_t^h]}{1 - \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^b / q_t^b]} \right)} = - \frac{d \log \left((q_t^b b_t) / (q_t^h h_t) \right)}{d \log \left(\frac{1 - \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^b / q_t^b]}{1 - \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^h / q_t^h]} \right)}$$

Define $v := \frac{q_t^b b_t}{q_t^h h_t}$ and $cy^j := 1 - \frac{\mathbb{E}_t[\xi_{t,t+1} x_{t+1}^j]}{q_t^j}$, and note that

$$\sigma := - \frac{\partial \log v}{\partial \log (cy_t^b / cy_t^h)} = \underbrace{- \frac{\partial \log v}{\partial \log (q_t^b / q_t^h)}}_{=:\varsigma} \cdot \frac{\partial \log (q_t^b / q_t^h)}{\partial \log (cy_t^b / cy_t^h)}$$

where ς is the elasticity of substitution w.r.t. relative prices. We will compute both σ and ς .

Implicit Function Theorem: When the relationship between $(q_t^b b_t)/(q_t^h h_t)$ and (cy_t^h/cy_t^b) is not explicit (like in BIU models), we need to choose which space we are projecting the funding advantage to. We consider two cases: decompose the funding advantage (orthogonally)

1. for a given q_t^e -path while keeping $(d, m) = (\bar{d}, 0)$
2. for a given (q_t^e, q_t^m) -path while keeping $d = \bar{d}$
3. for a given (q_t^e, q_t^m, q_t^d) -path

where we define the equity price $q_t^e = \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^e]$ and the (shadow) price of storage as $q_t^d = \mathbb{E}_t[\xi_{t,t+1} (1 + \psi \check{e}_{t+1}) (1 + \kappa^e \check{\nu}_{t+1}) \phi]$. That is, we will keep either (q_t^e, m_t, d_t) or (q_t^e, q_t^m, d_t) or (q_t^e, q_t^m, q_t^d) fixed and project the funding advantage onto “relative scarcity” point-by-point to get $1/\sigma$ (for any given point). The elasticity of substitution σ measures how $(q^b b_t)/(q^h h_t)$ changes in response to a change in relative convenience (cy_t^h/cy_t^b) , while keeping “certain things” constant. This is meant to isolate the “substitution effect” from the “funding cost effect” (how change in government funding advantage affects bank’s funding costs).¹⁹ We use the Implicit Function Theorem to uncover this relationship embedded in the bank’s Euler equations.

Strategy: We start with two observations about the two components of σ . First, the value ratio can be decomposed:

$$\log v = \log (q_t^b/q_t^h) + \log (b_t/h_t)$$

Differentiating with respect to $\log(q_t^b/q_t^h)$:

$$\frac{\partial \log v}{\partial \log(q_t^b/q_t^h)} = \frac{\partial \log(q_t^b/q_t^h)}{\partial \log(q_t^b/q_t^h)} + \frac{\partial \log(b_t/h_t)}{\partial \log(q_t^b/q_t^h)} = 1 + \frac{\partial \log(b_t/h_t)}{\partial \log(q_t^b/q_t^h)}$$

Therefore:

$$\varsigma = -\frac{\partial \log(b_t/h_t)}{\partial \log(q_t^b/q_t^h)} - 1 =: \bar{\varsigma} - 1$$

so for the elasticity w.r.t. relative prices, we need to compute $\bar{\varsigma}$. Second, for the elasticity w.r.t. funding advantage, we will also need to characterize $\frac{\partial \log(q_t^b/q_t^h)}{\partial \log(cy_t^h/cy_t^b)}$ along the iso-curves.

¹⁹Alternatively, we could keep the “purchasing power” $m_t + q_t^b b_t + q_t^h h_t - q_t^d d_t$ constant (in the spirit of Hicks vs Slutsky).

Chain rule leads to

$$\begin{aligned}
\frac{d \log(cy_t^b/cy_t^h)}{d \log(q_t^b/q_t^h)} &= \frac{-d \log(q_t^b/q_t^h) + d \log((q_t^b - \bar{q}_t^b)/(q_t^h - \bar{q}_t^h))}{d \log(q_t^b/q_t^h)} \\
&= -1 + \frac{\frac{\partial \log(q_t^b - \bar{q}_t^b)}{\partial \log q_t^b} d \log q_t^b - \frac{\partial \log(q_t^h - \bar{q}_t^h)}{\partial \log q_t^h} d \log q_t^h}{d \log q_t^b - d \log q_t^h} \\
&= -1 + \frac{\frac{1}{cy_t^b} d \log q_t^b - \frac{1}{cy_t^h} d \log q_t^h}{d \log q_t^b - d \log q_t^h}
\end{aligned} \tag{D.1}$$

The requirement that we need to stay on the iso-curves will determine $(d \log q^b, d \log q^h)$ as a function of the quantity differentials $(\partial \log m, \partial \log b, \partial \log h, \partial \log d)$. To this end, define the (inverse) price elasticities

$$\eta_{t,k}^j := \frac{\partial \log q_t^j}{\partial \log k} \quad \text{for } j, k \in \{b, h, d, m\}$$

for which we derive explicit formulas in Appendix D.1. The nature of our approximation is that the four quantity differentials must satisfy three constraints (I)-(III) so that we only have one degree of freedom to move which is v .

D.2.1 General Formula

The general formula of σ is built from the total derivative

$$\begin{aligned}
\partial \log q^j &= \mathbb{E}_t \left[\frac{\partial \log q^j}{\partial \check{v}_{t+1}} \left(\sum_{k \in \{b, h, d, m\}} \frac{\partial \check{v}_{t+1}}{\partial \log k} \partial \log k \right) \right] \\
&= -\mathbb{E}_t \left[\xi_{t,t+1} \left(\frac{(1 - \kappa^j \varrho)}{(1 - \mathcal{K}^j \check{v}_{t+1})^2} \frac{x_{t+1}^j}{q_t^j} \right) \frac{((1 - \mathcal{K}^h \check{v}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{v}_{t+1}) \Gamma_{t+1}^h) \partial M_{t+1}}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4 \mathfrak{C}_{t+1}}} \right]
\end{aligned}$$

where the second equality plugs in the partial derivatives from Appendix D.1 and

$$\partial M_{t+1} := \Xi_{t+1}^b \partial \log b + (1 - \Xi_{t+1}^b) \partial \log h + \Xi_{t+1}^m \partial \log m - \Xi_{t+1}^d \partial \log d$$

with

$$\begin{aligned}\Xi_{t+1}^b &:= \frac{(1 - \mathcal{K}^h \check{\nu}_{t+1})(1 - \varrho \kappa^b) x_{t+1}^b b_t}{(1 - \mathcal{K}^h \check{\nu}_{t+1})(1 - \varrho \kappa^b) x_{t+1}^b b_t + (1 - \mathcal{K}^b \check{\nu}_{t+1})(1 - \varrho \kappa^h) x_{t+1}^h h_t} \\ \Xi_{t+1}^m &:= \frac{\psi \phi m_t}{(1 + \psi \check{e}_{t+1})} + \frac{\kappa^e \phi m_t}{\hat{d}_{t+1}} \\ \Xi_{t+1}^d &:= 1 + \frac{\psi \check{e}_{t+1}}{(1 + \psi \check{e}_{t+1})} + \Xi_{t+1}^m\end{aligned}$$

The restrictions that we need to move along the iso-curves imposes constraints on the quantity partials in ∂M_{t+1} . Effectively, our three constraints allow us to express how quantities $(\partial \log h, \partial \log m, \partial \log d)$ move along the iso-curves as a function of $\partial \log b$:

$$\begin{pmatrix} \partial \log h \\ \partial \log m \\ \partial \log d \end{pmatrix} = - \begin{pmatrix} \beta_b^h \\ \beta_b^m \\ \beta_b^d \end{pmatrix} \partial \log b$$

We will discuss how the constraints determine $(\beta_b^h, \beta_b^m, \beta_b^d)$ as a function of the log-price elasticities in the following subsections. Plugging in, we get

$$\partial M_{t+1} = \underbrace{(\Xi_{t+1}^b - (1 - \Xi_{t+1}^b) \beta_b^h - \Xi_{t+1}^m \beta_b^m + \Xi_{t+1}^d \beta_b^d)}_{=: \mathcal{M}_{t+1}} \partial \log b$$

Then we use

$$\partial \log(b/h) = \partial \log b - \partial \log h = (1 + \beta_b^h) \partial \log b \quad \Rightarrow \quad \frac{\partial \log b}{\partial \log(b/h)} = \frac{1}{1 + \beta_b^h}$$

to get $\frac{\partial \log q_t^j}{\partial \log(b/h)}$ and also implying

$$\frac{\partial \log h}{\partial \log(b/h)} = -\frac{\beta_b^h}{1 + \beta_b^h}, \quad \frac{\partial \log m}{\partial \log(b/h)} = -\frac{\beta_b^m}{1 + \beta_b^h}, \quad \frac{\partial \log d}{\partial \log(b/h)} = -\frac{\beta_b^d}{1 + \beta_b^h}.$$

The elasticity ς is then

$$\begin{aligned}
\frac{d \log(q_t^b/q_t^h)}{d \log(b_t/h_t)} &= \left[\frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b} \right] \frac{\partial \log b}{\partial \log(b/h)} = \\
&= -\mathbb{E}_t \left[\xi_{t,t+1} \frac{\left((1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h \right) \mathcal{M}_{t+1}}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \right. \\
&\quad \left. \times \left(\frac{(1 - \kappa^b \varrho)}{(1 - \mathcal{K}^b \check{\nu}_{t+1})^2} \frac{x_{t+1}^b}{q_t^b} - \frac{(1 - \kappa^h \varrho)}{(1 - \mathcal{K}^h \check{\nu}_{t+1})^2} \frac{x_{t+1}^h}{q_t^h} \right) \right] \frac{1}{1 + \beta_b^h} \\
\Rightarrow \quad \varsigma_t &= -\frac{d \log(b_t/h_t)}{d \log(q_t^b/q_t^h)} - 1 = -\frac{\left[\frac{\partial \log b}{\partial \log(b/h)} \right]^{-1} + \left[\frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b} \right]}{\left[\frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b} \right]}
\end{aligned}$$

With symmetric regulation ($\kappa^b = \kappa^h$), $\varsigma_t = \infty$ as expected.

To get the elasticity with respect to the funding advantage, use formula (D.1) implies

$$\frac{\partial \log(q_t^b/q_t^h)}{\partial \log(cy_t^b/cy_t^h)} = \frac{\frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b}}{\frac{1 - cy_t^b}{cy_t^b} \frac{\partial \log q_t^b}{\partial \log b} - \frac{1 - cy_t^h}{cy_t^h} \frac{\partial \log q_t^h}{\partial \log b}}$$

Putting things together

$$\sigma = -\frac{1 + \beta_b^h + \frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b}}{\frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b}} \times \frac{\frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b}}{\frac{1 - cy_t^b}{cy_t^b} \frac{\partial \log q_t^b}{\partial \log b} - \frac{1 - cy_t^h}{cy_t^h} \frac{\partial \log q_t^h}{\partial \log b}} = -\frac{1 + \beta_b^h + \frac{\partial \log q_t^b - \partial \log q_t^h}{\partial \log b}}{\frac{1 - cy_t^b}{cy_t^b} \frac{\partial \log q_t^b}{\partial \log b} - \frac{1 - cy_t^h}{cy_t^h} \frac{\partial \log q_t^h}{\partial \log b}}$$

where the numerator is

$$\begin{aligned}
1 + \beta_b^h + \mathbb{E}_t \left[\xi_{t,t+1} \frac{\left((1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h \right) \mathcal{M}_{t+1}}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \right. \\
\left. \times \left(\frac{(1 - \kappa^b \varrho)}{(1 - \mathcal{K}^b \check{\nu}_{t+1})^2} \frac{x_{t+1}^b}{q_t^b} - \frac{(1 - \kappa^h \varrho)}{(1 - \mathcal{K}^h \check{\nu}_{t+1})^2} \frac{x_{t+1}^h}{q_t^h} \right) \right]
\end{aligned}$$

and denominator is

$$\begin{aligned}
\mathbb{E}_t \left[\xi_{t,t+1} \frac{\left((1 - \mathcal{K}^h \check{\nu}_{t+1}) \Gamma_{t+1}^b + (1 - \mathcal{K}^b \check{\nu}_{t+1}) \Gamma_{t+1}^h \right) \mathcal{M}_{t+1}}{\mathcal{K}^b \mathcal{K}^h \sqrt{\mathfrak{B}_{t+1}^2 - 4\mathfrak{C}_{t+1}}} \right. \\
\left. \times \left(\frac{1 - cy_t^b}{cy_t^b} \frac{(1 - \kappa^b \varrho)}{(1 - \mathcal{K}^b \check{\nu}_{t+1})^2} \frac{x_{t+1}^b}{q_t^b} - \frac{1 - cy_t^h}{cy_t^h} \frac{(1 - \kappa^h \varrho)}{(1 - \mathcal{K}^h \check{\nu}_{t+1})^2} \frac{x_{t+1}^h}{q_t^h} \right) \right]
\end{aligned}$$

D.2.2 Given (q_t^e, q_t^m, q_t^d) -path

The restrictions are

(I.) Keeping $q_t^d = 0$ fixed ($\partial \log q_t^d = 0$):

$$\partial \log q_t^d = \eta_{t,b}^d \partial \log b_t + \eta_{t,h}^d \partial \log h_t + \eta_{t,m}^d \partial \log m_t + \eta_{t,d}^d \partial \log d_t = 0$$

(II.) Keeping $q_t^m = 0$ fixed ($\partial \log q_t^m = 0$):

$$\partial \log q_t^m = \eta_{t,b}^m \partial \log b_t + \eta_{t,h}^m \partial \log h_t + \eta_{t,m}^m \partial \log m_t + \eta_{t,d}^m \partial \log d_t = 0$$

(III.) Keeping q_t^e fixed ($\partial \log q_t^e = 0$): Total differentiate

$$\partial \log q_t^e = \eta_{t,b}^e \partial \log b_t + \eta_{t,h}^e \partial \log h_t + \eta_{t,m}^e \partial \log m_t + \eta_{t,d}^e \partial \log d_t = 0$$

Using (I.)-(III.), we can express (assuming matrix H is invertible):

$$\begin{pmatrix} \partial \log h \\ \partial \log m \\ \partial \log d \end{pmatrix} = - \begin{bmatrix} \eta_{t,h}^e & \eta_{t,m}^e & \eta_{t,d}^e \\ \eta_{t,h}^m & \eta_{t,m}^m & \eta_{t,d}^m \\ \eta_{t,h}^d & \eta_{t,m}^d & \eta_{t,d}^d \end{bmatrix}^{-1} \begin{bmatrix} \eta_{t,b}^e \\ \eta_{t,b}^m \\ \eta_{t,b}^d \end{bmatrix} d \log b$$

assuming that the matrix is invertible.

D.2.3 Given q_t^e -path while keeping $(d, m) = (\bar{d}, 0)$

The restrictions are

(I.) Keeping $d = \bar{d}$ ($\partial \log d = 0$):

(II.) Keeping $m = 0$ ($\partial \log m = 0$):

(III.) Keeping q_t^e fixed ($\partial \log q_t^e = 0$): Total differentiate

$$\begin{aligned} \partial \log q_t^e &= \eta_{t,b}^e \partial \log b_t + \eta_{t,h}^e \partial \log h_t + \eta_{t,m}^e \partial \log m_t + \eta_{t,d}^e \partial \log d_t = 0 \\ &= \left(\frac{1}{q_t^e} \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^b] b_t \right) \partial \log b_t + \left(\frac{1}{q_t^e} \mathbb{E}_t[\xi_{t,t+1} x_{t+1}^h] h_t \right) \partial \log h_t = 0 \end{aligned}$$

where we used that (d_t, m_t) are fixed.

Use (I)-(III) to express

$$\partial \log h = -\frac{\eta_{t,b}^e}{\eta_{t,h}^e} \partial \log b$$

Plugging this into the total differential of $\log(b_t/h_t)$:

$$\begin{aligned} \partial \log(b_t/h_t) &= \partial \log b_t - \partial \log h_t \\ \partial \log(b_t/h_t) &= \left(1 + \frac{\eta_{t,b}^e}{\eta_{t,h}^e}\right) \partial \log b \end{aligned}$$

D.2.4 Given (q_t^e, q_t^m) -path while keeping $d = \bar{d}$

We have two regions. If $1 > q^m$, the storage technology is not used and $m_t = 0$ with $\partial \log m = 0$. In this region the elasticity formula from the previous section applies. We will fix the storage shadow price at $q^m = 1$. In this case, the restrictions are

(I.) Keeping $d = \bar{d}$ ($\partial \log d = 0$):

(II.) Keeping $q_t^m = 0$ fixed ($\partial \log q_t^m = 0$):

$$\partial \log q_t^m = \eta_{t,b}^m \partial \log b_t + \eta_{t,h}^m \partial \log h_t + \eta_{t,m}^m \partial \log m_t = 0$$

where we used that d_t is fixed.

(III.) Keeping q_t^e fixed ($\partial \log q_t^e = 0$): Total differentiate

$$\partial \log q_t^e = \eta_{t,b}^e \partial \log b_t + \eta_{t,h}^e \partial \log h_t + \eta_{t,m}^e \partial \log m_t = 0$$

where we used that d_t is fixed.

In matrix form

$$\begin{pmatrix} \partial \log h \\ \partial \log m \end{pmatrix} = - \begin{bmatrix} \eta_{t,h}^e & \eta_{t,m}^e \\ \eta_{t,h}^m & \eta_{t,m}^m \end{bmatrix}^{-1} \begin{pmatrix} \eta_{t,b}^e \\ \eta_{t,b}^m \end{pmatrix} d \log b$$

assuming that the matrix is invertible.